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A Multigroup Diffusion Nodal Scheme : Hybrid of AFEN and PEN Methods

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ABSTRACT

The good features of the analytic function expansion nodal (AFEN) method are utilized to develop a practical scheme for the multigroup diffusion problems, in combination with the polynomial expansion nodal (PEN) method. The thermal group fluxes exhibiting strong gradients are solved by the AFEN method[1-6], while the fast group fluxes that are smoother than the thermal group fluxes are solved by the PEN method[7-9]. The scheme is applied to a MOX-fuel loaded core with good results.

I. INTRODUCTION

The Analytic Function Expansion Nodal (AFEN) method was developed to overcome the limitations caused by the transverse integration in existing nodal methods, and successfully applied to the rectangular and hexagonal geometries in two-group problems.[1-6]

The AFEN method is based on decoupling the following multigroup diffusion equations

$$-\nabla \cdot \mathbf{D}^n \nabla \hat{\phi}^n(x,y) + \Sigma_r^n \hat{\phi}^n(x,y) = \{\frac{1}{k_{eff}} \chi^n \nu \Sigma_f^n + \Sigma_s^n\} \hat{\phi}^n(x,y) \qquad (1)$$

into the "mode-group" partial differential equations

$$-\nabla^2 \hat{\xi}_\mu^n(x,y) + \lambda_\mu^n \hat{\xi}_\mu^n(x,y) = 0, \quad \mu = 1,2,\cdots,G \qquad (2)$$

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by defining $\vec{\xi}^n(x,y) = (\mathbf{R}^n)^{-1}\vec{\phi}^n(x,y)$. Here, λ_μ^n's are the eigenvalues of matrix $(\mathbf{D}^n)^{-1}[\Sigma_r^n - \Sigma_s^n - (1/k_{eff})\chi^n\nu\Sigma_f^n]$ and $\mathbf{R}^n$ is the matrix with columns of the corresponding eigenvectors. As long as λ_μ^n's are real, derivation, formulation, and implementation of the AFEN method described in Ref. 3 can be retained for the case of the multigroup application. But, λ_μ^n's are not always real in actual multigroup problems. Some of them may be complex. This requires the AFEN method to treat complex variables in deriving the nodal coupling equations.

II. HYBRID OF AFEN AND PEN METHODS

To circumvent this problem, we decompose the multigroup diffusion equations (1) into a fast group block and a thermal group block :

$$\begin{aligned} -\mathbf{D}_{FF}^n\nabla^2\hat{\phi}_F^n(x,y) + \Sigma_{rFF}^n\hat{\phi}_F^n(x,y) =& \{\frac{1}{k_{eff}}\chi_F^n\nu\Sigma_{fF}^n + \Sigma_{sFF}^n\}\hat{\phi}_F^n(x,y) \\ &+\{\frac{1}{k_{eff}}\chi_F^n\nu\Sigma_{fT}^n + \Sigma_{sFT}^n\}\hat{\phi}_T^n(x,y) \end{aligned} \tag{3}$$

$$\begin{aligned} -\mathbf{D}_{TT}^n\nabla^2\hat{\phi}_T^n(x,y) + \Sigma_{rTT}^n\hat{\phi}_T^n(x,y) =& \{\frac{1}{k_{eff}}\chi_T^n\nu\Sigma_{fT}^n + \Sigma_{sTT}^n\}\hat{\phi}_T^n(x,y) \\ &+\{\frac{1}{k_{eff}}\chi_T^n\nu\Sigma_{fF}^n + \Sigma_{sTF}^n\}\hat{\phi}_F^n(x,y) \end{aligned} \tag{4}$$

where the subscripts F and T denote the matrix and vector decomposition into the fast and thermal group blocks.

Generally, the fast flux distributions are smooth so that they can be accurately modeled by a quartic polynomial. Thus, Eq. (3) is solved by the Polynomial Expansion Nodal (PEN) method[7,8,9], which is one of variations of the AFEN method. The PEN method retains all the features of the AFEN method (e.g. no transverse integration) except expanding the intranodal flux distribution into a nonseparable polynomial instead of the analytic function :

$$\hat{\phi}_F^n(x,y) = \mathbf{A}_F^{00} + \sum_{i=1}^{4} x^i\mathbf{A}_F^{i0} + \sum_{j=1}^{4} y^j\mathbf{A}_F^{0j} + \sum_{i=1}^{2}\sum_{j=1}^{2} x^iy^j\mathbf{A}_F^{ij} \tag{5}$$

In contrast to the fast fluxes, the thermal fluxes exhibit strong gradients near the material discontinuity. Therefore, Eq. (4) is solved by the AFEN method. The

thermal group decoupling matrix $(\mathbf{D}^n_{TT})^{-1}[\mathbf{\Sigma}^n_{rTT} - \mathbf{\Sigma}^n_{sTT} - (1/k_{eff})\chi^n_T \nu\mathbf{\Sigma}^n_{fT}]$ does not involve any complex eigenvalue. This is due to the fact that the fission spectrum χ^n_T is nearly null, and that without strong upscattering, the matrix $\Sigma^n_{rTT} - \Sigma^n_{sTT}$ becomes nearly lower triangular whose diagonal elements are real and positive.

The solution of the thermal group diffusion equation is divided into two parts : the particular solution and the homogeneous solution. The particular solution can be easily obtained by several arithmetic operations. The homogeneous solution is obtained by the AFEN method, i.e., by expanding the solution of the "mode–group" decoupled partial differential equation into eight analytic basis functions and an additional constant :

$$\begin{aligned}
&\hat{\xi}^n_T(x,y) = \\
&\mathbf{A}^{00}_T + \sinh\kappa^n_{TT}x\mathbf{A}^{10}_T + \cosh\kappa^n_{TT}x\mathbf{A}^{20}_T + \sinh\kappa^n_{TT}y\mathbf{A}^{01}_T + \cosh\kappa^n_{TT}y\mathbf{A}^{02}_T \\
&+ \sinh\frac{\sqrt{2}}{2}\kappa^n_{TT}x\sinh\frac{\sqrt{2}}{2}\kappa^n_{TT}y\mathbf{A}^{11}_T + \sinh\frac{\sqrt{2}}{2}\kappa^n_{TT}x\cosh\frac{\sqrt{2}}{2}\kappa^n_{TT}y\mathbf{A}^{12}_T \\
&+ \cosh\frac{\sqrt{2}}{2}\kappa^n_{TT}x\sinh\frac{\sqrt{2}}{2}\kappa^n_{TT}y\mathbf{A}^{21}_T + \cosh\frac{\sqrt{2}}{2}\kappa^n_{TT}x\cosh\frac{\sqrt{2}}{2}\kappa^n_{TT}y\mathbf{A}^{22}_T
\end{aligned} \tag{6}$$

where κ^n_{TT} is the square root of λ^n_{TT}.

To express the twenty-two coefficients of both fast and thermal flux expansions into the nodal unknowns, we need twenty-two constraints. Eighteen constraints are directly given by the nodal unknowns such as the node-average fluxes, the interface fluxes, and the corner-point fluxes for both fast and thermal groups. The four additional constraints are obtained by forcing the fast flux expansion (5) to obey the fast group equation in a weighted residual sense.

Once all the coefficients are expressed into the nodal unknowns, we build the nodal coupling equations to determine these nodal unknowns in the usual way : equations for the node-average fluxes, the interface fluxes, and the corner-point fluxes.

III. NUMERICAL RESULTS AND DISCUSSION

The accuracy and applicability of the AFEN/PEN hybrid method was tested on a four-group reactor core configuration which consists of three types of homogeneous fuel assemblies including MOX fuel assemblies(Fig. 1). The four-group cross sections for each fuel type are listed in Table I.

Two results of the hybrid method are compared with those of the PEN method and the VENTURE reference in Fig. 2 : one obtained by solving only one lowest energy group by AFEN and the other by solving two lowest energy groups by AFEN. The accuracy of the PEN method in predicting the flux distributions is very poor compared to the AFEN/PEN hybrid method. However, the AFEN/PEN hybrid method predicts the assembly powers and the multiplication factor very accurately. The higher accuracy achieved by the hybrid method is mainly attributed to the nonseparable analytic function expansion of the thermal flux distribution. Although solving only one energy group by the AFEN method can achieve high accuracy, increasing the number of energy groups solved by the AFEN method shows consistent improvement in the accuracy.

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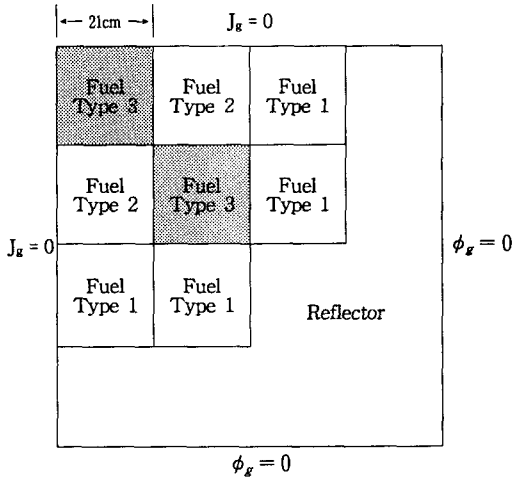

Fig. 1 Core configuration of benchmark problem

Table I Four group assembly cross section

	Group	D	Σ_a	$\nu\Sigma_f$	Σ_{1g}	Σ_{2g}	Σ_{3g}	Σ_{4g}
Fuel Type 1	1	2.21516E+00	4.25361E-03	9.64211E-03	8.20662E-02	6.28948E-02	2.69636E-04	0.0
	2	1.15164E+00	2.65013E-03	1.10902E-03	0.0	2.25524E-01	5.79330E-02	5.55973E-06
	3	8.00135E-01	2.64962E-02	1.56544E-02	0.0	0.0	3.49817E-01	4.83455E-02
	4	3.86141E-01	1.05000E-01	1.65259E-01	0.0	0.0	2.28771E-03	8.82576E-01
Fuel Type 2	1	2.21354E+00	4.15702E-03	9.37974E-03	8.19931E-02	6.29584E-02	2.69473E-04	0.0
	2	1.15110E+00	2.44987E-03	6.66079E-04	0.0	2.25543E-01	5.79858E-02	5.56534E-06
	3	8.05237E-01	2.28838E-02	9.62372E-03	0.0	0.0	3.47684E-01	4.99766E-02
	4	3.84402E-01	7.60164E-02	1.06461E-01	0.0	0.0	1.76582E-03	8.87859E-01
Fuel Type 3	1	2.29508E+00	4.22459E-03	9.89779E-03	7.95154E-02	6.08872E-02	2.65975E-04	0.0
	2	1.19024E+00	2.45412E-03	1.12288E-03	0.0	2.17329E-01	5.80414E-02	5.57578E-06
	3	8.01028E-01	3.62349E-02	1.68329E-02	0.0	0.0	3.51570E-01	4.01181E-02
	4	3.78890E-01	2.07011E-01	3.16155E-01	0.0	0.0	3.48093E-03	9.31181E-01
Reflector	1	2.6107E+0	2.5759E-4	0.0	5.4236E-2	7.2748E-2	4.3768E-4	0.0
	2	1.4261E+0	5.3796E-5	0.0	0.0	1.3949E-1	9.4192E-2	9.1953E-6
	3	8.7340E-1	1.5568E-3	0.0	0.0	0.0	3.0269E-1	7.7400E-2
	4	2.9194E-1	1.5267E-2	0.0	0.0	0.0	1.7345E-3	1.1248E+0
Fission spectrum (χ_g)					0.7595035	0.2403235	1.77519E-4	0.0

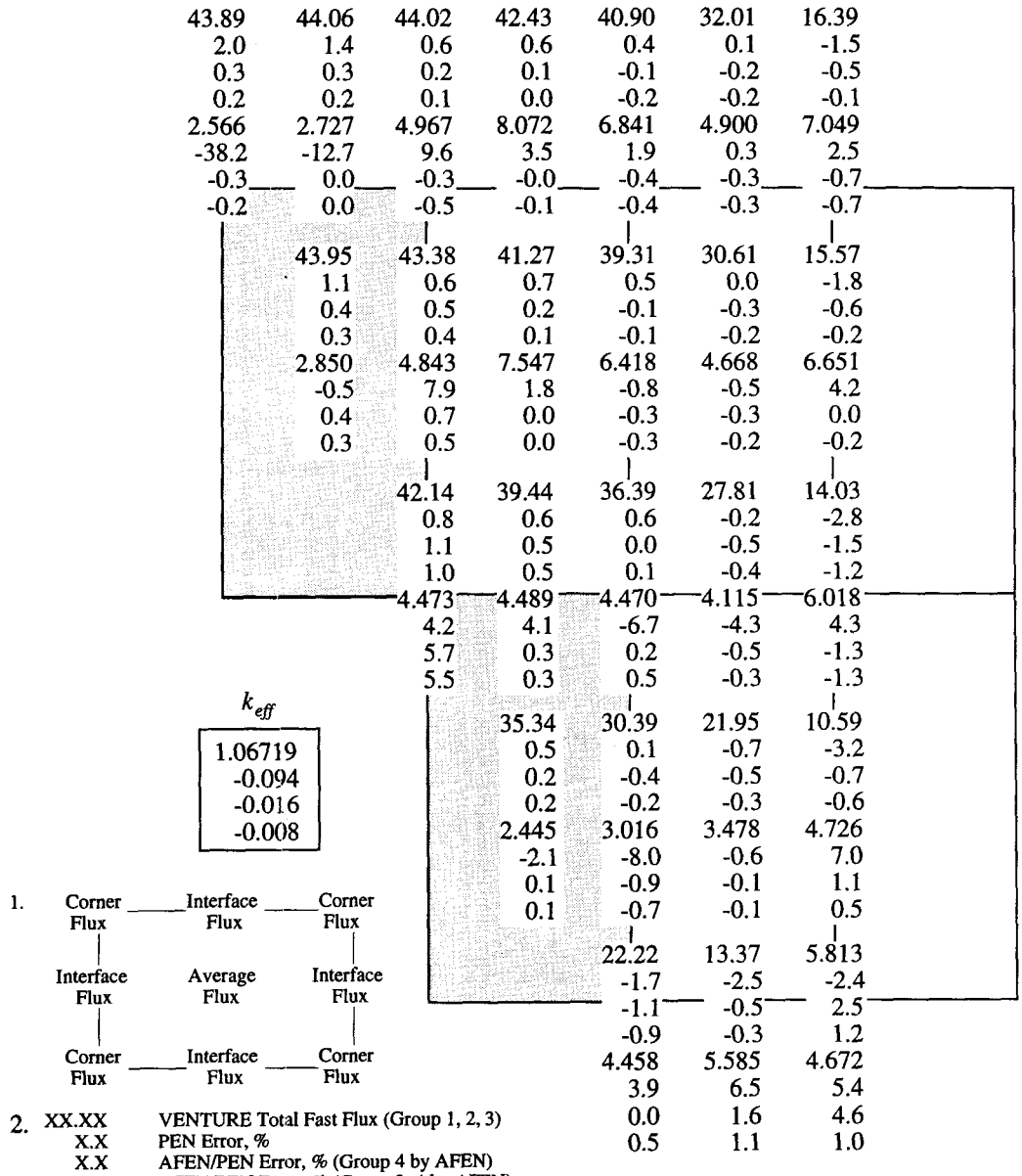

Fig. 2 Result of the PEN method and the AFEN/PEN hybrid method