# INVESTIGATION ON CRITERION OF SHOCK-INDUCED SEPARATION IN SUPERSONIC FLOWS

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**abstract**

A great number of experimental data indicating shock-induced separation(SIS) in internal or external supersonic flows were reviewed to make clear the mechanism of SIS and to present the criterion of turbulent boundary layer separation. The interesting conclusions were obtained for the considerably wide range of flow geometries that the incipient separation is almost independent of the flow geometries, and that it is relatively unaffected by changes in gas specific heat, and boundary layer Reynolds number. Furthermore, the pressure rise necessary to separate boundary layer in external flows was found to be applicable to SIS in overexpanded propulsion nozzles. This is due to the fact that the SIS phenomenon caused by the interaction between shock waves and turbulent boundary layers is processed through a supersonic deceleration. This is, the SIS in almost all of interacting flow fields is governed by the concept of free interaction, and criterion of SIS is only a function of upstream Mach number.

## INTRODUCTION

As the shock wave produces a retardation of the boundary layer flow, frequently the flow is separated from the shock foot, with a consequent serious fall of the performance of system. Under such circumstances, large scale instabilities can appear which are capable of including buffeting on supercritical wings or buzz in supersonic inlets, and as the results, can cause the fatigue fracture of flow components. Even if its consequences are not so extreme, that is, even when shock strength is not sufficient to induce the flow separation, the shock wave/boundary layer interaction often provokes an amplification of viscous effects to such an extent that the real flow may differ markedly from that corresponding to inviscid flow analysis, frequently used to define the shape of the body.

Boundary layer separation phenomenon is of considerable practical importance, since the advent of separation limits the performance of flow systems. It is thus of great interest to be properly predictable the onset of shock-induced separation(SIS). That is, if the onset of SIS is given in terms with known properties, i.e., boundary layer integral parameters, Reynolds number, Mach number, etc..., we can appropriately make use of it to apply to a variety of practical problems.

Historically, the study on the SIS in internal flows have carried out in the steam turbine nozzle experiment[1], about seventy years ago. The Second World War served as a major momentum to make the investigations on the rocket nozzle flows[2-5] more active. For maximum thrust at a given pressure ratio, the pressure in the nozzle exit plane should be theoretically equal to the ambient pressure(so-called correct expansion). For a high altitude vehicle operating at a constant reservoir pressure, there is only one altitude at which the flow can be correctly expanded for a fixed geometry. At other altitudes, the flow may not be fully expanded at exit (so-called underexpanded flow), or it may be expanded below the ambient pressure (overexpanded flow). At that time, most of the researchers realized that owing to the SIS occurring at off-design conditions some correction factor was required, and that further tests were necessary to determine what this correction should be.

Many later investigations have made by means of static pressure measurements or optical observations to detect the separation point and the conclusions reached were that the absolute pressure at which SIS took place was almost independent of the overall pressure ratio, kind of working fluid and divergence angle of nozzle employed.

In external flows past a body, the effect of SIS on the flow field has considerably advanced from the earlier time and enabled it to be predicted for most circumstances, or to be avoided by a suitable design when other considerations permit[6-7]. However the onset of SIS in such flow fields could not be successfully predicted in spite of many successive investigations.

Recently, the sustained interests[8-9] in predicting the onset of SIS were due to the phenomenological as well as aerodynamical and industrial importance of the mechanism causing the SIS. If the mechanism of SIS is fully understood and the governing parameters are appropriately found, the onset of SIS in the external flows may be used to correlate with the data obtained in the supersonic nozzle tests.

The purpose of this paper is to collect the existing experimental data and then to present criterion of the SIS. By doing so, it is then possible to understand the mechanism of SIS more clearly and to provide theoretical base about this phenomenon.

## FREE INTERACTION

A large body of experimental data in supersonic flows exists on boundary layer separation due both to obstacles and shock waves. The cases of forward-facing steps, curved surfaces, and compression corners or ramps have been comprehensively studied for laminar and turbulent flows.

Various theoretical attempts have been made in order to describe the mechanism and onset of the separation. We can find that in every experimental model investigated, reattachment of the boundary layer follows quite soon after the separation, and the early part of the pressure rise associated with the separation has features closely similar between all models, while the later part associated with the reattachment has not.

Mager[10-11] argued that the boundary layer does not know what combination of circumstances creates the pressure rise leading to the separation; It only knows what pressure rise is required at given conditions of Mach number and Reynolds number to cause the boundary layer to separate. Chapman et al.[12] used the termed free interaction with respect to regions of flow which are free from direct influences of downstream geometry, and in other words, independent of the mode of inducing the separation.

Many experimental results showed that, at least as far as the separation point (in the sense of two-dimensional flow) an incident shock wave and all the forms of the obstacle mentioned above are indeed free interaction. But, once separated, the effects of geometry which the boundary layer must negotiate during the process of reattachment put the later phase outside the category of the free interaction, and similarity of the various models ceases to be found.

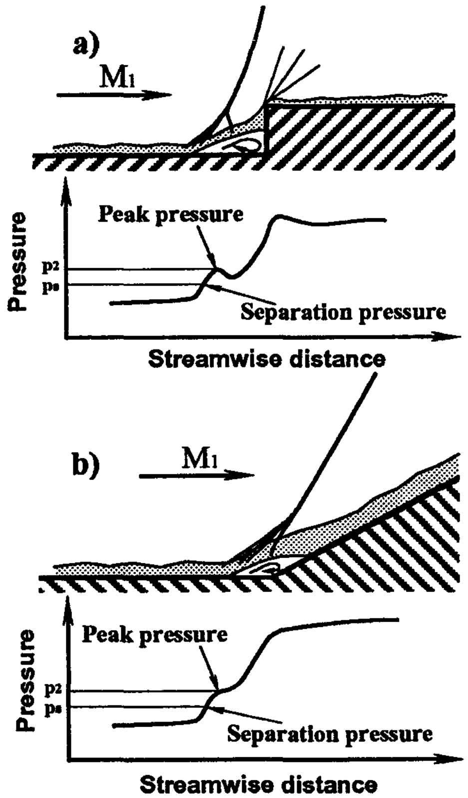


Fig.1 Schematics of interaction and wall pressure distributions for forward-facing step(a) and compression coner(b)

Fig.1 indicates schematics of wall pressure distributions of the interaction fields between shock waves and boundary layers over a forward-facing step and a compression ramp. In general, all the models investigated have been exhibited the pressure rise characteristics which correspond closely to one another under similar conditions of Mach and Reynolds numbers as far as the station 2 for laminar boundary layers, and up to $s$ for turbulent boundary layer flows. However a difficulty appears in further pressure rise from $s$ to 2 in the turbulent flow, where the detailed pressure distributions from various models are no longer identical. Therefore this region must be regarded to be outside the category of the free interaction.

The free interaction of shock wave with boundary layer came from the many earlier investigations for external flows and a great deal of experimental data are now well established. Typical experimental substantiations are the interaction on a supercritical wing surface.

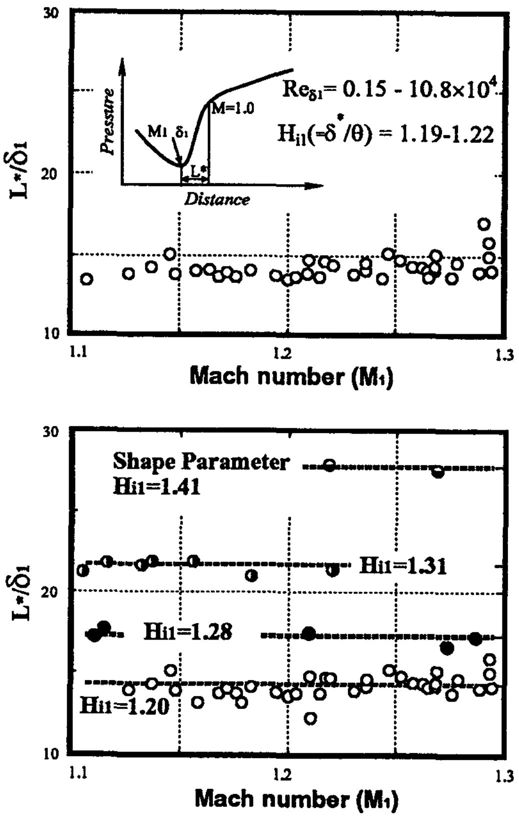


Fig.2 Influence of Reynolds number and shape parameter on interaction length

Fig.2 shows an excellent grouping of the experimental data[6-7,13], with a moderate scatter due to the difficulty of accurately defining the supersonic interaction length L* from the wall pressure distributions. For the range of Mach number of 1.09 to 1.30, the L* is normalized by the boundary layer displacement thickness $\delta_1^*$ at the start of the interaction, and the variation in Reynolds number based upon the boundary layer displacement thickness is between $0.15\times10^4$ and $1.08\times10^5$ and the value of the incompressible shape factor $H_{i1}$ for the whole set of data is close to about 1.20. We can find that the influence of the Reynolds number on the physical extent $L^*$ and on the thickness $\delta_1^*$ disappears when these two variables are normalized one by the other. For a given value of the shape parameter, the displacement thickness of the incoming boundary layer is a proper scale for the supersonic interaction length $L^*$, and the ratio $L^*/\delta_1^*$ is not very sensitive to the effect of the upstream Mach number $M_1$.

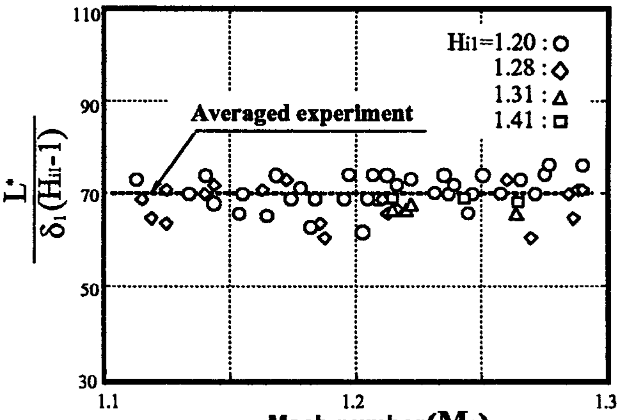


Fig.3 Correlation of supersonic interaction lengths

Although the scatter observed when the $M_1$ comes close to 1.30 corresponds to a situation in the verge of separation, the $L^*$ is practically independent of $M_1$. For different situations of the state of the incoming boundary layer, the experimental data shown are regularly spaced as a function of the shape parameter (see the lower figure). For example, we observe that the normalized interaction length increases twofold when the $H_{i1}$ increases from 1.20 to 1.41. This can be easily understood by considering that when the $H_{i1}$ is high, the boundary layer is less full and its subsonic part is thicker, and consequently the distance for the propagation of upstream influence is longer. Also the influence of shape parameter on the interaction length can be disappeared by using an appropriate variable (see Fig.3). As illustrated in these figures, the normalized interaction length at low supersonic speeds can be provided in terms of the boundary layer displacement thickness and shape parameter at the start of the interaction.

Similar conclusions for the other supersonic flows, i.e., the forward-facing steps, the compression ramps, or the incident shock waves can be also deduced by the wall pressure distributions. Fig.4 shows the pressure coefficient of the interaction fields, where $C_{ps}$ and $C_{p2}$ refer to the pressure coefficient at separation point and peak pressure point, respectively, as illustrated in Fig.1.

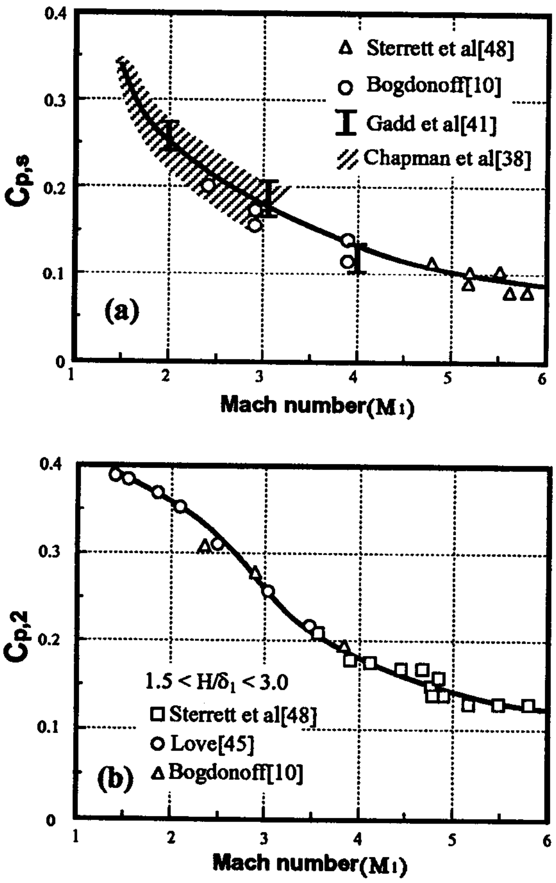


Fig.4 Variation of separation pressure with Mach number for various flow fields(a) and peak pressure for forward- facing steps(b)

The data shown refer to the flow conditions with a variety of interaction geo-metries but with a zero pressure gradient turbulent boundary layer. For the range of Mach number employed, the data of both the pressure coefficients seem to be collapsed onto a single curve. The pressure rises up to separation point or even up to peak pressure point are inde-pendent of an external action to cause themselves and are only a function of the upstream Mach number. This is just the concept of free interaction that has described earlier.

Many theoretical or semi-empirical equations concerned with the free interaction show good agreement with the experimental data obtained in the external flows. According to an analytical methods by Mager[10-11], it is postulated that the SIS occurs whenever the turbulent boundary layer finds itself at a certain pressure ratio. Under such a consideration, the pressure gradient in the ISWTBL was assumed to be sufficiently large compared with the friction force on the surface, and the pressure rise to the separation was thus a function of Mach number before and after the oblique shock wave. This was based upon Schuh's statemcnt[14] that separation in absence of surface friction can be predicted by use of the well-known Gruschwitz relations for the turbulent boundary layer This results in the following relations between Mach number $M_1$ just upstream of the interaction and Ms at separation point.

$$M_s^2 = K M_1^2 \qquad (1)$$

In order to determine the pressure ratio across the separation, the oblique shock approximation can be used as

$$\frac{p_s}{p_1} \cong 1 + \frac{\gamma (1-K)\frac{M_1^2}{2}}{1+(\gamma-1)\frac{M_1^2}{2}} \qquad (2)$$

,where K is constant to be assumed. Gadd et al.[15-16] considered a 1/7 power velocity profile on a supersonic plane flow and assumed some characteristic fraction J of the freestream velocity to be stagnated due to the pressure gradient due to shock wave. The friction force was also negligible by comparison with one due to the pressure gradient. In order to evaluate the pressure rise necessary to bring the flow on a streamline to rest isentropically, he drew Eq.(3) from one-dimensional gas dynamic equations,

$$\frac{p_s}{p_1} = \left\{ \frac{1 + \frac{(\gamma-1)}{2} M_1^2}{1 + \frac{(\gamma-1)}{2}\left(1-J^2\right) M_1^2} \right\}^{\frac{\gamma}{\gamma-1}} \qquad (3)$$

where $p_s/p_1$ corresponds to the pressure rise at the separation point. He employed 0.6 or 0.54 as the value of J to give quite good fit to experiment. A revise of this expression was proposed by Arens et al.[17-18] for the case in which the stagnation was conducted by an isentropic compressiona followed by the normal shock wave when the characteristic streamline was initially supersonic. This leads to the relation,

$$\frac{p_s}{p_1} = \frac{\left[\frac{(\gamma+1)}{2} M_1^2 J^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left\{1+\frac{(\gamma-1)}{2} M_1^2\left[1-J^2\right]\right\} \left\{\frac{M_1^2}{2}\left[(\gamma+1)J^2-\frac{(\gamma-1)^2}{(\gamma+1)}\right]-\frac{\gamma-1}{\gamma+1}\right\}^{\left(\frac{1}{\gamma-1}\right)}} \qquad (4)$$

where good agreement with experiments for the forward-facing steps, the compression ramps, and the incident shock waves was claimed to use the value J=0.56. Many other empirical relations for the pressure rise at the separation point were also available. The reader will be referred to References[19-22].

Fig.1.5 shows the pressure rise at the separation point against the Mach number, together with the averaged experimental data for the forward-facing steps, the compression ramps, and the incident shock waves. The value of specific heat ratio was taken by 1.4 throughout. We can find that when J=0.60, there is little difference between Eq.(3) and (4), and quite good agreement with the experimental data is given by Eq.(1) with K=0.67 and (4) with J=0.56. These data indicate that the flow process up to the separation point is dependent only on the Mach number and the pressure rise ($p_1/p_s$) decreases with increase in Mach number. This can be concerned with the separation process being in supersonic. Therefore the free interaction can be reasonably interpreted as being independent of the external actions or downstream geometries causing the pressure rise in interaction field between shock wave and boundary layer.

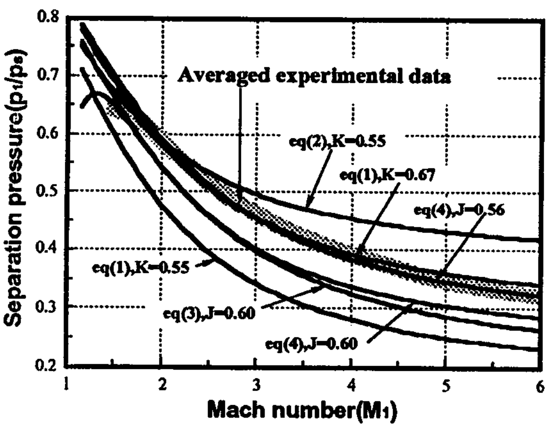

Fig.5 Comparison of separation pressures calculated by free interaction and measurements

## SIS IN EXTERNAL FLOWS

As shown in Fig.5, the boundary layer separation in external supersonic flows is well predictable by the concept of free interaction. When the external forces or downstream geometries causing the separation are, however, quite weak or small compared with the thickness scale of inner layer or viscous sublayer of the boundary under consideration, the separation is unlikely to occur and the concept of free interaction breaks down. This situation appears when the Mach number is so low that the pressure disturbance is not sufficient to significantly destabilize the boundary layer. Therefore it is necessary to determine the limit of the strength of disturbance applicable to the concept of free interaction. Unfortunately the authors cannot find any theoretical study or even any systematic experiment concerning with this problem. This section devotes to such a subject in external flows at low supersonic speeds.

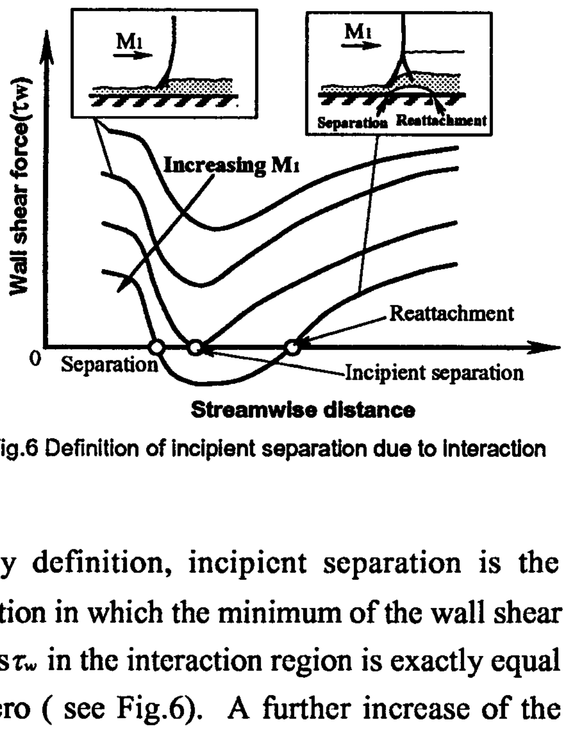

Fig.6 Definition of incipient separation due to interaction

By definition, incipient separation is the situation in which the minimum of the wall shear stress $\tau_w$ in the interaction region is exactly equal to zero ( see Fig.6). A further increase of the shock strength beyond that point leads to a change in the sign of the shear stress, the region where $\tau_w$ is negative being called separated. At laboratory the incipient separation is usually detected from visualizations such as surface oil-tracers, schlieren photographs, wall pressure distributions, and boundary layer velocity profiles, since the direct measurement of the wall shear stress is difficult and inaccurate under the circumstance where exists the strong pressure gradient.

According to the above definition of separation, the onset of SIS, in what follows, is the situation in which the streamwise distributions of surface friction coefficient has a minimum exactly equal to zero. We define the local pressures on the airfoil surface to characterize the separation onset. Fig.7 indicates the surface pressure distribution in the vicinity and downstream of the separation, together with a schematic description of interaction flow field.

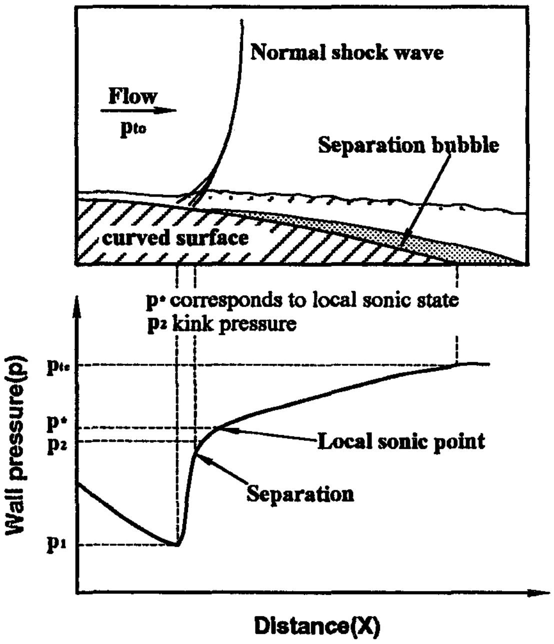


Fig.7 Definitions of wall pressures due to interaction

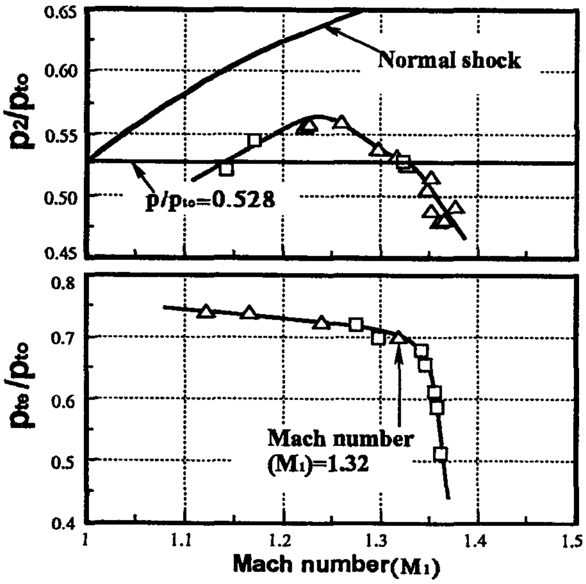


Fig.8 Criterion of shock-induced separation

The onset of SIS can be diagnosed by considering the variation of the normalized pressure $p_2/p_{to}$ and $p_{te}/p_{to}$ as functions of Mach number $M_1$, i.e., the shock strength.

Fig.8 shows that as the Mach number $M_1$ increases, the $p_2/p_{to}$ increases but it then decreases with further increase in the Mach number. Also, when the Mach number is increased, $p_{te}$ at the trailing edge decreases and abruptly drops off at Mach number of 1.32. Variation of $p_2/p_{to}$ at this point is related to the divergence of pressure at the trailing edge $p_{te}/p_{to}$, which is typical of the development of a large separation bubble. That is, the data shown in Fig.1.8 show that the dramatic change in the flow structure takes place when the kink pressure $p_2$ is equal to sonic value. From these considerations, the value of Mach number of 1.32 is interpreted as the bare minimum shock strength necessary to cause the separation.

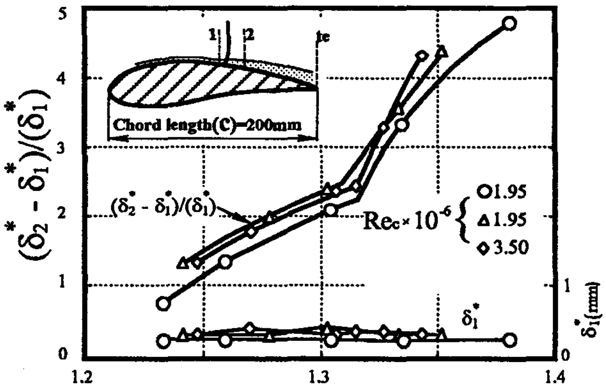


Fig.9 Criterion of shock-induced separation

The same conclusion can be seen in the separation measurements on an airfoil carried out by Stanewski[23-24]. Fig.9 shows the variation of the boundary layer displacement thickness at certain appropriate stations versus the Mach number $M_1$. In experiment, three stations on the airfoil upper surface (i.e., upstream of shock wave: 1, immediately downstream of it: 2, and at the trailing edge: te) were allocated as measurement position of the boundary layer displacement thickness. The data included refer to various transition techniques to trip the boundary layer and Reynolds number based upon the chord length of 1.95 to $3.5\times10^6$. In fact, the boundary layer tripping mainly affects the thickness of the boundary layer at the position of shock wave. The large difference between the corresponding variations in its thickness can constitute the scaling effect in viscous phenomena on the ISWTBL. However, the point of interest here is the kink in the curves which is observed for the Mach number $M_1$ of 1.32. The kink of variation in its thickness is related to the fast thickening due to the onset of SIS. Mach number at the kink remains constant, irrespective with variation of the state of incoming boundary layer. This is consistent with the conclusion drawn in Fig.8 and also compatible with the concept of free interaction.

Another way to determine the onset of SIS was made by Alber et al.[25]. They employed surface oil-tracers to detect separation. The conclusions were derived from the wall pressure distributions. In the vicinity of the shock and before the separation, these distributions exhibited a kink change in the pressure gradient.

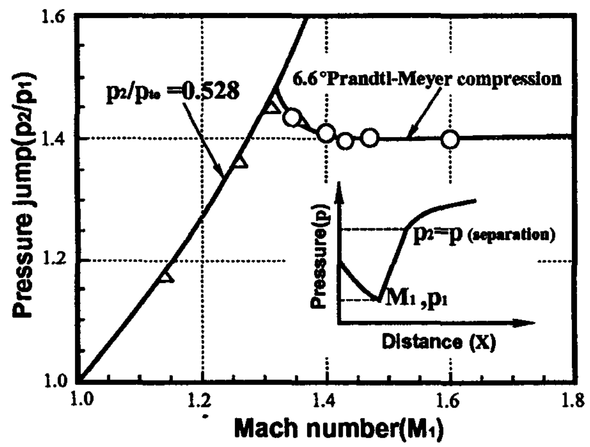

Fig.10 Alber's creterion of shock-induced separation

Fig.10 shows the situations in which the pressure rises are plotted as a function of Mach number at the start of interaction. The experimental data points fall exactly onto the curve corresponding to a sonic state. Furthermore, it can be shown that the turning angle of the inviscid outer flow during the supersonic part of the interaction process cannot exceed 6.6 deg. Once separation has occurred, the turning angle at the separation point remains nearly constant and equal to 6.6 deg. The pressure rise calculated from this turning angle by assuming a simple compression wave agrees well with experimental data. That is, the incipient separation occurs when the Mach number $M_1$ just upstream of the shock wave is 1.32 and the local outer Mach number at the separation point is sonic. Although this criterion of the SIS did not include a possible influence of Reynolds number, it is quite consistent with the two conclusions earlier.

Of course, the separation criterion can be influenced by the incoming boundary layer shape parameter which represents the fullness of the boundary layer, and the Reynolds number. The effect of Reynolds number is commonly included in the variation of the shape parameter. But these two parameters are not necessarily linked, since it is possible to change the shape parameter at a fixed Reynolds number by an external agent, i.e., pressure gradient, suction and blowing.

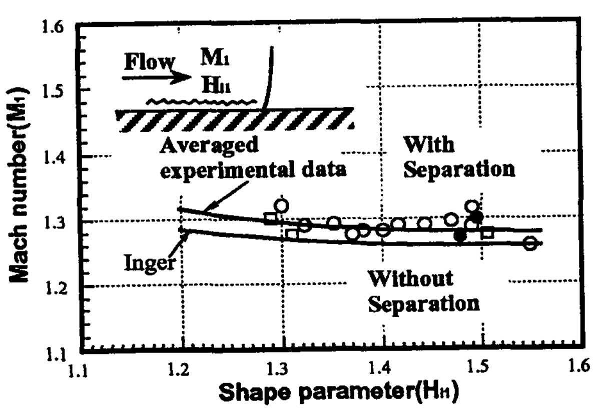

Fig.11 Effect of shape parameter on shock-induced separation

Fig.11 shows another data[26] representing the incipient separation. We find that the data all nearly collapse on a single curve defining the boundary between the interactions without and with separation. The averaged data decrease in the limit Mach number of separation when the shape parameter is increased. Such a trend can be interpreted since as the $H_{i1}$ is lower, the boundary layer velocity profile becomes fuller so that the resistance of the boundary layer to separation is greater. However the effect is not as significant as could be expected. When the boundary layer velocity profile becomes less filled, i.e., when the $H_{i1}$ increases, the interaction length L* will be increased rapidly. Such an increase in the interaction length reduces the intensity of streamwise pressure gradient, the supersonic part of the compression being spread over a longer distance. This reduction of the extent of the adverse pressure gradient allows the boundary layer separation to be delayed or avoided, in spite of the less filled velocity profile at the beginning of the interaction. This tendency of the shape parameter to the limit Mach number is well in agreement with other experimental[27-28] and theoretical data[29-32]. That is, the SIS is a purely supersonic process. Under this condition, the down-stream pressure level has no real importance on the separation phenomenon itself. But, of many applications to various practical problems of designs or performance evaluations, there are some unanswered problems to the criterion of SIS. For examples, the effects of surface curvature, condition of wall temperature, and the reacting flow on the criterion of SIS still remain unsolved. The reader will be referred to Reference[33].

# SIS IN INTERNAL FLOWS

Many earlier investigations[34-37] on the SIS in supersonic nozzles or diffusers have showed that it could be reasonably assumed that the boundary layer separates whenever the normal shock recompression exceeds the exit pressure. Generally the similar effects would occur if the divergence angle of nozzle were made too large giving rise to sizable adverse pressure gradients in the subsonic flow downstream of the shock wave.

As mentioned in the previous section, many experimental works on the ISWTBL in external flows have shown that the pressure rise to the separation point is apparently independent of the geometry of the interaction, and only dependent on the upstream Mach number. Such a deduction may be also applicable to overexpanded nozzle flows, where it is generally expected that a strength of oblique shock wave is required to separate the boundary layer. The present section devotes to a discussion of the onset of SIS in such overexpanded supersonic nozzles.

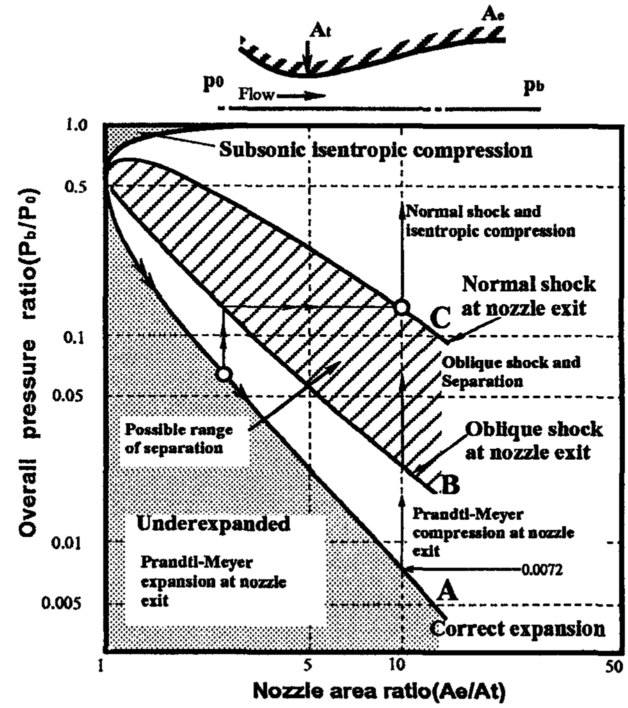


Fig.12 Flow pattern of supersonic nozzle operating at various conditions and possible range of separation

In what follows, the various ways in which the flow through a nozzle reacts in order to attain the back pressure conditions are presented in Fig.12. Curve A on the plot shows the relationship between the overall pressure ratio $p_b/p_0$ and the nozzle area ratio $A_e/A_t$ for the correct isentropic expansion conditions. If for a given nozzle the back or ambient pressure is reduced below the correct expansion value (curve A), the change is met by the formation of Prandtl-Meyer expansion waves at the nozzle exit. Conversely if the back pressure is increased, compression waves at the nozzle exit raise the flow pressure to meet the new boundary condition.

However there is a limit to the compression shock strength which can be maintained at the nozzle exit, and when the back pressure increases sufficiently to cause this limit to be exceeded, the shock wave moves back into the nozzle and the flow downstream separates from the nozzle wall to form a free-jet. For example, considering a nozzle of area ratio 10, a correct expansion at the nozzle exit is obtained with a pressure ratio $p_b/p_0$ of 0.0072. If this ratio is increased, for example, to 0.13, it is found in practice that isentropic expansion will only take place down the nozzle to an area ratio of about 2.6, where oblique shock waves raise the pressure ratio to the nozzle exit value on the curve B. Further flow down the nozzle will be separated from the nozzle wall. Thus the possible range of the SIS in this type of nozzle can be considered to be in the shaded area. It will be noted that the chosen overall pressure ratio of 0.13 can be attained by a normal shock wave at the nozzle exit, but this condition is seldom normally encountered.

Fig.13 shows comparison of various experimental values of separation pressure ratio $p_s/p_0$ versus overall pressure ratio $p_b/p_0$ for conical nozzles. The data indicated involves the separation pressure ratio $p_s/p_0$ for a variety of divergence angles of nozzle both with unheated and heated flows, and for different working fluids. Strictly speaking, the divergence angle varied from 15 to 30 degrees and the area ratio from 8 to 75.

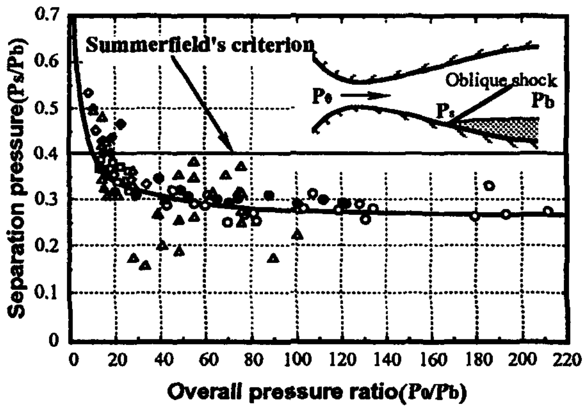

Fig.13 Experimental data on shock-induced separation in overexpanded supersonic nozzles

Summerfield et al.[38] argued that in an experiment of rocket nozzle the separation occurred when the flow was expanded to a static pressure approximately 0.4 times the back pressure. A similar result was also obtained by many other investigators[39-40]. In accordance with those experimental results, criteria for flow detachment in nozzles based upon a critical value of the ratio of the static pressure at the point of separation ($p_s$) to the back pressure ($p_b$) are often cited in the literature. However the critical pressure ratio $p_s/p_b$ is not a true constant, but rather may be a function of the overall pressure ratio prevailing during the experiment.

The experimental results[41], obtained in axially symmetric nozzles with the divergence half angles of 15 degrees, are again assembled in Fig.14, from which it can be seen that the conditions sufficient to cause the boundary layer separation are not well represented by the criterion $p_s/p_b$ = constant. Furthermore, the apparent lack of agreement between the different groups of data is such that only with some trepidation may a single empirical curve be fitted to all the points.

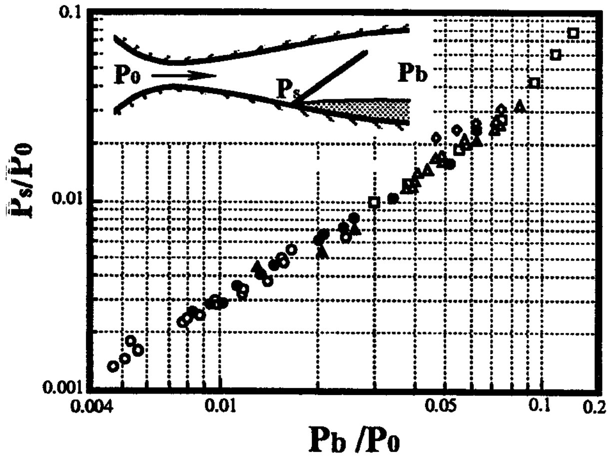


Fig.14 Experimental data on SIS in axial symmetric supersonic nozzles

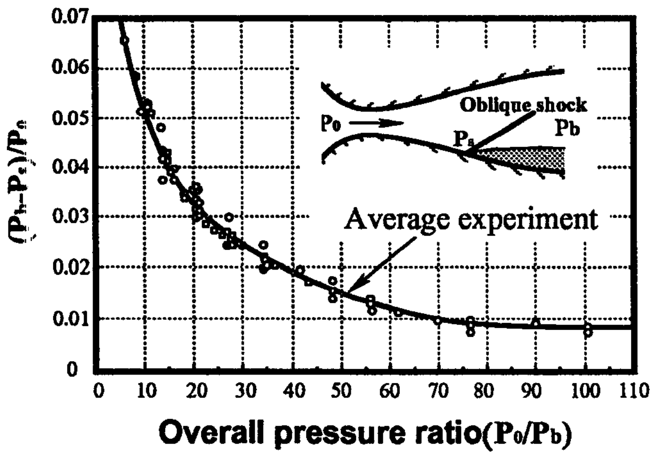


Fig.15 Experimental data on SIS in overexpanded supersonic nozzles

Fig.15 shows that these data on SIS permit a single empirical curve to be fitted with increased accuracy, if they are represented in terms of the ratio $(p_b-p_s)/p_o$. We can deduce that for a given value of overall pressure ratio $(p_o/p_b)$, the corresponding value of separation pressure ratio $(p_b/p_s)$ reduces to a function of $p_o/p_s$.

In order to compare the criterion of SIS with one due to the concept of free interaction in the previous external flows, the separation to back pressure ratio[68] is again plotted against the overall pressure ratio of the nozzle(see Fig.16). The data indicated represent two-dimensional and conical nozzles, nozzles with half angles between 7 and 30 degrees, and gas specific heat ratio between 1.2 and 1.4. The curve is presenting the separation pressure ratios as measured on forward facing steps, compression ramps, and incident shock waves, as shown earlier. The trend of these data is identical to that of the nozzle separation data for a wide range of Reynolds number applied. There does not seem to be a significant effect of Reynolds number on the separation pressure ratio.

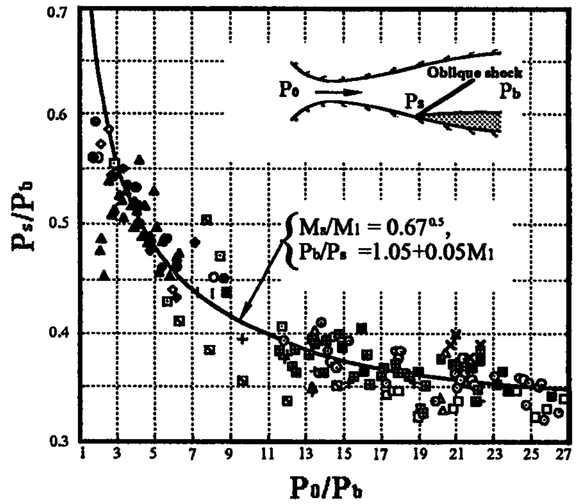


Fig.16 Comparison of theoretical and experimental separation pressures in two-dimensional and conical supersonic nozzles

It should be noted that within the accuracy of the data the pressure rise to separation is independent of the geometry causing the ISWTBL, depending only on the upstream Mach number. If these observations are interpreted as indicating that a supersonic turbulent boundary layer separates when subjected to a critical pressure jump, whose value depends only on the Mach number and not on the geometry of the interaction, it is then reasonable to expect that separation in such the internal flows as supersonic nozzles or diffusers should occur for pressure jumps of the same magnitude. The pressure rise due to separation in supersonic nozzles is only a function of the stream Mach number at the separation point.

Good agreement with the experimental data illustrated above can also be obtained by using the assumption[15-16] that the pressure rise must be sufficient to stagnate a characteristic velocity $u_s^*$ in the boundary layer. Assuming a constant stagnation temperature in the boundary layer, we obtain the separation pressure ratio from the following conservation laws ;

$$h_1 + \frac{u_1^2}{2} = h_s + \frac{u_s^2}{2} \tag{5}$$

$$\frac{T_s}{T_1} = \frac{h_s}{h_1} = 1 + \frac{u_1^2}{2h_1}\left\{1 - \left(\frac{u_s}{u_1}\right)^2\right\} = \left(\frac{a_s}{a_1}\right)^2 \tag{6}$$

$$M_s = \frac{u_s}{a_s} = \frac{u_s}{u_1} \cdot \frac{u_1}{a_1} \cdot \frac{a_1}{a_s} \tag{7}$$

where h is enthalpy. For $M_s$ less than unity, it is assumed that stagnation occurs isentropically. From isentropic relation between a state '1' and state 's', the pressure ratio was given by Eq.(3), where J means $u_s/u_1$. For $M_s$ larger than unity, it is assumed that an isentropic stagnation is preceded by a normal shock compression, similarly the required pressure rise was given by Eq.(4).

If the flow in the nozzle up to the separation point is assumed isentropic, then the separation Mach number is given by

$$\frac{p_0}{p_s} = \frac{p_0}{p_b} \cdot \frac{p_b}{p_s} = \left(1 + \frac{\gamma - 1}{2} M_s^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{8}$$

Neglecting the additional pressure rise in the mixing region downstream of the separation point, the ratio of the separation to back pressure is calculated as a function of overall pressure ratio of the nozzle. For $M_s$ less than unity, the separation pressure ratio is obtained explicitly as

$$\frac{p_s}{p_b} = \frac{p_0}{p_b} \times \left[\frac{1 - \left(\frac{u_s}{u_1}\right)^2}{\left(\frac{p_0}{p_b}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{u_s}{u_1}\right)^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{9}$$

, and the separation pressure ratio for $M_s$ larger than unity is given by

$$\left(\frac{p_s}{p_b}\right) = \frac{\left[1 + D \cdot \left\{1 - \left(\frac{u_s}{u_1}\right)^2\right\}\right] \cdot \left\{\frac{\gamma+1}{\gamma-1}\left(\frac{u_s}{u_1}\right)^2 \cdot D - \frac{\gamma-1}{\gamma+1} \cdot D - \frac{\gamma-1}{\gamma+1}\right\}^{\left(\frac{1}{\gamma-1}\right)}}{\left[\left(\frac{\gamma+1}{\gamma-1}\right) \cdot \left(\frac{u_s}{u_1}\right)^2 \cdot D\right]^{\left(\frac{\gamma}{\gamma-1}\right)}} \tag{10}$$

,where D is given by Eq. (11).

$$D = \left(\frac{P_0}{P_b} \cdot \frac{P_b}{P_s}\right)^{\frac{\gamma}{\gamma-1}} - 1 \tag{11}$$

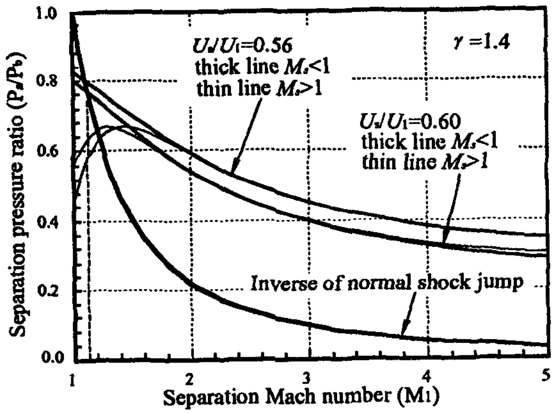


Fig.17 Variation of separation pressure ratio against separation Mach number in overexpanded nozzles($U_s/U_1$ denotes the extent of velocity stagnation)

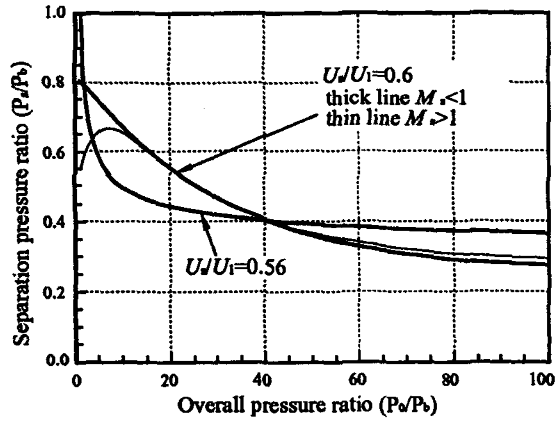


Fig.18 Variation of separation pressure ratio against overall pressure ratio of overexpanded nozzles($U_s/U_1$ denotes the extent of velocity stagnation)

The separation pressure ratios calculated from the above equations are presented against the Mach number (see Fig.17) and overall pressure ratio ( see Fig.18). For Mach number lower than 1.13, the pressure jump required to separate the boundary layer is larger than that provided by normal shock compression. Therefore, this situation is not real. The overall pressure ratio ($p_0/p_b$) of the nozzle necessary for the boundary layer separation, which is caused by oblique shock wave at a Mach number of 1.13, is calculated by about 1.7 for $u_s/u_1$=0.6, neglecting the pressure rise due to mixing downstream of the separation point. It is unlikely that the oblique shock separation will occur for overall pressure ratios lower than 1.7.

For nozzles with very small exit to throat area ratios, normal shock flow can be obtained for nozzle pressure ratios up to 1.89, the limiting value corresponding to ideally expanded convergent nozzle flow. For rather large area ratios, however, oblique shock flow will be established for pressure ratios exceeding 1.7. It should be noted that, for flow characterized by a normal shock at the limiting Mach number, the nozzle pressure ratio is lower than the nozzle pressure ratio required for separated flow at the limiting Mach number.

# CONCLUSIONS

This paper refers to the onset of shock-induced separation appearing in supersonic internal/ external flows. A great number of experimental data indicating the shock-induced separation in internal or external flows were reviewed to make clear the mechanism of shock-induced separation and to present the criterion of turbulent boundary layer separation. The interesting conclusions were obtained for the considerably wide range of flow geometries that the incipient separation is almost independent of the flow geometries, and that it is relatively unaffected by changes in gas specific heat, and boundary layer Reynolds number. Furthermore, the pressure rise necessary to separate boundary layer in external flows could be applied to the shock-induced separation in overexpanded nozzles. This is due to the fact that the shock-induced separation phenomenon caused by the interaction between shock waves and turbulent boundary layers is processed through a supersonic deceleration. This is, the shock-induced separation in almost all of interacting flow fields is governed by the concept of free interaction. Under circumstances of turbulent boundary layer, criterion of shock-induced separation is only a function of upstream Mach number. However, physical scales of the produced separation are not independent of the downstream flow field.

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