IGNITION OF REACTIVE SOLIDS WITH ROUGH SURFACE BY CONSTANT HEAT FLUX

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ABSTRACT

The ignition characteristics of a reactive solid with rough surface by constant heat flux were studied. The geometry of surface was represented by a set of identical protrusions having a shape of wedge based on the block of reactive solid. Several regimes of ignition were found, depending on the ratio of the protrusion length and the depth of the heated layer, formed in course of ignition process: 1) when the substance is ignited as the massive block, and the effect of roughness is not pronounced; 2) when ignited are the individual protrusions; and 3) in the intermediate region between the first two. Critical ignition conditions: ignition time and ignition criterion, are determined for the three regimes. The results are compared with the results for the one-dimensional ignition of the semi-infinite body. It is shown, that the effect of geometry on ignition results in the considerable reduction of ignition delay, and the amount of energy required for the successful ignition is less compared to the one-dimensional case.

Introduction

Ignition of explosives and solid propellants by an external energy source often starts with the initiation of the chemical reaction in a hot point. The hot point is formed locally in the area where the energy input rate is maximal, temperature is highest, and the conditions for the *initiation of the chemical reaction* in the solid are the most favorable. Recently the understanding of the events leading to hot spot formation attracted much interest [1,2].

Among the possible causes resulting in the formation of the hot points, one can note the non-uniformity of heat flux, the composition and structure of the propellant itself, and the complexity of the surface geometry, that is the presence of protrusions, sharp edges, roughness and the like irregularities. Whatever factor is dominant, consequently the temperature distribution in the solid phase becomes multidimensional, and the characteristics of ignition will differ from those for the substance with the smooth surface. It is well known, that the ignition delays, depending upon the intensity of the heat flux applied to the surface, can vary greatly, from seconds to milliseconds. For the typical values of heat conductivity of solid propellants, $\alpha \approx 10^{-3}$ cm^2/s, the penetration depth of the thermal wave for the ignition delays t* equal to 10^{-3} s is approximately equal to $d \approx (\alpha t^*)^{0.5} = 10^{-3}$ cm. The value of the characteristic thickness of the reacting layer, where the chemical reactions take place, would be even smaller. For such small values of the heated layer, usually the surface of propellants should be considered as rough. Since the multidimensionality of the heat transfer is the key feature of this type of problems, their adequate treatment requires the development of the multidimensional ignition theory.

Much progress has been achieved in the development of the modern ignition theory concerning the modeling of one-dimensional ignition of bodies with regular shape, such as semi-infinite body, cylinder or ball [3,4]. However, multidimensional ignition problems have been considered until now only in few publications. By means of direct two-dimensional numerical integration of the solid-phase governing equation, Vorsteveld and Hermance [5,6] studied the ignition of wedge by constant heat flux applied to the surface and found, that the ignition delay was decreased 3.6 times for a square-angle wedge, and even more for some acute angles considered, compared to the ignition delay of the semi-infinite body. In our previous papers [7-10] the ignition of acute wedges and cones was studied, the analytical two-dimensional solutions of the inert (without chemical reaction) heating of wedge and cone were obtained and utilized for the analysis of the ignition in the whole range of angles. For the very acute angles, the method of temperature averaging was developed, because this problem was too complicated for the direct numerical solution, and the ignition characteristics were found as a function of the apex angle and other parameters. The configurations considered by now do not represent fully the geometry of real surface, since the real surface protrusions are of finite height.

In the present paper the ignition of reactive solids with rough surface by constant heat flux provided, for example, by a laser source, is studied. Protrusions of the surface are represented as identical wedges of finite height based on the block of reactive solid. Homogeneous ignition model [3,4] is used, and the ignition regimes and the ignition characteristics are found as a function of apex angle and the height of the protrusions.

Model Description

A diagram of the model is shown in Fig.1a. A massive block of reactive solid with surface protrusions in shape of identical wedges of height L with apex angle $2\varphi_0$ was the geometry considered. Constant heat flux of intensity q is exposed to a surface. If the heat flux is supplied by a laser source, then the normal component of the heat flux depends upon the apex angle of the protrusions and is equal to $q\sin\varphi_0$. The dependence of the value of heat flux on the mutual orientation of the energy source and the substance is the important feature of the multidinesional ignition, and previous researches [5-9] did not take it into account. One-step, irreversible homogeneous chemical reaction can occur in the substance, generating heat in course of ignition process. Within the framework of the solid-phase ignition theory, the temperature dependence of the reaction rate W(T) is described by the Arrhenius law: W(T) = Z exp(-E/RT). Due to the symmetry of the problem it is sufficient to consider one protrusion, as it is shown on Fig.1b.

Mathematical formulation of the problem is given by energy equation for temperature distribution along with initial and boundary conditions. Within the framework of the homogeneous ignition theory [3,4], the reaction depletion being neglected, the governing equations in dimensional form for the element of the solid shown on Fig.1b are written down as follows:

$$c\rho\frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) + QZ\,exp(-E/RT) \qquad (1a)$$

Variables and domain:

$T = T(t, x, y);\ t > 0;\ x > 0;\ |y| < x\tan\varphi_0$ for $0<x<L$; $|y|<L\tan\varphi_0$ for $x>L$.

Initial and boundary conditions:

$T(0, x, y) = T_0;$ $\qquad$ $T(t, \infty, y) = T_0;$ (1b)

at $y = x \tan\varphi_0$: $\quad \lambda\, \partial T/\partial n = q \sin\varphi_0$, where n - vector, normal to the surface;

at $y = L \tan\varphi_0$ and $y = 0$: $\quad \partial T/\partial y = 0.$

The introducing the dimensionless variables merits some consideration, because in the case of ignition by heat flux we do not know in advance the characteristic temperature in the reaction zone, to be used as the predetermined ignition scale temperature. It is commonly accepted, that the Frank-Kamenetsky expansion of the exponent [11], with the dimensionless temperature θ defined as $\theta = E(T-T^*)/RT^{*2}$, would significantly simplify the problem. Therefore, we will use this expansion with the post-determined scale temperature T*. The value of T* is to be obtained in course of the solution. For the dimensionless scales for time and coordinates we have:

$\xi = x / x_0$; $\eta = y / x_0$; $u = t / t_0$; where $x_0 = \lambda R T^{*2}/q \sin\varphi_0 E$; $t_0 = x_0^2 c\rho/\lambda$.

So, the governing equations and initial and boundary conditions are rewritten in the dimensionless form as follows:

$$\frac{\partial\theta}{\partial u} = \frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial\eta^2} + \omega \exp(\theta) \qquad (2a)$$

$\theta = \theta(u, \xi, \eta)$; $u > 0$; $\xi > 0$; $|\eta| < \xi \tan\varphi_0$ for $0 < \xi < L_0$; $|\eta| < L_0 \tan\varphi_0$ for $\xi > L_0$;

$\theta(0, \xi, \eta) = \theta_0;$ $\qquad$ $\theta(u, \infty, \eta) = \theta_0;$ (2b)

at the surface $\eta = \xi \tan\varphi_0$: $\quad \partial\theta/\partial n = 1$, ($n$ - dimensionless normal vector);

at the symmetry lines $\eta = L_0 \tan\varphi_0$ and $\eta = 0$: $\partial\theta/\partial\eta = 0$.

Four dimensionless parameters, along with φ_0, appear after non-dimensionalization:

$\theta_0 = E(T_0 - T^*)/RT^{*2}$, $\omega = \lambda R T^{*2} QZ \exp(-E/RT^*) E^{-1} (q \tan\varphi_0)^{-2}$; $L_0 = L/x_0$.

Frank-Kamenetski's expansion of the exponent based on the ignition scale temperature T* determined *a posteriori* allows to minimize the dependence on small parameter $\beta = RT^*/E$, which otherwise should have been taken into account. Therefore, one obtains from the analysis of dimensionless parameters, that the temperature distribution and solution behaviour will be a function of four dimensionless parameters: $\theta = \theta$ (u, ξ, η, θ_0, ω, L_0,

φ_0), the ignition criterion will be determined from a relationship between the dimensionless parameters: $\omega^*=\omega^*(\theta_0, L_0, \varphi_0)$; and the ignition time will read as follows: $u^*=u^*(\theta_0,\omega^*,L_0,\varphi_0)$.

Analysis of Inert Heating

Due to the complexity of the geometry, the full analytical solution of two-dimensional inert problem (system (2), $\omega = 0$), describing the temperature distribution due to the heating by energy flux in the absence of chemical reaction, is not available. However, one can seek for asymptotic solutions and obtain the solution for early stages of heating, when the temperature waves from the both sides of the protrusions have not merged yet, that is, in dimensionless units, for $u \ll (L_0 \sin\varphi_0)^{0.5}$, as well as for later stages of heating, when the depth of the heated layer is much more than the height of protrusions, $u \gg L_0^{0.5}$. For the former case, the two-dimensional analytical solution was obtained in [7]; for the latter case it is evident, that the temperature distribution will asymptotically tend to one-dimensional distribution, same as it would be for the heating of smooth surface by a constant heat flux of intensity q. The equation for the protrusion apex temperature is as follows:

$$\theta(u, 0, 0) = \theta_0 + B\, u^{0.5},$$

where $B = \pi^{0.5}\sin\varphi_0/\varphi_0$ for small u, and $B = 2/\pi^{0.5}$ for large u.

Therefore, for the two limit regimes of ignition of protrusions, the dimensional ignition time can be estimated preliminary as follows:

for long protrusions, or intense heating:

$$t^* = \frac{\pi}{4}\frac{\lambda c\rho\left(T^*(\varphi_0) - T_0\right)^2}{q^2}\left(\frac{2\varphi_0}{\pi \sin\varphi_0}\right)^2 \quad (3a)$$

for short protrusions, or non-intense heating:

$$t^* = \frac{\pi}{4}\frac{\lambda c\rho\left(T^*(\varphi_0) - T_0\right)^2}{q^2} \quad (3b)$$

One can notice that both of these equations give the known result for the ignition delay of the semiinfinite body, when $\varphi_0=\pi/2$ [4]. The equations (3) are directly applicable for the quantitative estimation of ignition time, if the value of T* is determined. They provide qualitatively correct dependence of ignition time on angle, because dependence of T* on φ_0 is much less pronounced than that for t*. In the limit of small angles the equations (3a) and (3b) give the different values of ignition delays, by the factor approximately equal to $\pi^2/4\approx2.5$. This can be attributed to the effect of geometry on ignition and will be discussed later.

The effect of the protrusion length is noticeable at the intermediate regimes of heating, when the depth of the heated layer is of the same order of magnitude as the protrusion length. This effect is studied later for the limit case of small apex angles.

The Treatment of the Small Angles: Temperature Averaging Method.

For the treatment of the small angles the direct numerical integration of (2) is hardly possible, and we will take the advantage of the method of temperature averaging along the isotherms, developed in our previous studies [7-10]. For the thin body, when the thermal boundary layers in the protrusion converge, the heat transfer occurs mainly in the axial (x) direction, while in the transverse (y) direction it can be neglected. The isotherms are approximately perpendicular to the axis of symmetry, and the temperature may be averaged along the isotherms. The procedure of temperature averaging gives correct results for the small angles φ_0, and moreover, in [7] it was proved, that for the infinitely long wedge the results remain valid in the whole range of angles $0<\varphi_0<\pi/2$. It allows to reduce the number of variables and parameters, as well as to find the functional relationships among the system parameters in the limit case of small angle φ_0.

Upon performing the temperature averaging: $\overline{\theta}=\int_0^{\xi \tan\varphi_0}\theta\, d\eta/(x\ \tan\varphi_0)$ for $0<\xi<L_0$, and $\overline{\theta}=\int_0^{L_0 \tan\varphi_0}\theta\, d\eta/(L_0 \tan \varphi_0)$ for $\xi>L_0$, (the averaging symbol later will be omitted) and introducing new dimensionless scales, the equation for temperature distribution in the body along with initial and boundary conditions becomes as follows:

for $0<\zeta<\zeta_0$,: $$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\zeta^2}+\frac{1}{\zeta}\frac{\partial\theta}{\partial\zeta}+\frac{1}{\zeta}+\Omega\cdot\exp\theta \qquad (4a)$$

for $\zeta>\zeta_0$,: $$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\zeta^2}+\Omega\cdot\exp\theta \qquad (4b)$$

$$\theta=\theta(\tau,\zeta);\ 0<\zeta<\infty;\ \tau>0;$$

$$\theta(0,\zeta)=\theta(\tau,\infty)=\theta_0; \qquad \frac{\partial\theta}{\partial\zeta}(\tau,0)=-1 \qquad (4c)$$

$$\theta(\tau,\zeta_0-0)=\theta(\tau,\zeta_0+0); \qquad \frac{\partial\theta}{\partial\zeta}(\tau,\zeta_0-0)=\frac{\partial\theta}{\partial\zeta}(\tau,\zeta_0+0);$$

where: $\zeta=x/x_1$, $\tau=t/t_1$, $t_1=c\rho x_1^2/\lambda$; $x_1=x_0\sin\varphi_0.=\lambda RT^{*2}/qE$; $\zeta_0=L/x_1$; $\Omega=\omega\sin^2\varphi_0=\lambda RT^{*2}QZ\exp(-E/RT^*)/Eq^2$. It is evident, that due to the temperature averaging the dependence on angle φ_0 is incorporated into the dimensionless variables and parameters.

The analysis of the inert heating (system (4) with Ω=0) can be simplified with the use of the Laplace transformation. Namely, we introduce a function $v(x,p)=\int_0^\infty\theta(x,\tau)\exp(-p\tau)d\tau$, and the governing equations along with initial and boundary conditions are transformed into the set of ordinary differential equations for v(x, p):

for $0<\zeta<\zeta_0$,: $$\frac{d^2v}{dx^2}+\frac{1}{\zeta}\frac{dv}{d\zeta}+\frac{1}{\zeta p}-pv+\theta_0=0 \qquad (5a)$$

for $\zeta>\zeta_0$,: $$\frac{d^2v}{d\zeta^2}-pv+\theta_0=0 \qquad (5b)$$

with the boundary conditions:

$dv/d\zeta\,(\zeta=0)=-1/p$; $\qquad v(\zeta=\infty)=\theta_0/p$;

and with the conjugation conditions at $\zeta=\zeta_0$:

$v(\zeta_0-0)=v(\zeta_0+0)$; $\qquad dv/d\zeta\,(\zeta_0-0)=dv/d\zeta\,(\zeta_0+0)$.

System (5) has the analytical solution which can be expressed in terms of modified Bessel functions; for our purposes the solution can presented in the following form:

$$v=\theta_0/p+p^{-1.5}f(z,Z_0),$$

where function f depends on variable $z = \zeta\, p^{0.5}$ and one parameter $Z_0 = \zeta_0\, p^{0.5}$.

For the apex temperature, the solution will give $v\,(\,\zeta = 0\,) = p^{-1.5}\, F(Z_0)$, where

$$F(Z_0) = -\int_0^{Z_0} \frac{K_0(s)ds}{s(I_0(s)K_0'(s) - K_0(s)I_0'(s))} + \frac{K_0(Z_0)+K_0'(Z_0)}{I_0(Z_0)+I_0'(Z_0)} \int_0^{Z_0} \frac{I_0(s)ds}{s(I_0(s)K_0'(s) - K_0(s)I_0'(s))}$$

I_0 and K_0 are the modified Bessel functions. By examining the asymptotics at small and large values of Z_0, one can obtain the equations for the apex temperature in limit cases of initial and late stages of inert heating, corresponding respectively to the heating of the sharp apex and the body as a whole block. Therefore, the apex temperature will read as follows:

$$\theta\,(\,\tau, 0) = \theta_0 + \pi^{1/2}\tau^{1/2} + \;, \qquad \zeta_0\tau^{-1/2} \gg 1;$$

$$\theta\,(\,\tau, 0) = \theta_0 + 2\pi^{-1/2}\tau^{1/2} + \ldots, \qquad \zeta_0\tau^{-1/2} \ll 1.$$

These expressions, rewritten in dimensional units, will result in the equations (3) for the estimations of the ignition delay. The inert heating is illustrated on Fig.2, where the dependence of the body apex temperature on time for $\Omega = 0$, $\theta_0 = 0$ and $\zeta_0 = 1$ is shown, and the regimes of heating up like infinite wedge (at the initial stages of heating), like the semi-infinite slab (at the later stages), are clearly seen. The intermediate regime, when the value of the heated layer is approximately equal to the protrusion length,. corresponds to the values of dimensionless time about 1, and the transition from the upper curve (wedge apex temperature) to the lower curve (slab surface temperature) occurs in that region.

Solution Behaviour and Definition of the Critical Ignition Conditions

System (4) was solved numerically. Solution behaviour can be described as follows. At the beginning, after the application of the constant heat flux, temperature inside the substance is not enough to initiate the chemical reaction, and the inert heating due to the external heat flux takes place. After some period of initial heating, when the temperature is high enough for the reaction to occur, both factors of heating are important; and finally, with the further increase of temperature, the chemical heating becomes dominant, and the rapid temperature runaway is observed. The evolution of the temperature distribution with time for the typical values of the parameters is shown on Fig. 3. In course of the heating process, the apex temperature remains the highest. The dependence of apex temperature on time is shown on Fig. 4.

It is known, that the definition of ignition criterion cannot be done unambiguously; for example, Kulkarny, Kumar and Kuo [3] reviewed 14 definitions of ignition criteria used in studies on ignition of solid propellants. Here the ignition moment is defined as the moment of the most rapid rate of temperature runaway (see Fig. 4), that is, in the absence of depletion, $\tau \to \tau^*$, $\theta(\tau, 0) \to \infty$; and the scale ignition temperature is defined as the apex temperature of inert body at the ignition moment: $\tau=\tau^*$, $\theta_{in}(\tau, 0) = 0$, that is, expressed in dimensional units, $T^* = T_{in}(\tau^*, 0)$.

Results

The effect of the protrusion length ζ_0.

The dependence of ignition time τ^* on the protrusion length ζ_0 is shown on Fig. 5. For the very small values of protrusion length the ignition delay is approximately equal to that of the smooth surface; but with the increase of protrusion length the ignition delay experiences significant decrease, and for the large values of ζ_0 the ignition delay asymptotically tends to its value for the infinitely long protrusion. The ignition criterion is increased with the increase of protrusion length, but not much, varying approximately 2-3 times from the small values of ζ_0 to the large values of ζ_0 (Fig. 6). Since the ignition criterion Ω^* depends exponentially on the ignition scale temperature T*, $\Omega^* \sim \exp(-E/RT^*)$, the ignition scale temperature does not vary much with the increase of protrusion length; e.g., if the ignition criterion is increased twice, the corresponding increase of the ignition scale temperature is given by the value of the characteristic temperature interval: $T_0 = RT^{*2}/E \approx$ 20-30 K for typical solid propellants. Therefore, the effect of the protrusion length on the ignition delay is to be attributed primarily to the more effective energy distribution when it is transferred into the substance: much energy is concentrated in the apex region, quickly heating the hot points up to the ignition temperature, while the whole substance remains relatively cold compared to the case of heating the sample with the smooth surface. At the later stages of ignition, much heat is generated by chemical reaction, and the reaction itself becomes the heat source providing enough heat for self-sustained combustion.

At the given initial temperature θ_0 the dependence of ignition delay on the protrusion length can be presented with a good accuracy by the following formula:

$$\tau^* = \tau_\infty + (\tau_0 - \tau_\infty) \exp(-\zeta_0/D), \qquad (6)$$

where τ_∞ - the ignition delay of the infinite wedge, τ_0 - the ignition delay of the semi-infinite slab, and D - parameter, depending on initial temperature. The calculation showed, that the dependence of D on initial temperature is linear, being described by the following equation:

$$D(\theta_0) = -\,0.46 + 0.52\ |\theta_0| \qquad (7)$$

The effect of the initial temperature θ_0.

With the increase of the absolute value of the initial temperature θ_0 the ignition delay is increased (Fig. 7). For the long protrusions at the lower initial temperatures the ignition delay is equal to that of the infinite wedge. The length of the protrusion which may be considered as "long" can be estimated as follows: in course of the ignition the depth of the heated layer δ is proportional to the square root of the ignition delay: $\delta \sim \tau^{*1/2}$. Ignition delay is proportional to the squared initial temperature, namely, for the infinite wedge: $\tau^* = \theta_0^2/\pi$, i.e., $\delta \sim \theta_0$, with the constant of proportionality of the order of 1, as the calculations showed. Therefore, the parameter $D(\theta_0)$, introduced in the previous section, can be also interpreted as a characteristic value, corresponding to the penetration depth of thermal wave in course of ignition, and the expression for the ignition delay as a function of dimensionless parameters will read as follows:

$$\tau^*(\theta_0, \zeta_0) = \theta_0^2 \left\{ 1/\pi + (\pi/4 - 1/\pi) \exp\left[-2\zeta_0/(|\theta_0|-1)\right] \right\}, \qquad (8),$$

adopting the approximate values for $D(\theta_0)$ instead of specified in (7).

The dependence of the ignition criterion Ω^* on the initial temperature is not pronounced at all It is presented at Fig. 8 in the coordinates Ω^* versus $|\theta_0|^{-2/3}$, where the dependence is almost linear. Again, one can notice, that for the longer protrusions at the lower initial temperatures the curves for the ignition criterion tend to merge, converging to its value for the infinite wedge. For the values of $\zeta_0 < 16$ it can be expressed by the following equation:

$$\Omega^* (\theta_0, \zeta_0) = [3.64 - 2.39 \exp(-\zeta_0/3)] \, |\, \theta_0 \,|^{-2/3}, \qquad (9),$$

and for larger values of ζ_0 the expression (9) may also be used for the evaluation of Ω^*, because the errors in the numerical value of Ω^* due to the inaccuracy of (9) in that region of parameters are negligible. Thus, equations (8) and (9) provide the required characteristics of ignition of the protrusion in homogeneous ignition model, since (9) is the implicit equation for obtaining the ignition scale temperature T* as a function of angle and other parameters.

The treatment up to now has been done in dimensionless formulation. For the analysis of the effect of the dimensional parameters, such as heat flux q, protrusion length L etc., one should distinguish between the two possible modes of heating: the first, by uniform heat flux q being independent of the angle φ_0, and the second, by plane-parallel heat flux with the intensity of given by its component, normal to the body surface: q $\sin\varphi_0$. In the latter case, this value of heat flux should be substituted into the equations (8) and (9).

The ignition time t* strongly depends upon the angle (proportional to the approximately second power), and the decrease of ignition delay in two-dimensional case compared to one dimensional is quite considerable. For the long protrusions, the ratio of ignition delays by the same intensity heat flux in the two-dimensional and the one- dimensional cases is expressed by the following equation in case of heating by uniform heat flux:

$$\frac{t^*_{2D}}{t^*_{1D}} = \left(\frac{T^*(\varphi_0) - T_0}{T^*(\pi/2) - T_0} \right)^2 \left(\frac{2\varphi_0}{\pi} \right)^2 \qquad (10a)$$

and by the following equation in case of heating by plane-parallel heat flux:

$$\frac{t^*_{2D}}{t^*_{1D}} = \left(\frac{T^*(\varphi_0) - T_0}{T^*(\pi/2) - T_0} \right)^2 \left(\frac{2\varphi_0}{\pi \sin \varphi_0} \right)^2 \qquad (10b)$$

The dependence of the ignition ratios on the angle for the both cases is shown at Fig.2. For the plane-parallel heating case, the ignition delay for the thin bodies with the apex angle less than 60° becomes constant, approximately equal to 0.25. One should note, that the ignition delay is reduced four times compared to that for the smooth surface, though the intensity of the heat flux remains the same. On the other hand, if the ignition delay remains the same for the smooth surface and for the rough surface in the shape of the set of the wedges, the heat flux or the energy required for the successful ignition is decreased two times due to the effect of geometry.

Discussion

In previous sections the model geometrical configuration was considered. It is expedient to discuss some limitations which one should bear in mind as far as the application of the presented results to the analysis of ignition of reactive solids in practical condidtions is concerned.

Since the solid-phase ignition model adopted herein does not account for the gas-phase reactions which usually are of importance in course of ignition of real substances, the characteristics of ignition in the presence of gas-phase reactions might be diffferent from the obtained in the present study. However, the roughness of surface must also influence significantly the ignition characteristics in that case, since the protrusions are heated up earlier, resulting in decomposition of the solid phase and preparation of the initiation of gas-phase reactions. Also it turned out to be useful to perform the temperature averaging of the gas-phase model equations. That immediately yields the main part of the dependence of the ignition characteristics on angle. Therefore, the dependence of ignition time on angle remains similar, i.e., ignition delay t* is proportional to the second power of angle for long protrusions, whereas the ignition criterion is inversely proportional to the first power of angle. The calculation of proportionality coefficients requires additional research.

Our model deals with sharp protrusions. Practically, any protrusion has a finite curvature radius. The point of maximum heat release is located in depth of the substance, which can be estimated adopting the linear law of temperature decrease near the apex, $\theta(x) = \theta(0) - x$ for wedge. The location of maximum heat release should be calculated from the maximum of the function $x \cdot \exp(-x)$, proportional to the product of body thickness and the chemical reaction rate, giving $x_m=1$. The results for sharp protrusion are valid if curvature radius is much less than x_m.

It is a problem of great interest, whether the protrusions, being ignited, are able to ensure the self-sustained ignition after the removal of ignition source. We can note, that in our experiments of polymer ignition (which is believed to be better described by heterogeneous ignition model), the ignition of sharp edges resulted in effective stable combustion [12].The problem of stability of ignition is not discussed in this paper, since it demands the application of methods of non-stationary combustion theory, and even for the one-dimensional cases it has not been resolved until now [13].

Conclusions

The present research provides an estimation of ignition characteristics of surface irregularities. Analytical solution of the inert problem (without chemical reaction) was utilized to find out the regimes of heating, depending on the ratio of the thickness of the heated layer and the length of the protrusions. With the use of the temperature averaging procedure multidimensional ignition problem was reduced to effectively one-dimensional, and the functional relationships between the parameters were established. The effect of geometry on ignition was found to be pronounced even at small values of protrusion length, due to the effective heating of hot points instead of the substance as a whole, as it would be the case for the heating of the smooth surface. The ignition delays were reduced considerably with the increase of the protrusion length, and they also proved to be very sensitive to the apex angle of the body. The law of dependence of ignition delay on angle of the protrusion is approximately quadratic. The energy required for the ignition of the rough surface is at least two times lower than that for the smooth surface, if the ignition delay is the same, and this is achieved exclusively due to the geometry factors.

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Nomenclature

q - heat flux

Q - heat of reaction

Z - pre exponential factor

c - specific heat

ρ - density

λ - thermal conductivity

E - activation energy

R - universal gas constant

T - temperature

t - time

L - protrusion length

r and φ - radial and angular variables of cylindrical system of coordinates

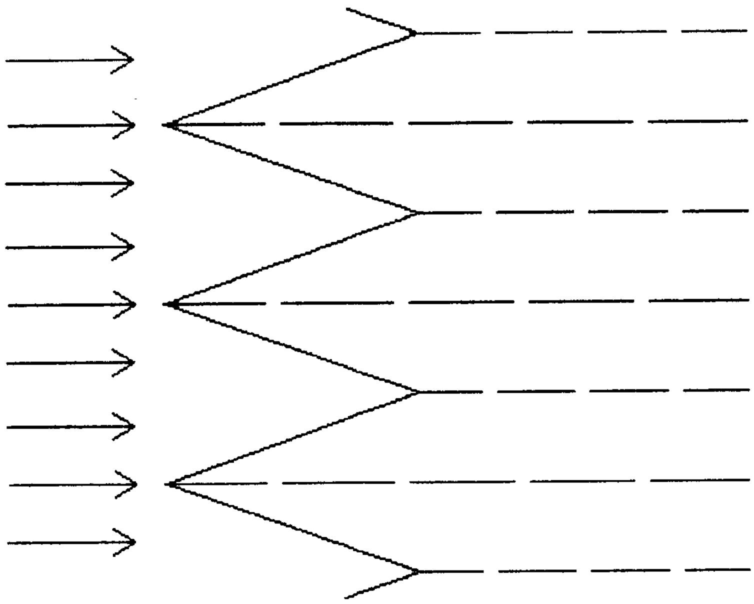
φ_0 - protrusion semiangle

T_0 - initial temperature

T^* - ignition scale temperature

θ - dimensionless temperature

a)



b)

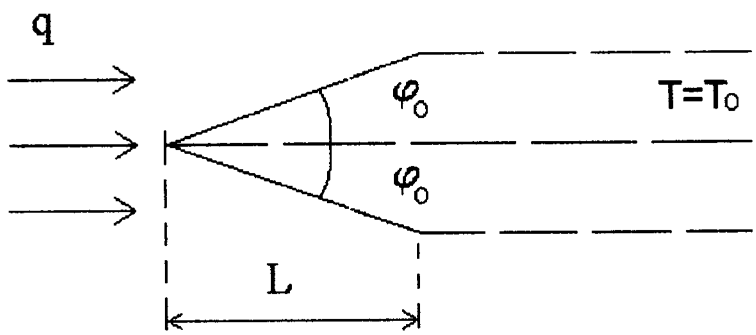


Fig. 1. The Model Diagram: a) Ignition of Rough Surface by Constant Heat Flux

b) a Single Protrusion.

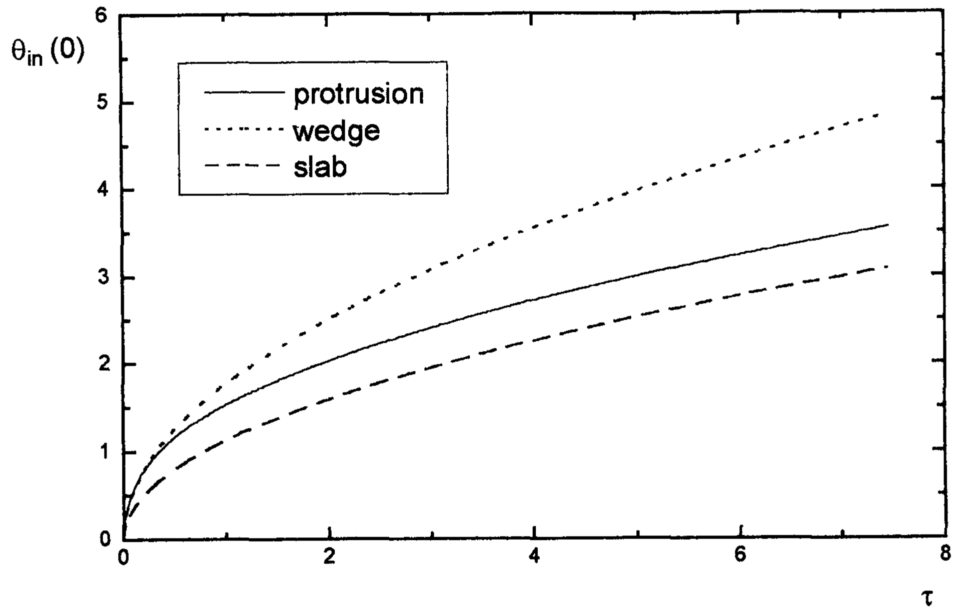


Fig. 2. Apex temperature versus time in course of inert heating, ζ_0=1.

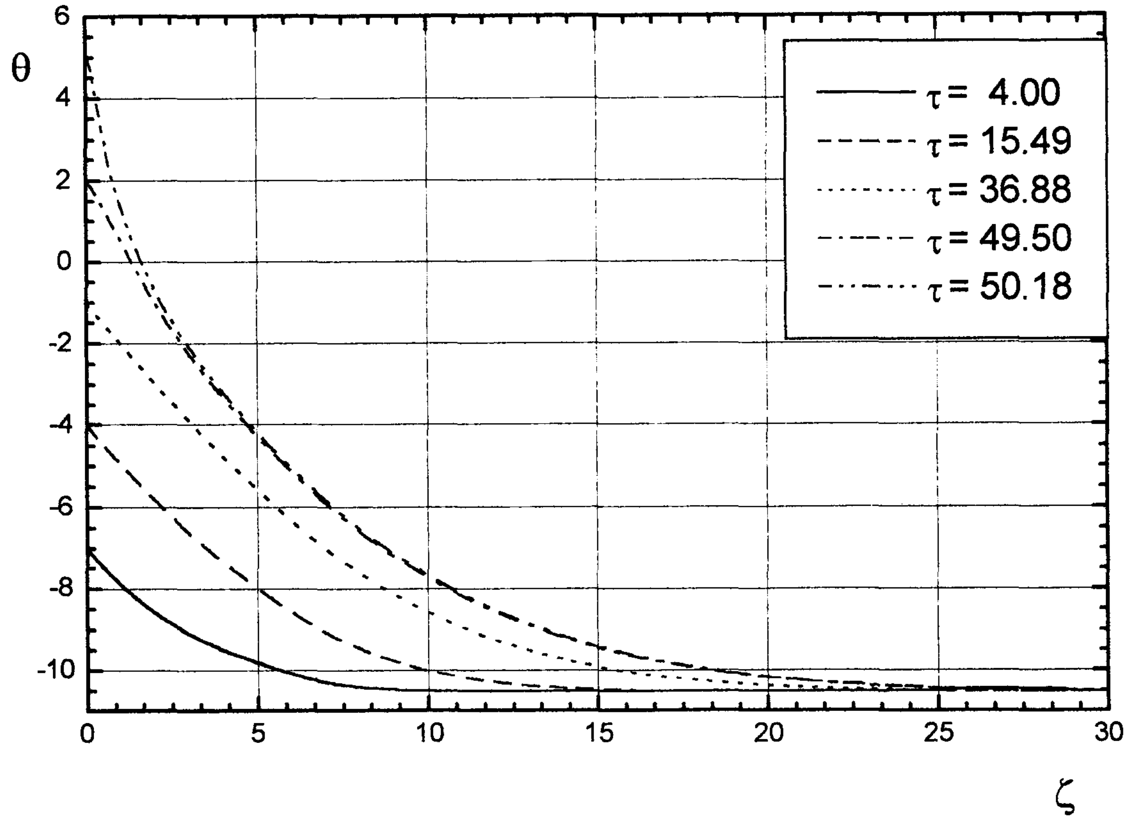


Fig. 3. Evolution of temperature distribution in the protrusion with time.

$\zeta_0 = 6$; $\theta_0 = -10.53$; $.\Omega^* = 0.686$.

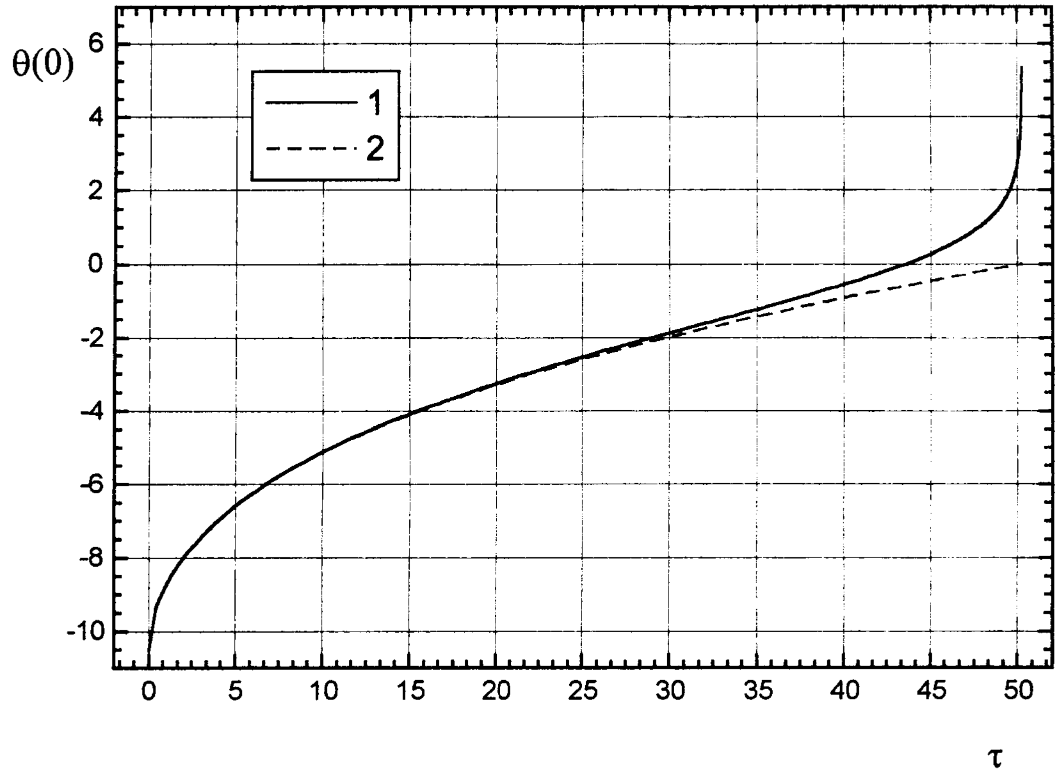


Fig. 4. Apex temperature versus time (curve 1) in comparison with the of inert heating (curve 2), $\zeta_0 = 6$; $\theta_0 = -10.53$; .$\Omega^* = 0.686$.

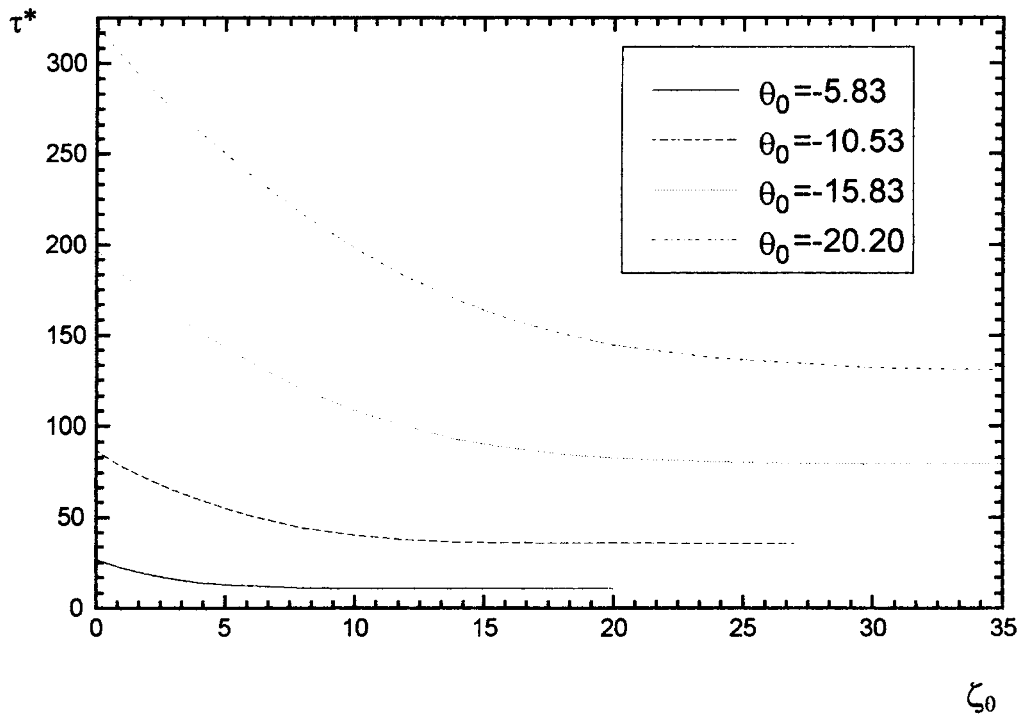


Fig.5. Dependence of ignition time τ^* on protrusion length ζ_0 at various initial temperatures θ_0

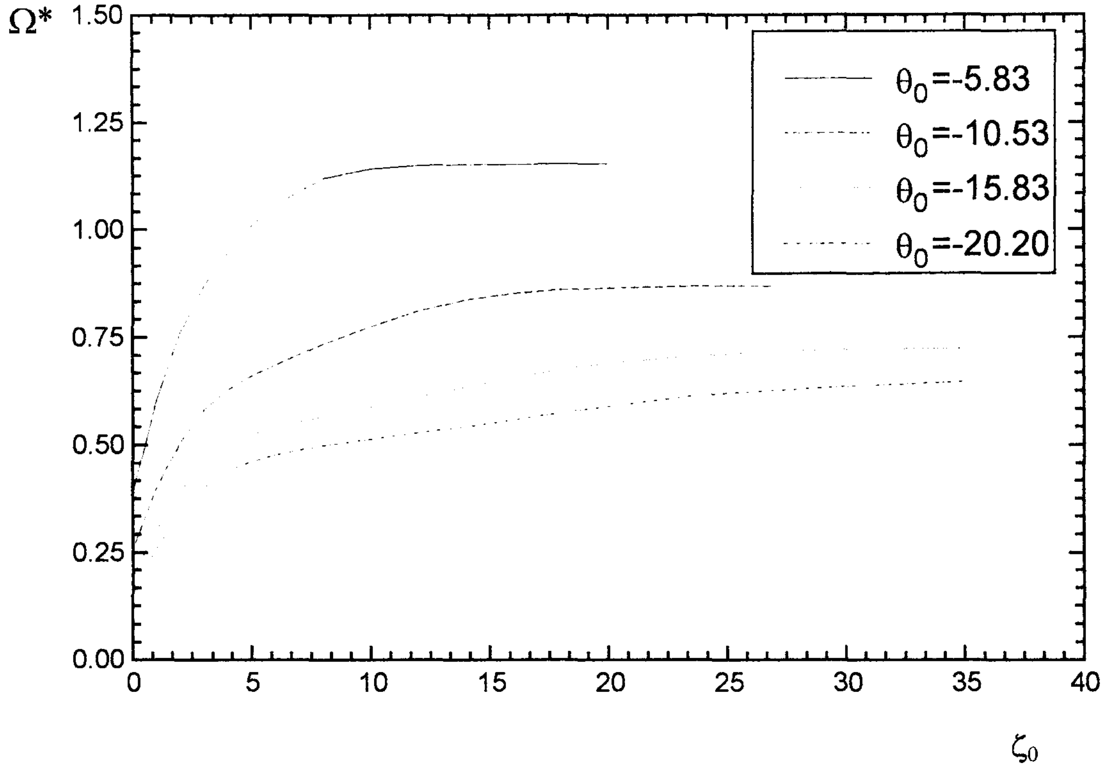


Fig. 6. Dependence of ignition criterion Ω^* on protrusion length ζ_0 at various initial temperatures θ_0

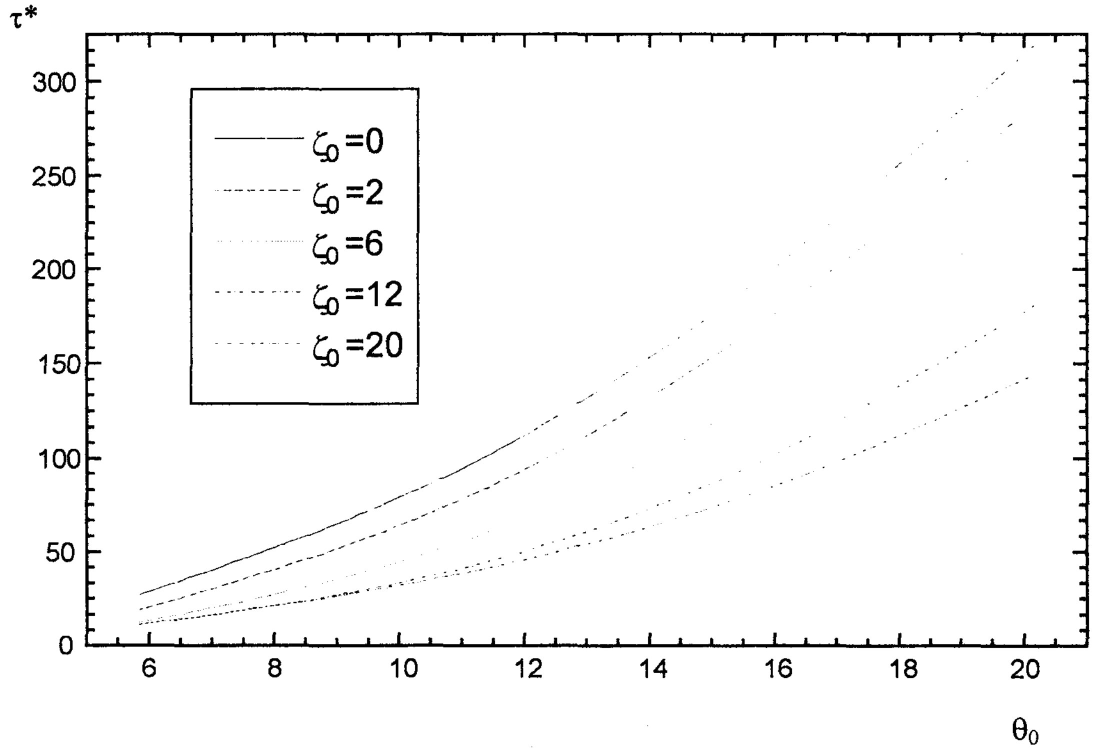


Fig. 7. Dependence of ignition time τ^* on initial temperature θ_0 at various protrusion lengths ζ_0

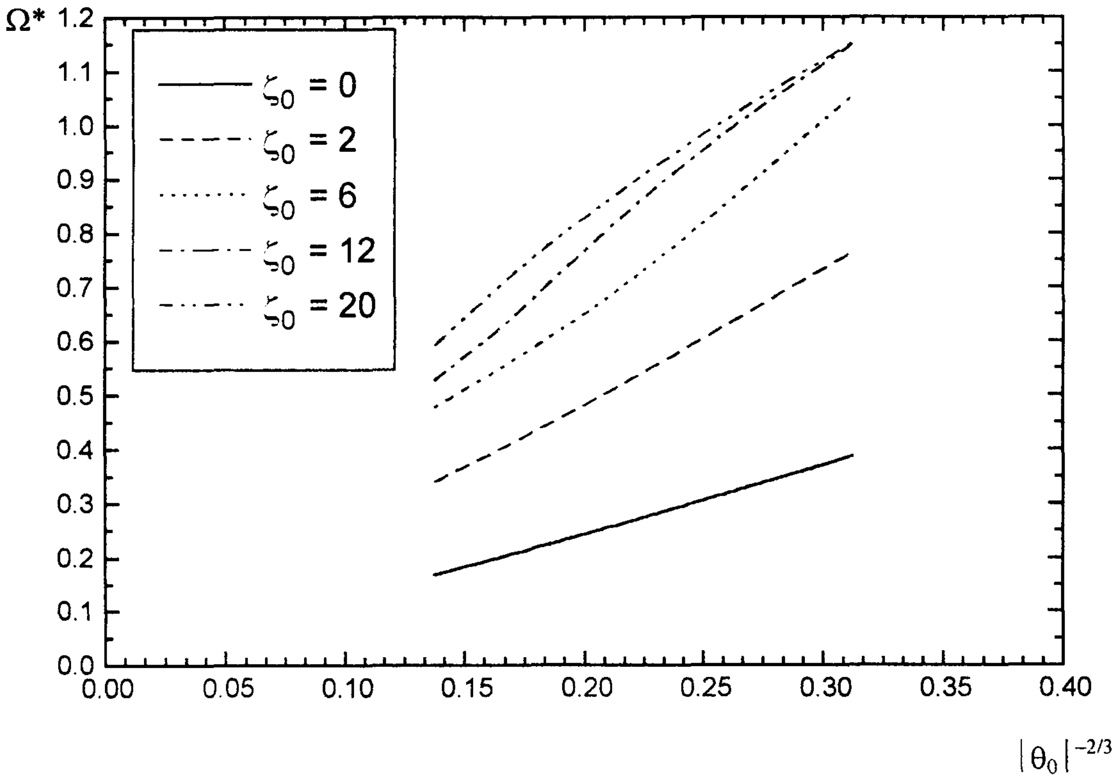


Fig. 8. Dependence of ignition criterion Ω^* on initial temperature θ_0 at various protrusion lengths ζ_0