변위제어하에서 콘크리트의 점소성 연화거동해석

Analysis of Viscoplastic Softening Behavior of Concrete under Displacement Control

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ABSTRACT

The softening behaviors of concrete have been the object of numerous experimental and numerical studies, because the load carrying capacity of cracked concrete structure is not zero. Numerical studies are devoted to the investigation of three-dimensional softening behaviors of concrete on the basis of a viscoplastic theory, which may be able to represent the effects of plasticity and also of rheology. In order to properly describe material behaviors corresponding to different stress levels, two surfaces in stress space are adopted; one is a yield surface, and the other is a failure or bounding surface. When a stress path reaches the failure surface, it is considered that the softening behaviors are initiated as micro-cracks coalesce and are simulated by assuming that the actual strain increments in the post-peak region are less than the equivalent viscoplastic strain increment. The experimental studies and the finite element analyses have been carried out under the displacement control. Numerically simulated results indicate that the model is able to predict the essential characteristics of concrete behaviors such as the non-linearity, stiffness degradation, different behaviors in tension and compression, and specially dilatation under uniaxial compression.

INTRODUCTION

Concrete is a composite made up of hydrated cement paste, sand and stone aggregate, air voids, free water, and some chemical additives. In order to develop a mathematical theory with the aim of modeling its constitutive behavior, it is necessary to understand and rationalize the various characteristics recorded in the laboratory experiments. The degree of stress-strain non-linearity depends upon the stress level to which material is subjected. Under low stress levels, it exhibits almost linear behavior and becomes increasingly nonlinear with increasing stress level. The stress-strain curve in compression exhibits more non-linearity than that in tension. This non-linearity is due not only to the composite action of the material, but also to the strength of the cement paste-aggregate bond. Concretes with low bond strength exhibit strongly nonlinear curves, whereas higher strength concretes derive their higher strength partially from bond strength, which delays micro-cracking and makes the stress-strain curves more linear. As a result, the theory of viscoplasticity and the two surfaces theory are adopted to simulate the three dimensional behavior of concrete structure properly. One of the two surfaces is the initial yield surface and the other is the bounding failure surface (Voyiadjis and Abu-Lebdeh, 1993).

Owing to the assumption that the viscoplastic properties of the material manifest only after the passage to the plastic state and that these properties are not essential in elastic state, the viscoplastic state can be determined using the yield function corresponding to material used in the analysis. When the state of stress is within the yield surface in the principal stress space, its state can be represented by the theory of elasticity. But when the state stress is beyond the yield surface, it may be expressed by the theory of viscoplasticity or plasticity.

As the applied load increases, the stress path reaches its limit, called ultimate strength or failure, and the softening behavior initiates, which means that the stress decreases with increasing strain. Once the stress path reaches to the failure surface or bounding surface expressed by the failure criterion, the macro-cracks appear in a body and the strain energy stored starts to dissipate, and at the same time the

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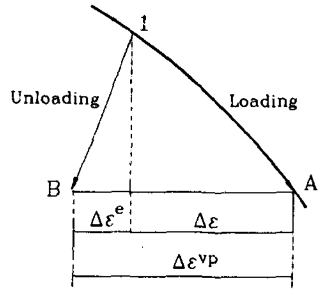

Fig. 1 Strain increments in descending part;
$\Delta\varepsilon^e$=elastic, $\Delta\varepsilon^{vp}$= equivalent viscoplastic.

failure surface also shrinks in size as the softening behavior continues.

If the incremental stress-strain law based on the theory of plasticity is employed to describe the softening behaviors, the explicit constitutive equation have been formulated by using the tension softening and fracture concept from which the negative crack moduli are obtained.

However, in this study the viscoplastic constitutive equation is utilized to describe the softening behavior of concrete by adjusting the equivalent viscoplastic strain increments. Considering the descending parts of stress-strain curves in Fig. 1, it may be possible to speculate that the actual or total strain increment is less than the equivalent viscoplastic strain increment, because the elastic increment which may be obtained from the unloading path is not contained in the actual strain increment. As a result, the softening behavior can be described by increasing the viscoplastic strain rate as long as strain increases. The reason will be explained in detail.

The proposed expression has been implemented in the finite element analysis program and evaluated by simulating the uniaxial, biaxial and triaxial behaviors of concrete blocks subjected to the imposed displacements on boundaries displacement control.

STRESS-STRAIN RELATIONS

It is difficult to simulate the behavior of concrete up to failure under various loading conditions. Herein, the theory of viscoplasticity and the two surfaces theory are employed to attempt this task. The simulations are carried out in three regions in stress space, which correspond to the linear, nonlinear and descending behaviors. Three regions can be determined by two surfaces, the yield and bounding failure surfaces.

Inside the yield surface, i.e. in the elastic region, concrete behavior can be described by the theory of linear elasticity, because micro-cracking and the resulting strains are relatively small, and concrete can be assumed to have negligible viscous properties.

Elastic strains are completely reversible under unloading and reloading conditions and obey Hooke's law,

$$\varepsilon_{ij}=\frac{S_{ij}}{2G}+\frac{1-2\nu}{E}\sigma_m\delta_{ij} \tag{1}$$

where $S_{ij}=\sigma_{ij}-\delta_{ij}\sigma_m$ is the deviatoric stress tensor, $\sigma_m=\frac{\sigma_{ii}}{3}$ is the hydrostatic stress, and G, E and ν are the shear modulus, elastic modulus and Poisson's ratio, respectively.

If a stress path passes through the yield surface but has not yet reached the failure surface, the state of stress is in the viscoplastic region, and viscoplastic straining occurs. In this region, cracks are not ignored and are represented by a viscoplastic strain. In this case, considering material non-linearity only, the total strain can be decomposed into two parts as

$$\varepsilon_{ij} = \varepsilon^{e}_{ij} + \varepsilon^{vp}_{ij} \quad (2)$$

in which ε^{e}_{ij} is the elastic strain tensor of Eq.(1), and ε^{vp}_{ij} is the viscoplastic strain tensor. The viscoplastic strains are the result of both viscous and plastic behavior.

Failure is initiated once the stress path reaches the failure surface, and then strains due to cracks are taken into account. In the smeared crack model, the softening behavior of concrete structure has been numerically analyzed with the consideration of crack modulus according to fracture mechanics concepts.

Instead of that, one assumes that the softening or descending behavior can be simulated by activating the viscoplastic straining more in the post-peak region, and then the viscoplastic strain increments are assumed to be greater than the actual strain increments, resulting stress decrements against strain increments.

Viscoplastic Straining

The viscoplastic strain rate is expressed in its most general form as

$$\dot{\varepsilon}^{vp}_{ij} = \psi(\sigma_{ij}, \varepsilon^{vp}_{ij}) \quad (3)$$

With the introduction of the potential concept, a specific form of Eq.(3) due to Perzyna (Perzyna, 1966) retains wide generality for material description, but at the same time defines the viscoplastic strain rate in the same form as employed in the conventional plasticity theory,

$$\dot{\varepsilon}^{vp}_{ij} = \gamma \langle \Phi(Y) \rangle \frac{\partial Q(\sigma_{ij}, \varepsilon^{vp}_{ij})}{\partial \sigma_{ij}} \quad (4)$$

in which the notation < > indicates that viscoplastic straining occurs only for the yield function Y>0, and γ is a fluidity parameter. The fluidity parameter can be considered to be inversely proportional to the viscosity of material. Accordingly, at the same stress level, the viscoplastic strain rates of soft materials are generally greater than those of hard materials, and also the viscoplastic strain rate increases with increasing stress level. The scalar function Q can be interpreted as a viscoplastic potential which plays the same role as the flow potential in fluid flow (Kim, 1993).

Substitution of the chosen yield function into Eq.(4) for the associated flow rule gives

$$\dot{\varepsilon}^{vp}_{ij} = \gamma \langle \Phi(Y) \rangle \frac{\partial Y}{\partial \sigma_{ij}} \quad (5)$$

or in matrix form,

$$\dot{\varepsilon}^{vp} = \Gamma \sigma \quad (6)$$

where γ < $\Phi(Y)$> is absorbed in the matrix.

The relationship between stress and strain can be derived by approximating the viscoplastic strain rate. The viscoplastic strain increment during the time step Δt_n can be expressed in the form,

$$\Delta\varepsilon^{vp}_{ij} = [\,(1-\alpha)\dot{\varepsilon}^{vp}_{ij} + \alpha\dot{\varepsilon}^{vp}_{ij}]\,\Delta t_n \;; \quad 0 \le \alpha \le 1 \quad (7)$$

where α is a parameter that influences the integration accuracy and stability. Eq.(7) represents an explicit, explicit/implicit, or implicit integration scheme for $\alpha = 0$, $0 < \alpha < 1$, and $\alpha = 1$, respectively.

The major factor that relates to the viscoplastic deformation is the state of stress. Then the viscoplastic strain rate at time t_{n+1} can be approximated by the first terms of a Taylor series expansion

$$\dot{\varepsilon}^{vp}_{n+1} \simeq \dot{\varepsilon}^{vp}_{n} + \left(\frac{\partial \dot{\varepsilon}^{vp}}{\partial \sigma}\right)_n \Delta\sigma_n \quad (8)$$

Substituting Eq.(8) into Eq.(7), the viscoplastic stain increment can be formulated by the variables at the current time station as

$$\Delta\varepsilon^{vp}_{n} = \left[\,\dot{\varepsilon}^{vp}_{n} + \alpha\left(\frac{\partial \dot{\varepsilon}^{vp}}{\partial \sigma}\right)_n \Delta\sigma_n\right] \Delta t_n \quad (9)$$

The stress increment vector $\Delta\sigma$ is related to the elastic strain increment vector $\Delta\varepsilon^{e}$ through the elastic stiffness matrix D^{e} as

$$\Delta\sigma = D^{e}\Delta\varepsilon^{e} \quad (10)$$

where

$$\Delta\varepsilon^{e} = \Delta\varepsilon - \Delta\varepsilon^{vp} \tag{11}$$

Substituting Eqs. (9) and (11) into Eq.(10), we may obtain the relationship between the incremental stress and the incremental strain :

$$\Delta\sigma = D^{ev}(\Delta\varepsilon - \dot{\varepsilon}^{vp}\Delta t) \tag{12}$$

where

$$D^{ev} = \left((D^{e})^{-1} + \alpha \frac{\partial \dot{\varepsilon}^{vp}}{\partial \sigma} \Delta t \right)^{-1} \tag{13}$$

The stress increment is expressed in terms of the strain increment, time step length, viscoplastic strain rate and state of stress, and thus the relation would be suitable for the rate dependent materials such as concrete and rock. Although Eq.(13) does not have the common form of stress-strain relationships for either linear elastic or plastic material, the implicit time integration scheme can be applicable by treating the term containing $\Delta t\dot{\varepsilon}^{vp}$ as a pseudo-load.

Softening Behavior

After the stress path reaches the bounding failure surface, the softening behavior starts and stress decreases even when strain increases. In the finite element analysis of concrete structures, the softening behaviors have been represented by two types of cracking. One is the discrete representation of cracks and the other is the smeared cracking model (Comi et al., 1992). In the discrete model, the major shortcoming for implementing in the finite element program are the change of connectivities and the constraint on the direction of crack propagation. For the distributed fracture, the smeared crack models have been widely used. This approach treats the cracked material as an equivalent continuum.

Although the smeared crack models have been widely used in analyzing the softening behavior of concrete structures, they may not cover the various loading conditions because of the lack of representative constitutive models and of the spurious mesh-size sensitivity relating to the fracture energy concept.

For a plastic material, the incremental stress-strain law can be written in the form,

$$d\sigma = D^{p} d\varepsilon$$

where D^{p} is the plastic moduli matrix. This constitutive equation means that stress increases with increasing strain up to its peak and that at peak, the tangent moduli matrix D^{p} becomes a null matrix because stress increments are zero for finite strain increments. The null moduli matrix can be obtained from crack moduli matrix.

In Fig. 1, point 1 on the descending part of stress-strain curve moves to point A when strain increases, while it moves to B in case of unloading. The total strain increment is the sum of elastic and inelastic strain increments and the unloading curve can be used to determine the elastic strain increment. It is clear that the actual strain increment does not contain the elastic increment during descending behavior. Hence, one can speculate that the actual strain increment is less than the nonlinear and irreversible strain increment. Then the descending behavior can be described by the viscoplastic constitutive equation Eq(12) as

$$\Delta\sigma = D^{ev}(\Delta\varepsilon - \dot{\varepsilon}^{vp}\Delta t)$$

The ascending and descending parts can be described by

$$|\Delta\varepsilon| - |\dot{\varepsilon}^{vp}\Delta t| \begin{cases} > 0 & \text{; for ascending} \\ = 0 & \text{; at ultimate stress point} \\ < 0 & \text{; for descending} \end{cases}$$

This expression indicates that in the viscoplastic region, the equivalent viscoplastic strain increment $\dot{\varepsilon}^{vp}\Delta t$ increases with increasing stress level and that in the post-peak region, $\dot{\varepsilon}^{vp}\Delta t$ continues to increase. In a word, the equivalent viscoplastic strain increment increases as long as strain increase.

Therefore, it is possible to numerically simulate not only the hardening behavior but also the softening behavior of concrete structures by the application of a viscoplastic theory alone, without a cracking model.

EQUATIONS OF MOTION

For the numerical solution of an elastic-viscoplastic problem, both explicit and implicit time integration schemes can be employed to integrate the rate dependent constitutive equations. The equations of motion in the finite element system have been formulated by applying the principle of virtual work and the constitutive equations in semi-discrete form (Moran, 1987) as

$$M\ddot{X}+C\dot{X}= F^{ext}- F^{int} \tag{14}$$

where M and C are the global mass and damping matrices,

$$M = \int_V \rho\, N^T N\, dV$$
$$C = \int_V c\, N^T N\, dV$$

and F^{ext} and F^{int} denote the externally applied load vector and the internally resisting nodal force vector as

$$F^{ext} = \int_A N^T T\, dA + \int_V N^T f\, dV$$
$$F^{int} = \int_V B^T \sigma\, dV$$

N and B are the matrices of interpolation functions and their derivatives, respectively.

In some cases the excitation is caused by prescribed displacement at the supports or boundaries of the structure. The column matrix of displacement X then can be partitioned into the prescribed displacements X_B and the displacements of remaining nodes X_I giving

$$X=\begin{pmatrix} X_I \\ X_B \end{pmatrix}$$

Partition of Eq. (14) in a similar manner gives

$$\begin{pmatrix} M_{II} & M_{IB} \\ M_{BI} & M_{BB} \end{pmatrix}\begin{pmatrix} \ddot{X}_I \\ \ddot{X}_B \end{pmatrix} + \begin{pmatrix} C_{II} & C_{IB} \\ C_{BI} & C_{BB} \end{pmatrix}\begin{pmatrix} \dot{X}_I \\ \dot{X}_B \end{pmatrix} = \begin{pmatrix} F_I^{ext} \\ F_B^{ext} \end{pmatrix} - \begin{pmatrix} F_I^{int} \\ F_B^{int} \end{pmatrix} \tag{15}$$

Usually there are no external loads applied to the interior of a structure, $F_I^{ext}=0$, and the external loads applied at the prescribed nodes are not well determined.

Separating out Eq. (15) corresponding to the internal nodal points gives

$$M_{II}\ddot{X}_I + C_{II}\dot{X}_I = -F_I^{int} - M_{IB}\ddot{X}_B - C_{IB}\dot{X}_B \tag{16}$$

Since the prescribed displacements, X_B, are given by the right hand side can be calculated.

Integration of equations of motion (16) is performed by means of the explicit central difference scheme which can be considered as a special case of the well-known Newmark family of algorithms. When the explicit integration scheme is employed, the lumped mass matrix in general could be enough to appropriately estimate the dynamic behavior of structures. However, in this study, the consistent mass matrix must be computed because M_{IB} is used to calculate the load vector in Eq.(16).

The essential feature of any time integration scheme is the approximation of the velocity and acceleration by difference equations in time. For the Newmark family of algorithms, the difference equations are given by

$$\begin{aligned} X_{n+1} &= X_n + \dot{X}_n \Delta t + \frac{\Delta t^2}{2}\{(1-2\beta)\ddot{X}_n + 2\beta\ddot{X}_{n+1}\} \\ \dot{X}_{n+1} &= \dot{X}_n + \Delta t[(1-\eta)\ddot{X}_n + \eta\ddot{X}_{n+1}] \end{aligned} \tag{17}$$

with the initial conditions, X_0 and $\dot{X}_0$. The acceleration $\ddot{X}_{n+1}$ are obtained from the solution of Eq.(16), and the velocity $\dot{X}_{n+1}$ and displacement X_{n+1} are computed through Eq. (17). The parameters β and η determine the stability and accuracy of solutions. The central difference scheme is obtained by setting $\beta=0$ and $\eta=\frac{1}{2}$.

NUMERICAL SIMULATION

The formulations, constitutive model and the equation of motion, developed in the preceding sections have been implemented in the finite element program to simulate the behavior of concrete structures under the triaxial stress states. In this section, a procedure of the computer implementation will be described, followed by numerical examples. The solutions will be compared with recorded responses of a few selected laboratory experiments, in order to demonstrate the applicability of the formulations developed herein.

Computer Implementation

The essential steps outlining the implementation of the solution processes can be summarized as follows. Solution to the problem must begin from the known initial conditions at t=0. At this moment, the displacements, velocities and stresses (or strains) are known, but the viscoplastic strains do not exist in the structure. The solution flow adopted is as follows;

1. Initial conditions (n=0) ; X_0, $\dot{X}_0$
   At first, stresses or strains are obtained, and then solve for the accelerations of internal nodes at t=0 using Eq.(16) as

$$\ddot{X}_0 = M_{II}^{-1}(-F_0 - C_{II}\dot{X}_0 - M_{IB}\ddot{X}_0 - C_{IB}\dot{X}_0)$$

2. Using the central difference method, compute the displacements in Eq.(17) at the $n+1^{th}$ time station.
3. Compute strains and viscoplastic strain-rate if viscoplastic straining occurs

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i})$$
$$\dot{\varepsilon}^{vp} = \gamma\langle Y\rangle\frac{\partial Y}{\partial\sigma}$$

4. Compute stress increment and update stresses

$$\Delta\sigma = D^{ev}(\Delta\varepsilon - \dot{\varepsilon}^{vp}\Delta t)$$
$$\sigma_{n+1} = \sigma_n + \Delta\sigma$$

   where the moduli matrix D^{ev} depends on the corresponding behavior.
5. Compute the internally resisting force vector

$$F_{I,n+1}^{int} = \int_V B^T\sigma_{n+1}\,dV$$

6. Calculate the displacements of controlled nodes

$$X_{n+1}^B = X_n^B + \Delta X^B$$

   and their velocities $\dot{X}_{n+1}^B$ and accelerations $\ddot{X}_{n+1}^B$ and compute the associated load in Eq.(16).
7. Solve Eq. (16) for accelerations of internal nodes, and then calculate their velocities from Eq. (17).
8. If n+1 is less than the total number of steps N, go to 2. Otherwise stop.

Numerical Results

A constitutive model developed in this study has been implemented in the finite element analysis program. In order to validate the model, the behaviors of concrete under uniaxial, biaxial and triaxial states of loadings have been simulated by imposing the prescribed displacements to the finite element model as shown in Fig. 2. The material data are given as follows;

Young's Modulus $E = 25,357\ MPa$ Compressive strength $f_c = 27.94\ MPa$

Tensile strength $f_t = 2.8\ MPa$ Biaxial strength $f_{bc}/f_c = 1.16$

The five parameters model of Willam-Warnke (Willam and Warnke, 1974) that is based on five experimental data points which uniquely define the meridional and deviatoric planes are chosen to describe the behavior of concrete. From that the yield function is formulated according to the isotropic hardening rule.

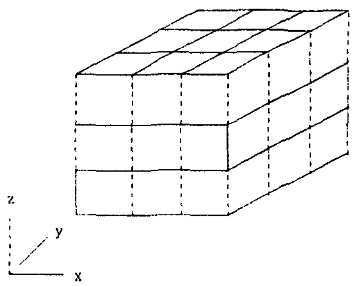

Fig. 2 The finite element model of concrete brick

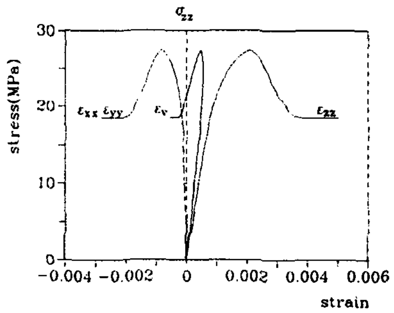

Fig.3 Uniaxial compressive stress-strain curves under the displacement imposed in the z-direction; ε_v=volumetric strain.

An uniaxial compressive stress-strain behavior is simulated by the monotonically imposed displacement in the z-direction, and the result is shown in Fig. 3. The figure shows a uniaxial compressive stress-strain curve($\sigma_{zz} - \varepsilon_{zz}$), and the lateral and volumetric strains versus stress σ_{zz}. The lateral strains are ε_{xx} and ε_{yy}, and the volumetric strain is ε_v. The uniaxial stress-strain curve, $\sigma_{zz} - \varepsilon_{zz}$ reproduces well the conventional compressive stress-strain curve, but the descending part of $\sigma_{zz} - \varepsilon_{zz}$ curve does not approximate as well as the ascending part. Also, the stress vs. the volumetric strain curve, $\sigma_{zz}-\varepsilon_v$ shows the effects of dilatation well. Hence, it is possible to analyze the softening behavior not only on the material level but also on the structural level by adjusting the amount of the equivalent viscoplastic strain increment to the proper measure such as stress level.

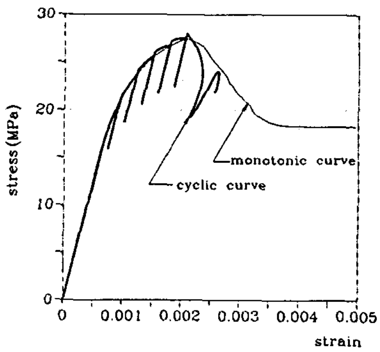

Fig. 4 Cyclic response enveloped by a monotonic curve

A cyclic displacement is imposed on the upper face of the model to investigate the behaviors of unloading and reloading in both of ascending and descending parts. The cyclic stress-strain response is enveloped by the monotonic curve in Fig. 4. The response during ascending is well reproduced, but the descending part does not approximate the cyclic behavior. As expected, the energy dissipation during ascending does not occur because the unloading and reloading are assumed to be linear elastic. The responses corresponding to reloading are greater than the monotonic responses. This result may be attributed to the effects of stress wave or inertia.

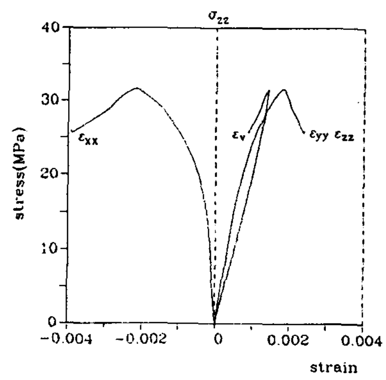

Fig. 5 Biaxial curves; $\sigma_{xx}/\sigma_{yy}/\sigma_{zz}=0/1/1$

When the uniform displacements are imposed at the nodes on the upper surface of the model, stresses at integration points in the static analysis should be same, but stresses in this analysis are not same due to the effect of inertia or stress wave. Biaxial and triaxial stress-strain curves are shown in Figs. 5 and 6, respectively. From these results, it is realized that the three-dimensional softening behavior of concrete structures can be reproduced by the viscoplastic constitutive equation with the assumption that the equivalent viscoplastic strain increments in the post-peak region are greater than the actual strain increments.

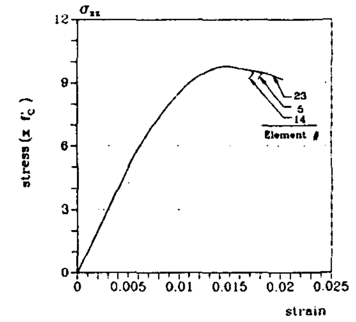

Fig. 6 Triaxial behaviors of elements 5, 14, 24;
$\sigma_{xx}/\sigma_{yy}/\sigma_{zz} = 0.25/0.25/1.0$

CONCLUSIONS

The softening of concrete has been reproduced by expanding viscoplastic region to failure and by assuming that the equivalent strain increment in the post-peak region is greater than the actual strain increment. Considering the simulated results under various loading conditions, it is felt that the model developed herein predicts the response of concrete structures with a degree of accuracy which is sufficient for practical purposes.

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