동특성 추정을 이용한 구조물의 손상도 추정
Damage Estimation of Structures Incorporating Structural Identification

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ABSTRACT

The problem of the structural identification becomes important, particularly with relation to the rapid increase of the number of the damaged or deteriorated structures, such as highway bridges, buildings, and industrial facilities. This paper summarizes the recent studies related to those problems by the present authors. The system identfication methods are generally classified as the time domain and the frequency domain methods. As time doamin methods, the sequential algorithms such as the extended Kalman filter and the sequential prediction error method are studied. Several techniques for improving the convergences are incorporated. As frequency domain methods, a new frequency response function estimator is introduced. For damage estimation of existing structures, the modal perturbation and the sensitivity matrix methods are studied. From the example analysis, it has been found that the combined utilization of the measurement data for the static response and the dynamic (modal) properties are very effictive for the damage estimation.

TIME DOMAIN IDENTIFICATION METHODS

Time domain identification is divided into two categories. One refers to the batch algorithm, and the other the sequential algorithm.The batch algorithm is generally known to be very time-consuming, especially when nonlinear optimization techniques should be used. In this paper, the extended Kalman filter and the sequential prediction error methods are discussed, which are sequential algorithms.

Extended Kalman Filetering Techniques (Ref. 6 and 8)

The Kalman filtering technique is the method of the state etsimation for a linear system based on the measurement squence of the system output which is usually contaminated with noises. It is accomplished by way of minimizing the mean square estimation error. Its extension to the nonlinear system is called as the extended Kalman filtering(EKF), which consists of linearization of the nonlinear state equation and state estimation using the Kalman filter at each time step. By introducing an augmented state vector composed of the state variables and the unknown parameters to be identified, the EKF technique can be used for the parameter estimation of the linear as well as the nonlinear systems. Since the EKF is basically for nonlinear systems, other methods such as the sequential prediction error methods may be more efficient for linear systems.

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Sequential Prediction Error Method (Ref. 3,7,9 and 10)

For the parameter estimation of the a linear structure, the equation of motion may be transformed into an auto-regressive and moving average with auxiliary input(ARMAX) model. It is in terms of the measured time histories of the response and the excitation. Then the prediction error model can be constructed as

$$y(k) = W(k)^T\theta + e(k,\theta) \tag{1}$$

where $y(k)$ is the response measurement vector at $t = k\Delta t$, θ is the vector of the unknown parameters, $W(k)$ is the regression matrix composed of the measurement data up to $t = k\Delta t$, and $e(k,\theta)$ is the error in the response prediction.

The unknown parameters can be determined by minimizing the summation of the prediction error as

$$V_{k+1}(\theta) = V_k(\theta) + \frac{1}{2}e(k+1,\theta)^T e(k+1,\theta) \tag{2}$$

which results in the following sequential algorithm for θ

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\Psi(k)\{e(k+1,\hat{\theta}(k))\} \tag{3}$$

$$F(k+1)^{-1} = F(k)^{-1} + \Psi(k)\Psi(k)^T \quad with \quad F(0) = \beta \cdot I \ (\beta > 0) \tag{4}$$

$$\Psi(k) = -\left.\frac{de(k+1,\theta)}{d\theta}\right|_{\theta=\hat{\theta}(k)} \tag{5}$$

where $\hat{\theta}(k)$ denotes the estimates of θ at $t = k\Delta t$; $\Psi(k)$ is the matrix of the negative gradient vectors of the prediction error $e(k)$; $F(k)$ is the adaptation gain matrix which is an inverse of the second derivative matrix of the criterion function $V_k(\theta)$.

Techniques for Improving Convergences (Ref. 3,7,9 and 10)

Many techniques were developed to improve the convergence and the robustness of the sequential estimation methods. Among them the exponential data weighting and square root estimation of adaptation gain matrix are found to be very effective.

Exponential Data weighting It is generally true that the prediction error associated with the recent estimate is more informative for the next estimation than the past ones. The concept can be incorporated into the criteria function by introducing a weighting $a(k)$ as

$$V_{k+1}(\theta) = a(k)\, V_k(\theta) + \frac{1}{2}e(k+1,\hat{\theta})^T e(k+1,\hat{\theta}) \tag{6}$$

where $a(k) < 1$. Using the above criteria function, an enhanced algorithm for the adaptation gain matrix can be obtained as

$$F(k+1)^{-1} = a(k)\, F(k)^{-1} + \Psi(k)\, \Psi(k)^T \tag{7}$$

Global Data weighting The data weighting technique can be also applied globally,then a modified algorithm for $F(k)$ is obtained as

$$F(N_0+1)^{-1} = wF(N_0)^{-1} + \Psi(N_0)\Psi(N_0)^T \tag{8}$$

where N_0 is the number of the measured time data points in a record, and w is the weighting parameter ($w < 1$).

Square Root Estimation of Adaptation Gain Matrix With the sequential algorithm, inversions of the matrices with the dimension of the output are required as in Eq. 4. Series of the matrix inversions may cause the adaptation gain matrix to lose the positive definiteness, especially when the quality of the measurement data and/or the concurrent estimates for the unknown parameters are poor. If the positive definiteness of the matrix is lost, the sequential prediction error algorithm could diverge to the direction of increasing the prediction error. In this study, the square root estimation algorithm is employed to insure the positive definiteness of the adaptation gain matrix. In this technique, the matrix inversion is carried out by using the triangular factorization, in which the diagonal elements are estimated by taking square roots of the positive qualities. Details of the technique can be found in *References* 7,9 and10 .

Example Analysis

In order to investigate the effectiveness of the techniques presented in this study, example studies are carried out for an idealized case with 2 degrees of freedom. Artificial time histories of two excitations and two acceleration responses are generated based on the assumed exact ARMAX parameters. The parameters are estimated by the sequential algorithm incorporating the techniques described in the previous sections, and the results are compared with the assumed exact values. The initial guesses for the parameters and the adaptation gain matrix are taken as ; $\hat{\theta}(0) = 0$ and $F(0) = 100I$. *Figures* 1 and 2 show the processes of the sequential parameter estimations by incorporting the techniques above. From the estimated parameters, it can be observed that, by using the exponential data weighting technique, the better estimates can be obtained. It is noted that divergence problem has been experienced as shown in *Figure 2*, if the square root estimation technique is not utilized. The experimental study is also performed for a 3 story shear building model as shown in *Figure 3.* The estimated modal parameters from different sets of measurement data are found to be very consistent as shown in *Table 1.*

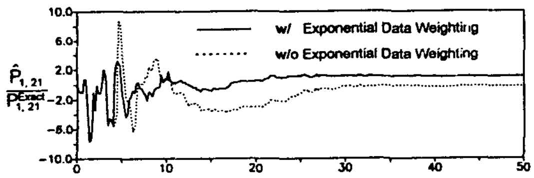


Fig. 1 Effect of Exponential Data Weighting on ARMAX parameters for a 2-DOF Case

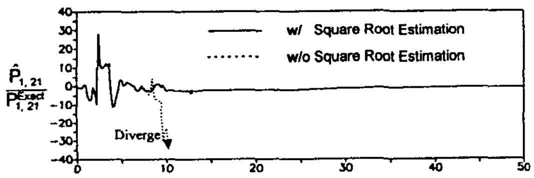


Fig. 2 Effect of Square Root Estimation on ARMAX parameters for a 2-DOF Case

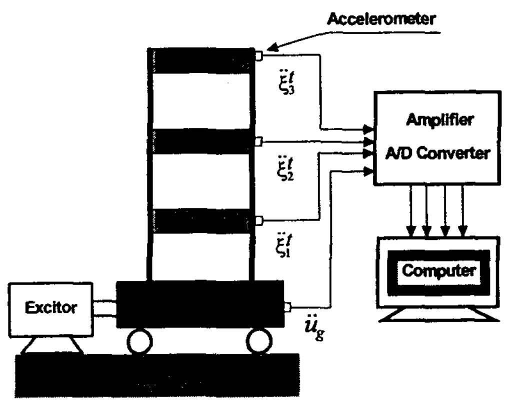


Fig. 3 Experimental Set-up of a Building Model

Table 1 Estimated Modal Parameters of Building Model

Modal Properties		Experiment I		Experiment II		Experiment III		Analysis Using Shear Building Model	
f_1	ξ_1	3.585	0.0051	3.671	0.0083	3.596	0.0175	4.06	—
f_2	ξ_2	9.979	0.0013	9.979	0.0042	9.969	0.0054	11.32	—
f_3	ξ_3	14.779	0.0029	14.838	0.0000	14.784	0.0028	16.49	—
Ψ_1	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	0.814	(8)	0.783	(1)	0.819	(3)	0.802	(0)
	1F	0.507	(1)	0.403	(11)	0.421	(26)	0.445	(0)
Ψ_2	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	0.470	(176)	0.323	(200)	0.412	(160)	0.554	(180)
	1F	1.177	(180)	1.004	(195)	1.082	(175)	1.250	(180)
Ψ_3	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	2.119	(177)	1.734	(161)	2.076	(157)	2.242	(180)
	1F	1.495	(3)	1.546	(3)	1.520	(15)	1.801	(0)
Input Types		Implusive		Long Duration		Implusive			

Note : 1. The unit of f_i is Hz

2. Values in parentheses denote the phase angles of the complex modes in degrees

FREQUENCY DOMAIN IDENTIFICATION METHODS

Frequency Response Function Estimators (Ref. 1,2 and 4)

Frequency response functions (FRF) are the most fundamental data for the frequency domain identifications of structural systems. Several techniques have emerged for the estimation of FRF's in such a way to minimize the effect of measurement noise. By converting many samples of measured time history records into the frequency domain, the conventional FRF's, which usually designated as $H_1(f)$ and $H_2(f)$[1], are obtained from the relations between the averaged power spectra and the cross spectra of the excitation and response measurement records. In a recent paper, Fabunmi and Tasker [2] introduced a new estimator $H_3(f)$, which is based on the minimization of the measurement noises. More recently, an improved FRF estimator $H_4(f)$ was proposed by the current author [4] as

$$H_4(f) = (1 - W(f))\, H_1(f) + W(f)\, H_2(f) \tag{8}$$

where $W(f)$ is a weighting function. Considering that $H_2(f)$ is more accurate at resonances while $H_1(f)$ is better at antiresonances, $H_4(f)$ is designed to have the characteristics that it approaches $H_2(f)$ at resonances and $H_1(f)$ at antiresonances by selecting the weighting function appropriately. An exponential weighting function is used, and the shape of the weighting function is determined by way of minimizing the resolution bias error over the significant frequency range.

Example Analysis

Numerical investigation is performed for a 3-span bridge model as shown in *Figure* 4. The results in Figure 5 indicate that $H_4(f)$ is more accurate than the other FRF estimators.

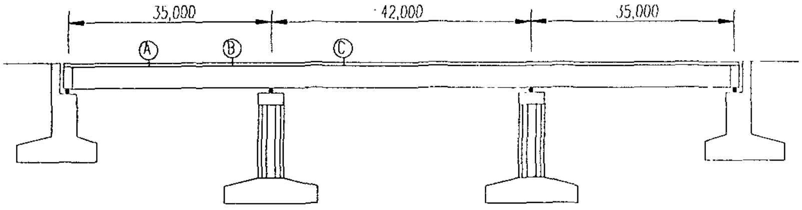
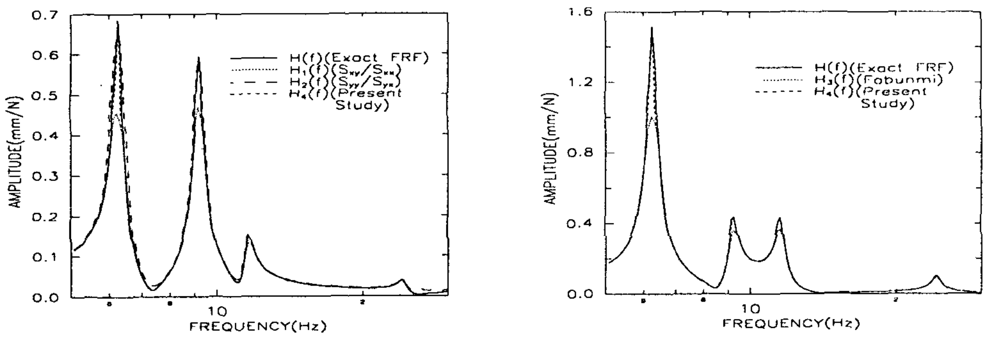


Fig 4. Sketch of a continuous 3-span bridge model (in mm)



(a) FRF's at Point B (b) FRF's at Point C

Fig 5 . Frequency response functions of a 3-span bridge model at Points B and C for an exciation at Point A (using Hanning Window)

DAMAGE ESTIMATION OF STRUCTURES

For the safety evaulation of the existing structures, it is very important to estimate the degree of damage or deterioration. Advanced system identification techniques were extensively applied to the aero-space structures. However, for the civil engineering structures, the safety evaluation has relied upon conventional methods, such as visual inspection, non-destructive material tests on critical locations, monitoring of static strain and deflection. The information on the changes of dynamic characteristics of the structures may be incorporated for the improvement of the safety evaluation. In this paper, the inverse modal perturbation and the sensitivity methods are discussed.

Inverse Modal Perturbation Method (Ref. 5)

The properties of a structure which has suffered damage can be defined by introducing perturbations to the original values as

$$K' = K + \Delta K, \quad M' = M + \Delta M$$
$$\Lambda' = \Lambda + \Delta\Lambda, \quad \Phi' = \Phi + \Delta\Phi \qquad (9)$$

where K, M, Λ and Φ are the matrices of the stiffness, mass, eigenvalues (or squares of natural frequencies), and eigenvectors (or mode shapes) of the original structure, respectively, and K', M', Λ', and Φ' are those of the damaged structure. Δ denotes the corresponding perturbation due to damage. The second order perturbation equation for the eigenvalue problem can be derived as

$$\Phi^T \Delta K \Phi' = M(C^T\Lambda' - \Lambda' C^T + \Delta\Lambda) \qquad (10)$$

where $\mathcal{M} = \Phi^T M \Phi$ is the generalized mass matrix of the original structure, the term C_{ij} relates the participation of the j-th mode of the original structure to the change in the i-th mode i.e. $\Delta\phi_i = \sum_{j=1}^{n} C_{ij}\,\phi_j$. If only small perturbations which are of order Δ are considered, the first order perturbation equation can be obtained from equation (10) as

$$\Phi^T \Delta K \Phi = \mathcal{M}(C^T\Lambda - \Lambda C^T + \Delta\Lambda) \tag{11}$$

To deal with the damage estimation problem effectively, a practical interpretation must be given to the structural changes due to the structural damage, ΔK. The stiffness changes may be decomposed into l-element stiffness changes, where l is the total number of the element stiffness components which are suspected to get damage. Then, ΔK becomes

$$\Delta K = \sum_{e=1}^{l} K_e\,\alpha_e \tag{12}$$

where α_e represents the fractional change(damage coefficient) in the e-th element stiffness component : $-1 \leq \alpha_e \leq 0$.

This perturbation equation can be solved by way of minimizing the estimation error with respect to the damage coefficient α_e's. The objective function may be taken as the sum of squared errors among the modal perturbation matrix equations, which are related to the changes of the natural frequencies in the perturbation equation;

$$\text{min. } J = \sum_{k=1}^{n_k}\left\{\sum_{e=1}^{l}\phi_k^T K_e\,\phi_k^l\alpha_e - \mathcal{M}_k(1 + C_{kk})\Delta\lambda_k\right\}^2 \tag{13}$$

where n_k is the number of the measured natural frequencies. On the other hand, the constraint equations for the above quadratic optimization problems may consist of the equations related to the mode shape changes in the perturbation equation.

Sensitivity Method

This method uses the sensitivity matrices which relate the changes of structural properties of interest to the changes of structural parameters related to damage. For the case of natural frequencies, mode shapes and static deflection, the relations can be written as

$$\{\delta\omega^2\} = [\bar{S}]\{\delta p\} \tag{14}$$

$$\{\delta\Phi_k\} \cong [\tilde{S}]_k\,\{\delta p\} \tag{15}$$

$$\{\delta u\} \cong [\hat{S}]\{\delta p\} \tag{16}$$

where $\{\delta p\}$ is the small variation of the parameters, $[\bar{S}]$, $[\tilde{S}]$ and $[\hat{S}]$ are the sensitivity matrices as follow

$$\bar{S}_{ij} = \frac{\sum_{l=1}^{N_{dof}}\sum_{m=1}^{N_{dof}}\sum_{e=1}^{N_e}\left(\phi_{li}\phi_{mi}\frac{\partial K_{lm}^e}{\partial p_j}\right) - \omega_i^2\sum_{l=1}^{N_{dof}}\sum_{m=1}^{N_{dof}}\sum_{e=1}^{N_e}\left(\phi_{li}\phi_{mi}\frac{\partial M_{lm}^e}{\partial p_j}\right)}{\sum_{l=1}^{N_{dof}}\sum_{m=1}^{N_{dof}}\sum_{e=1}^{N_e}\left(\phi_{li}M_{lm}^e\phi_{mi}\right)} \tag{17}$$

$$\tilde{S}_{nkj} = \sum_{q=1}^{N_{sps}} \left(\frac{\sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lq} \phi_{mk} \phi_{nq} \frac{\partial K^e_{lm}}{\partial p_j} \right)}{(\omega_k^2 - \omega_q^2) \sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lq} M^e_{lm} \phi_{mq} \right)} (1 - \delta_{qk}) \right)$$

$$- \sum_{q=1}^{N_{sps}} \left(\frac{\omega_k^2 \sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lq} \phi_{mk} \phi_{nq} \frac{\partial M^e_{lm}}{\partial p_j} \right) (1 - \delta_{qk})}{(\omega_k^2 - \omega_q^2) \sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lq} M^e_{lm} \phi_{mq} \right)} + \frac{\sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lk} \phi_{mk} \phi_{nq} \frac{\partial M^e_{lm}}{\partial p_j} \right) (\delta_{qk})}{2 \sum_{l=1}^{N_{dof}} \sum_{m=1}^{N_{dof}} \sum_{e=1}^{N_e} \left(\phi_{lk} M^e_{lm} \phi_{mk} \right)} \right) \quad (18)$$

$$[\hat{S}] = -[K]^{-1} [\frac{\partial K}{\partial p}] \{u\} \quad (19)$$

Using a similar procedure to those in the perturbation method, the objective function and constraint conditions can be constructed. Then the parameters can be determined by way of minimizing the objective function.

Example Analysis

Numerical investigations were carried out for a truss shown in *Figure 6*. The damage coefficients estimated by the first and second order perturbation equations indicated that the results by the first order perturbation do not improve significantly, though the number of the natural frequencies included in the analysis increases. On the other hand, the results by the second order perturbation get improved significantly even in the case using only a few natural frequency measurements. *Figure 7* shows the estimated element damages of a case with 5 damaged elements. The results were obtained using the sensitivity methods iteratively. In general, the estimates are found to improve significantly, if the static deflection measurements and the modal property changes are incorporated.

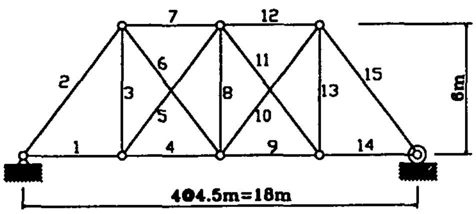


Figure 6 Truss model

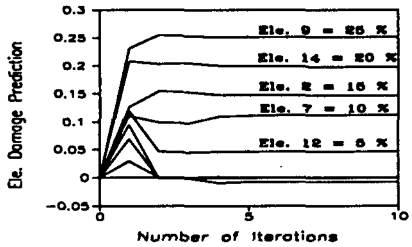


Figure 7 Estimates of Damage Parameters

CONCLUDING REMARKS

In this paper, time and frequency domain identification methods of structural systems and their application to the damage estimation of the existing structures are discussed. Several techniques for improving the estimation algorithm are also investigated. From the numerical example studies, it has been found that a combined utilization of the static and the dynamic response measurement data is very effective for the estimation of structural damage.

Table 2. Comparison of Methods for Damage Assessment

Methods		Perturbation		Sensitivity		
Element	Exact	1st Order	2nd Order	Dynamic	Dynamic + Static	
# of Mode Shape		(5)*	(5)*	(5)*	(2)*	(5)*
1	0.00	0.00	0.00	0.01	0.00	0.00
2	**0.20**	**0.00**	**0.20**	**0.20**	**0.19**	**0.20**
3	0.00	0.00	0.00	0.01	0.00	0.00
4	0.00	0.00	0.05	-0.03	0.00	0.00
5	**0.10**	**0.00**	**0.10**	**0.13**	**0.10**	**0.10**
6	0.00	0.00	0.00	-0.01	-0.02	0.02
7	**0.30**	**0.00**	**0.30**	**0.26**	**0.31**	**0.30**
8	**0.10**	**0.10**	**0.10**	**0.10**	**0.11**	**0.09**
9	0.00	1.00	0.07	0.02	0.00	0.00
10	0.00	0.06	0.00	-0.01	-0.02	-0.01
11	**0.20**	**0.17**	**0.20**	**0.21**	**0.20**	**0.21**
12	0.00	0.00	0.00	0.08	0.06	0.00
13	0.00	0.00	0.00	-0.01	0.00	0.00
14	0.00	0.00	0.00	0.01	0.05	-0.02
15	0.00	0.01	0.00	-0.04	-0.05	0.00
Error(%)	**0.00**	**602.42**	**3.89**	**6.58**	**5.10**	**0.58**

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