Fuzzy Symmetic Groups

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Abstract :
We study fuzzy symmetric subgroups and obtain some properties of fuzzy symmetric subgroups of symmetric groups.

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1. Introduction

Symmetric groups S_n are classical in some point of view ([3] is listed in References for symmetric groups S_n) and infinite symmetric groups $S(X)$ ([4], [5]) are interesting (X denote an infinite set). After Zadeh [8, Fuzzy sets], Rosenfeld [6] initiated fuzzy groups. There are some papers (for instance, [1], [2] and [7]) on fuzzy subgroups. We define a fuzzy symmetric subgroup of symmetric group S_n and study fuzzy symmetric subgroups of symmetric groups S_n.

2. Definitions

For the sake of convenience, we recall some definitions and elementary results. G denotes a group with e as the identity element.

Definition 1. (i)[8]. A fussy subset of a set A is a function $f: A \to [0,1]$.
(ii)[6]. Let G be a group. A fuzzy subset f of G is said to be a fuzzy subgroup of G if for every $x,y \in G$, (1) $\min\{f(x), f(y)\} \leq f(xy)$ and (2) $f(x^{-1}) = f(x)$.

Theorem 1. [6]. *If f is a fuzzy subgroup of G, then $f(x) \leq f(e)$ for every $x \in G$, where e denotes the identity of the group G.*

Theorem 2. [6]. *A fuzzy subset f of G is a fuzzy subgroup of G if and only if $\min\{f(x), f(y)\} \leq f(xy^{-1})$, for every $x,y \in G$.*

Notation 1. [2] Let G be a group. Let f be a fuzzy subset of G. We introduce a notation f_t: $f_t = (x \in G:\ f(x) \geq t\}$, where $t \in [0,1]$.

Theorem 3. [2]. *Let f be a fuzzy subset of G. f is a fuzzy subgroup of G if and only if f_t is a subgroup of G, for every t with $0 \leq t \leq f(e)$.*

Definition 2. S_n denotes the symmetric group on $\{1,2,\ldots,n\}$. e denotes the identity of S_n.

(i) Let $\pi \in S_n$. $C(\pi)$ denotes the set of all λ in S_n such that $\lambda = x(\pi)x^{-1}$, $x \in S_n$. $C(\pi)$ is called the conjugacy class of S_n containing π. (See [3, page 10] for the definition of a conjugacy class.)

(ii) $F(G)$ denotes the set of all fuzzy subgroups of G.

(iii) $|A|$ denotes the cardinality of a set A.

(iv) Let $f \in F(S_n)$. We define $f(C(\pi)) = \{f(x): x \in C(\pi)\}$.

(v) $A = \{f(x_1), f(x_2), \cdots, f(y_k)\} = B$ means that $f(x) \leq f(y)$, for $f(x) \in A$, $f(y) \in B$ (Similarly, we define $A < B$ to mean that $f(x) < f(y)$.)

(vi) Let $t \in [0,1]$. $f(C(\pi)) = t$ mean that $f(x) = t$ for all $x \in C(\pi)$. This is the case, we say that $f(C(\pi))$ is constant.

(vii) Let $f \in F(G)$. Im $(f) = \{f(x): x \in G\}$ denotes the image set of f

(viii) Let $f, g \in F(S_n)$. If $|\mathrm{Im}(f)| < |\mathrm{Im}(g)|$, then we write $f < g$. Following this rule, we define $\max F(S_n)$.

(ix) Let $f = \max F(S_n)$. Then we say that f is a fuzzy symmetric subgroup (or group) of S_n.