퍼지 완전 전-irresolute와 약 완전 전-irresolute 사상

FUZZY COMPLETELY PRE-IRRESOLUTE AND WEAKLY COMPLETELY PRE-IRRESOLUTE MAPPINGS

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ABSTRACT

In this paper, we introduce fuzzy completely pre-irresolute and fuzzy weakly completely pre-irresolute mappings between fuzzy topological spaces, and study these mappings in relation to some other types of already known mappings.

1. INTRODUCTION AND PRELIMINARIES

As weaker form of fuzzy continuity, fuzzy pre-continuity were introduced and studied in [2]. Park and Park [14] introduced fuzzy pre-irresolute mapping which is stronger than fuzzy pre-continuity and is independent of the fuzzy continuity. Recently, Park and Ha [12] introduced fuzzy strongly pre-irresolute and fuzzy weakly pre-irresolute mappings which are stronger and weaker respectively than fuzzy pre-irresolute mapping, and investigated relationships between those mappings and fuzzy continuous mapping. In this paper, we introduce two new classes of mappings between fuzzy topological spaces, under the terminologies fuzzy completely pre-irresolute and fuzzy weakly completely pre-irresolute mappings, and give some characterizing theorems for these two types of mappings. Finally, the concept of fuzzy pre-connected set, by the help of fuzzy pre-closure, is introduced and studied in section 4.

Throughout this paper, by (X,τ) (or simply X) we mean a fuzzy topological space in Chang's [3] sense. A fuzzy point in X with support $x \in X$ and value α $(0 < \alpha \leq 1)$ is denoted by x_α. For a fuzzy set A in X, $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and $1 - A$ will respectively denote the closure, interior and complement of A, whereas the constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X, respectively. A fuzzy set A in X is said to be q-coincident with a fuzzy set B, denoted by AqB, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [15]. It is known [15] that $A \leq B$ if and only if A and $1 - B$ are not q-coincident, denoted by $A\bar{q}(1-B)$. For definitions and results not explained in this paper, the reader is referred to [1, 2, 6, 8, 9, 15, 17] in the assumption they are well known.

2. FUZZY COMPLETELY PRE-IRRESOLUTE MAPPINGS

Definition 2.1. A mapping $f : X \to Y$ is said to be fuzzy completely pre-irresolute if $f^{-1}(V)$ is fuzzy regularly open in X for each fuzzy preopen subset V in Y, or equivalently, $f^{-1}(V)$ is fuzzy regularly closed in X for each fuzzy preclosed subset V in Y.

Example 2.2. Let $X = [0,1]$, $\tau_1 = \{1_X, 0_X, U_1, 1-U_1, U_1 \wedge (1-U_1), U_1 \vee (1-U_1)\}$ and $\tau_2 = \{1_X, 0_X, U_1, U_2, U_1 \vee U_2\}$, where

$$U_1(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ 2x-1, & \frac{1}{2} \le x \le 1, \end{cases} \qquad U_2(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{4} \\ -4x+2, & \frac{1}{4} \le x \le \frac{1}{2} \\ 0, & \frac{1}{2} \le x \le 1. \end{cases}$$

Let us consider the mapping $f : (X, \tau_1) \to (X, \tau_2)$ defined by $f(x) = \frac{1}{2}x$ for all $x \in X$. Then f is fuzzy completely continuous but not fuzzy completely pre-irresolute.

Example 2.3. Let $X = \{a, b, c\}$, $\tau_1 = \{1_X, 0_X, U_1, U_2, U_1 \vee U_2\}$ and $\tau_2 = \{1_X, 0_X, U_1, U_3\}$, where

$$\begin{aligned} &U_1(a) = 0.4, \quad U_1(b) = 0, \quad U_1(c) = 0, \\ &U_2(a) = 0, \quad U_2(b) = 0.4, \quad U_2(c) = 0, \\ &U_3(a) = 0.4, \quad U_3(b) = 0.4, \quad U_3(c) = 0. \end{aligned}$$

If we consider mapping $f : (X, \tau_1) \to (X, \tau_2)$ defined by $f(a) = b, f(b) = a$ and $f(c) = c$, then f is fuzzy pre-irresolute but not fuzzy completely pre-irresolute.

Theorem 2.4. *If a mapping $f : X \to Y$ is fuzzy completely pre-irresolute, then the following are equivalent:*

(a) For each fuzzy point x_α in X and each fuzzy preopen pre-q-nbd V of $f(x_\alpha)$, there exists a fuzzy regularly open q-nbd U of x_α such that $f(U) \le V$.

(b) For each fuzzy set A in X, $f([A]_\delta) \le pCl(f(A))$.

(c) For each fuzzy set B in Y, $[f^{-1}(B)]_\delta \le f^{-1}(pCl(B))$.

(d) For each fuzzy preclosed set V in Y, $f^{-1}(V)$ is a fuzzy δ-closed set in X.

(e) For each fuzzy preopen set V in Y, $f^{-1}(V)$ is a fuzzy δ-open set in X.

Theorem 2.5. *If $f : X \to Y$ is fuzzy completely pre-irresolute and $g : Y \to Z$ is fuzzy pre-irresolute, then $g \circ f : X \to Z$ is fuzzy completely pre-irresolute.*

Theorem 2.6. *If $f : X \to Y$ is a fuzzy completely pre-irresolute surjection and X is fuzzy nearly compact space, then Y is fuzzy strongly compact.*

3. Fuzzy weakly completely pre-irresolute mappings

Definition 3.1. A mapping $f : X \to Y$ is said to be fuzzy weakly completely pre-irresolute if $f^{-1}(V)$ is fuzzy regularly open in X for each fuzzy pre-θ-open subset V in Y, or equivalently, $f^{-1}(V)$ is fuzzy regularly closed in X for each fuzzy pre-θ-closed subset V in Y.

Example 3.2. Let $X = \{a, b\}$, $\tau_1 = \{1_X, 0_X, U_1, U_2, U_1 \wedge U_2\}$ and $\tau_2 = \{1_X, 0_X, U_3\}$, where $U_1(a) = \frac{2}{3}$, $U_1(b) = 1$, and $U_2(a) = 1$, $U_2(b) = 0$, and $0 < U_3(a) \le \frac{1}{3}$, $U_3(b) = 0$. If we consider identity mapping $f : (X, \tau_1) \to (X, \tau_2)$, then f is fuzzy weakly completely pre-irresolute but not fuzzy completely pre-irresolute.

Example 3.3. Let $X = \{a,b\}$, $\tau_1 = \{1_X, 0_X, U_1\}$ and $\tau_2 = \{1_X, 0_X, U_1, U_2, U_1 \vee U_2\}$, where $U_1(a) = 0$, $U_1(b) = 1$ and $U_2(a) = \frac{1}{4}$, $U_2(b) = 0$. If we consider identity mapping $f : (X, \tau_1) \to (X, \tau_2)$, then f is fuzzy weakly pre-irresolute but not fuzzy weakly completely pre-irresolute.

Theorem 3.4. *If a mapping $f : X \to Y$ is fuzzy weakly completely pre-irresolute, then the following are equivalent:*

(a) For each fuzzy point x_α in X and each fuzzy pre-θ-nbd V of $f(x_\alpha)$, there exists a fuzzy regularly open q-nbd U of x_α such that $f(U) \leq V$.

(b) For each fuzzy set A in X, $f([A]_\delta) \leq [f(A)]_{p\text{-}\theta}$.

(c) For each fuzzy set B in Y, $[f^{-1}(B)]_\delta \leq f^{-1}([B]_{p\text{-}\theta})$.

(d) For each fuzzy pre-θ-closed set V in Y, $f^{-1}(V)$ is a fuzzy δ-closed set in X.

(e) For each fuzzy pre-θ-open set V in Y, $f^{-1}(V)$ is a fuzzy δ-open set in X.

Theorem 3.5. *If a mapping $f : X \to Y$ is fuzzy weakly completely pre-irresolute and Y is fuzzy p-regular, then f is fuzzy completely pre-irresolute.*

4. Fuzzy pre-connectedness

Definition 4.1. Two non-null fuzzy sets A and B in a fts X are said to be fuzzy pre-separated if $p\mathrm{Cl}(A)\bar{q}B$ and $p\mathrm{Cl}(B)\bar{q}A$.

Theorem 4.2. *Let A and B be non-null fuzzy sets in a fts X.*

(a) If A and B are fuzzy pre-separated, and A_1 and B_1 are non-null fuzzy sets such that $A_1 \leq A$ and $B_1 \leq B$, then A_1 and B_1 are also fuzzy pre-separated.

(b) If $A\bar{q}B$ and either both are fuzzy preopen or both are fuzzy preclosed, then A and B are fuzzy pre-separated.

(c) If A and B are either both fuzzy preopen or both fuzzy preclosed and if $C_A(B) = A \wedge (1 - B)$ and $C_B(A) = B \wedge (1 - A)$, then $C_A(B)$ and $C_B(A)$ are fuzzy pre-separated.

Theorem 4.3. *Two non-null fuzzy sets A and B are fuzzy pre-separated if and only if there exist two fuzzy preopen sets U and V such that $A \leq U$, $B \leq V$, $A\bar{q}V$ and $B\bar{q}U$.*

Definition 4.4. A fuzzy set which can not be expressed as the union of two fuzzy pre-separated sets is said to be a fuzzy pre-connected set.

Example 4.5. Let $X = \{a,b\}$ and $\tau = \{1_X, 0_X, A\}$, where A is fuzzy set in X defined by $A(a) = 0.7$, $A(b) = 1$. We consider two fuzzy sets B and C in X given by $B(a) = 0.5, B(b) = 0$ and $C(a) = 0.4, C(b) = 0$. Since B and C are fuzzy pre-separated, $B = B \vee C$ is not fuzzy pre-connected. But B is fuzzy semi-connected and thus B is fuzzy connected.

Example 4.6. Let $X = \{a,b\}$ and $\tau = \{1_X, 0_X, A\}$, where A is fuzzy set in X defined by $0 < A(a) \leq \frac{1}{4}$, $A(b) = 0$. Now we consider two fuzzy sets B and C in X given by $B(a) = \frac{1}{2}$, $B(b) = 0$ and $C(a) = \frac{1}{3}$, $C(b) = 0$. Since B and C are fuzzy semi-separated, $B = B \vee C$ is not fuzzy semi-connected. But B is fuzzy pre-connected.

Theorem 4.7. *Let A be a non-null fuzzy pre-connected set in a fts X. If A is contained in the union of two fuzzy pre-separated sets B and C, then exactly one of the following conditions (a) and (b) holds:*

(a) $A \leq B$ and $A \wedge C = 0_X$.

(b) $A \leq C$ and $A \wedge B = 0_X$.

Theorem 4.8. *Let $f : X \to Y$ be a fuzzy completely pre-irresolute surjection. If A is fuzzy connected subset in X, then $f(A)$ is also fuzzy pre-connected in Y.*

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