Robust Iterative Learning Control Algorithm

YONG-TAE KIM AND ZEUNGNAM BIEN
Korea Advanced Institute of Science and Technology

Abstract : In this paper are proposed robust *iterative learning control*(ILC) algorithms for both linear continuous time-invariant system and linear discrete-time system. In contrast to conventional methods, the proposed learning algorithms are constructed based on both time-domain performance and iteration-domain performance. The convergence of the proposed learning algorithms is proved. Also, it is shown that the proposed method has robustness in the presence of external disturbances and the convergence accuracy can be improved. A numerical example is provided to show the effectiveness of the proposed algorithm.

1. Introduction

Ever since Arimoto [1] suggested ILC methodology, there have been a number of efforts to improve and apply ILC method. In fact, ILC can be easily applied to the repetitive tasks that is in many robotic industrial operations since it requires less *a priori* knowledge about the controlled system in the controller design phase and it has the capability of modifying an unsatisfactory control input signal based on the knowledge of previous operations of the same task [2, 3, 4, 5, 8, 10, 12, 16]. Also, ILC is known to guarantee an eventual uniform tracking performance as the algorithm repetitively applies.

External disturbances such as state disturbances, measurement noise and error of initialization are inevitable in the real control systems. This disturbances can have an effect on the ILC system and make the system diverge by its iterative property. So, the robustness problem of ILC has been studied by many researchers. Heinzinger *et al.* [11] have studied the robustness properties of a class of learning control algorithm for the nonlinear system. Saab [13, 15] proved the convergence and the robustness of both P-type learnig control for the nonlinear time varing system and D-type learning control for the linear discrete-time system. Bien and Huh [7] proposed the higher-order ILC method that utilise more than one past error history contained in the trajectories generated at prior iterations and showed that the higher-order ILC can improve the convergence performance and the robustness to the disturbances by using the multiple past-history data pairs at the expense of additional storage. However, this ILC method can be applied to the dynamic system that has the direct linkage between the input and the output and there may arise some difficulty in finding the suitable weighting matrices satisfying the convergence conditions, especially when the number of past-history data pairs is large [7, 14].

In this paper, we propose new ILC algorithms based on both time-domain performance and iteration-domain performance for both linear continuous time-invariant dynamic system and linear discrete-time dynamic system. The control law based on the iteration-domain performance can improve the robustness to the disturbances by using the past-history data pair like higher-order ILC algorithm [7]. The convergence of the proposed algorithms is proved and a numerical example is given to show that the proposed method has robustness in the presence of the external disturbances and the convergence property according to parameters change is presented.

In the sequel, the following notational convention is adopted : k is the iteration number; $x(t)$, $x(i)$ are state vectors, $u(t)$, $u(i)$ are control input vectors and $y(t)$, $y(i)$ are output vectors for continuous and discrete-time systems respectively; I_r is $r \times r$ identity matrix; $||x||$ denotes the Euclidian norm of a vector x; $||A||$ denotes the induced matrix norm of a matrix A; $||f(t)||$ denotes $(f(t)^T f(t))^{\frac{1}{2}}$ for a time function $f : [0,T] \rightarrow R^n$ and $||f(t)||_\infty$ denotes $\sup_{t\in[0,T]} ||f(t)||$; and the following norms are definded:

Definition 1 *We define the λ_c norm for a time function $f : [0,T] \rightarrow R^n$*

$$||f(\cdot)||_{\lambda_c} = \sup_{t\in[0,T]} e^{-\lambda t}||f(t)||,$$

where $\lambda > 0$.

Definition 2 *We define the λ_d norm for a time function $g : [0,N] \rightarrow R^n$*

$$||g(\cdot)||_{\lambda_d} = \sup_{i\in[0,N]} a^{-\lambda i}||g(i)||,$$

where $\lambda > 0$ if $a > 1$, and $\lambda < 0$ if $a < 1$.

Remark 1 From above definitions, it is obvious that $||f||_{\lambda_c} \leq ||f||_\infty \leq e^{\lambda T}||f||_{\lambda_c}$ and $||f||_{\lambda_d} \leq ||f||_\infty \leq e^{\lambda T}||f||_{\lambda_d}$, implying that the defined λ_c, λ_d norm and $||\cdot||_\infty$ norm are equivalent [17]. Therefore, the convergence can be proved employing the defined λ_c norm and λ_d norm.

2. ILC for linear continuous time-invariant dynamic system

In this section, we present a ILC algorithm for linear continuous time-invariant dynamic systems.

Consider the linear time-invariant dynamical system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \quad (1) \\ y(t) &= Cx(t) \end{aligned}$$

where $x \in R^n, u \in R^r$ and $y \in R^r$ denote the state vector, input vector and output vector respectively. A, B and C are constant matrices with appropriate dimensions and it is assumed that CB is nonsingular.

Let x_d be the desired state trajectory which is continuously differentiable on $[0,T]$, and, W.L.O.G, assume that

$$x_d(0) \equiv 0. \quad (2)$$

Then we consider an ILC algorithm based on both time-domain performance and iteration-domain performance. At first, we consider PD-type control law in the time domain such as Oh *et al.* [6], Bien *et al.* [8] and Hwang *et al.* [9] as follows :

$$u_{k+1}(t) = u_k(t) + \Gamma[\delta \dot{y}_k(t) + \Lambda\delta y_k(t)]. \quad (3)$$

Also, we consider PD-type control law in the iteration domain as follows :

$$u_{k+1}(t) = u_k(t) + \Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t))]. \quad (4)$$

Above iteration-domain control law use the past-history data pairs like 2nd-order ILC algorithm [7]. We propose a new type of ILC of the form

$$\begin{aligned} u_{k+1}(t) &= u_k(t) + \Gamma[\delta \dot{y}_k(t) + \Lambda\delta y_k(t) \\ &\quad +\Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t))], \quad (5) \end{aligned}$$

where

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k(t), \quad (6) \\ y_k(t) &= Cx_k(t), \\ y_d(t) &= Cx_d(t), \quad (7) \\ \delta y_k(t) &= y_d(t) - y_k(t). \quad (8) \end{aligned}$$

If it is assumed that

$$||I_r - \Gamma CB|| \leq \rho < 1 \quad (9)$$

and

$$y_k(0) = y_d(0) \equiv 0 \quad k = 0,1,2,\cdots, \quad (10)$$

Arimoto's control law [1] can make the error between $y_k(t)$ and $y_d(t)$ approache to zero as $k \rightarrow \infty$. In this paper, we assume the conditions (9), (10).

Theorem 1 *Suppose that we can choose Γ such that (9), (10) holds, and that the learning law (5) is repetively applied to (1). Then, for a given desired output $y_d(t), 0 \leq t \leq T$, the learning law (5) quarantees that, for each $t \in [0,T]$*

$$\lim_{k\rightarrow\infty} y_k(t) = y_d(t). \quad (11)$$

Proof

Let $u^*(t)$ be a control input such that

$$y_d(t) = Ce^{At}x_0 + C\int_0^t e^{A(t-\tau)}Bu^*(\tau)d\tau. \quad (12)$$

The proof is completed if one can show $\lim_{k\to\infty} u_k(t) = u^*(t)$.

For this, define

$$\delta u_k(t) \triangleq u^*(t) - u_k(t). \quad (13)$$

Then it follows from (5) and (12) that

$$\begin{aligned}
\delta u_{k+1}(t) &= u^*(t) - u_k(t) + \Gamma[\delta \dot{y}_k(t) + \Lambda\delta y_k(t) \\
&\quad +\Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t))] \\
&= u^*(t) - u_k(t) + \Gamma[\delta \dot{y}_k(t) \\
&\quad +(\Lambda + \Phi)\delta y_k(t) - \Phi\Theta\delta y_{k-1}(t))] \\
&= (I - \Gamma CB)\delta u_k(t) \\
&\quad -\Gamma CA\int_0^t e^{A(t-\tau)}B\delta u_k(\tau)d\tau \\
&\quad -\Gamma(\Lambda + \Phi)C\int_0^t e^{A(t-\tau)}B\delta u_k(\tau)d\tau \\
&\quad +\Gamma\Phi\Theta C\int_0^t e^{A(t-\tau)}B\delta u_{k-1}(\tau)d\tau \\
&= (I - \Gamma CB)\delta u_k(t) - \Gamma(CA + \Lambda C \\
&\quad +\Phi C)\int_0^t e^{A(t-\tau)}B\delta u_k(\tau)d\tau \\
&\quad +\Gamma\Phi\Theta C\int_0^t e^{A(t-\tau)}B\delta u_{k-1}(\tau)d\tau
\end{aligned} \quad (14)$$

Taking the norm $\|\cdot\|$ on both side of (14), we have

$$\begin{aligned}
\|\delta u_{k+1}(t)\| &\leq \|I - \Gamma CB\| \cdot \|\delta u_k(t)\| \\
&\quad +\|\Gamma(CA + \Lambda C + \Phi C)\| \\
&\quad \cdot\int_0^t \|e^{A(t-\tau)}\|\|B\|\|\delta u_k(\tau)\|d\tau \\
&\quad +\|\Gamma\Phi\Theta C\| \\
&\quad \cdot\int_0^t \|e^{A(t-\tau)}\|\|B\|\|\delta u_{k-1}(\tau)\|d\tau \\
&= \rho\|\delta u_k(t)\| \\
&\quad +h_0\int_0^t e^{a(t-\tau)}\|\delta u_k(\tau)\|d\tau \\
&\quad +h_1\int_0^t e^{a(t-\tau)}\|\delta u_{k-1}(\tau)\|d\tau \quad (15)
\end{aligned}$$

where $\rho \triangleq \|I - \Gamma CB\|$, $h_0 \triangleq \|\Gamma(CA + \Lambda C + \Phi C)\| \cdot \|B\|$, $h_1 \triangleq \|\Gamma\Phi\Theta C\| \cdot \|B\|$, $a \triangleq \|A\|$.

By multiplying both side of (15) by $e^{-\lambda t}$ and taking the norm $\|\cdot\|_{\lambda_c}$,

$$\begin{aligned}
\|\delta u_{k+1}(t)\|_{\lambda_c} &= \sup_{t\in[0,T]} e^{-\lambda t}\|\delta u_{k+1}(t)\| \\
&\leq \rho\|\delta u_k(t)\|_{\lambda_c} \\
&\quad +h_0 \sup_{t\in[0,T]}\int_0^t e^{(a-\lambda)(t-\tau)} \\
&\quad \sup_{\tau\in[0,T]} e^{-\lambda\tau}\|\delta u_k(\tau)\|d\tau \\
&\quad +h_1 \sup_{t\in[0,T]}\int_0^t e^{(a-\lambda)(t-\tau)} \\
&\quad \sup_{\tau\in[0,T]} e^{-\lambda\tau}\|\delta u_{k-1}(\tau)\|d\tau \\
&= (\rho + h_0\frac{1 - e^{(a-\lambda)T}}{\lambda - a}))\|\delta u_k(t)\|_{\lambda_c} \\
&\quad +(h_1\frac{1 - e^{(a-\lambda)T}}{\lambda - a}))\|\delta u_{k-1}(t)\|_{\lambda_c}, \\
&\quad , \text{for} \lambda \neq a. (16)
\end{aligned}$$

Now, it is not diffcult to show that $\lim_{k\to\infty} \|\delta u_k(t)\|_{\lambda_c} = 0$, if

$$[\rho + h_0\frac{1 - e^{(a-\lambda)T}}{\lambda - a}] + [h_1\frac{1 - e^{(a-\lambda)T}}{\lambda - a}] < 1. \quad (17)$$

Noting that the inequality (16) can be represented by a non-negative sequence x_k, $k = 1, 2, 3, \cdot$, with the property

$$x_{k+2} \leq r\ x_{k+1} + s\ x_k, \quad (18)$$

where $r, s > 0$ and the convergence condition is equivalent to the condition that eigenvalues of $x_{k+2} = rx_{k+1} + sx_k$ are all in the unit circle in the complex plane, we can easily show that the above sequence converges to zero, if $r + s < 1$ holds.

Since $0 \leq \rho < 1$ by assumption, it is possible to choose λ sufficiently large so that

$$\rho + h_0\frac{1 - e^{(a-\lambda)T}}{\lambda - a} + h_1\frac{1 - e^{(a-\lambda)T}}{\lambda - a} < 1. \quad (19)$$

Thus,

$$\lim_{k\to\infty} \|\delta u_k(t)\|_{\lambda_c} = 0.$$

By definition of $\|\cdot\|_{\lambda_c}$, this implies

$$\lim_{k\to\infty} u_k(t) = u^*(t).$$

This completes the proof. ■

Theorem 1 shows that the proposed learning algorithm (5) guarantees convergence of the output in tracking as k increases.

Remark 2 The proposed algorithm (5) looks more complex than 1st-order methods [1, 2, 8, 9, 6, 16]. However, it gives more freedoms for adjustment of both convergence speed and tracking accuracy. With $\Phi = 0$, the proposed method is much the same as the learning law proposed by Oh *et al.* [6], Bien *et al.* [8] and Lee *et al.* [16]. Also, when $\Phi = 0$ and $\Lambda = 0$, the proposed method is essentially the same as the learning law proposed by Arimoto *et al.* [1]. Thus, the proposed method can be considered as a generalization of the previous works [1, 6, 8, 16]. The convergence conditions in theorem 1 are similar to the previous works [1, 6, 8, 16] and more simple than the higher-order ILC method [7].

3. ILC for linear discrete-time dynamic system

In this section, a robust ILC algorithm for linear discrete-time dynamic system is proposed.

Consider the linear discrete-time dynamical system described by

$$\begin{aligned} x(i+1) &= Ax(i) + Bu(i) \\ y(i) &= Cx(i) \end{aligned} \tag{20}$$

where $x \in R^n, u \in R^r$ and $y \in R^r$ denote the state vector, input vector and output vector respectively. A, B and C are constant matrices with appropriate dimensions and it is assumed that CB is nonsingular.

Then the ILC control law for the system(20) can be described as follows :

$$\begin{aligned} u_{k+1}(i) &= u_k(i) + \Gamma[\delta y_k(i+1) + \Lambda \delta y_k(i) \\ &\quad +\Phi(\delta y_k(i) - \Theta \delta y_{k-1}(i))] \end{aligned} \tag{21}$$

where

$$\delta y_k(i) = y_d(i) - y_k(i) \quad i = 0, 1, \cdots, N. \tag{22}$$

We assume that

$$\|I_r - \Gamma CB\| \le \rho < 1, \tag{23}$$

and

$$y_k(0) = y_d(0) \equiv 0 \quad k = 0, 1, 2, \cdots. \tag{24}$$

Theorem 2 *Suppose that we can choose Γ such that (23), (24) holds, and that the learning law (21) is repetively applied to (20). Then, for a given desired output $y_d(i), i = 0, 1, \cdots, N$, the learning law (21) quarantees that, for each $i \in [0, N]$*

$$\lim_{k\to\infty} y_k(i) = y_d(i). \tag{25}$$

Proof

Let $u^*(i)$ be a control input such that

$$y_d(i) = CA^i x_0 + C\sum_{j=0}^{i-1} A^{i-j-1} Bu^*(j). \tag{26}$$

The proof is completed if one can show $\lim_{k\to\infty} u_k(i) = u^*(i)$.

For this, define

$$\delta u_k(i) \triangleq u^*(i) - u_k(i). \tag{27}$$

Then it follows from (21) and (26) that

$$\begin{aligned} \delta u_{k+1}(i) &= u^*(i) - u_k(i) + \Gamma[\delta y_k(i+1) + \Lambda \delta y_k(i) \\ &\quad +\Phi(\delta y_k(i) - \Theta \delta y_{k-1}(i))] \\ &= u^*(i) - u_k(i) + \Gamma[\delta y_k(i+1) \\ &\quad +(\Lambda + \Phi)\delta y_k(i) - \Phi\Theta\delta y_{k-1}(i))] \\ &= (I - \Gamma CB)\delta u_k(i) \\ &\quad -\Gamma CA\sum_{j=0}^{i-1} A^{i-j-1} B\delta u_k(j) \\ &\quad -\Gamma(\Lambda + \Phi)C\sum_{j=0}^{i-1} A^{i-j-1} B\delta u_k(j) \\ &\quad +\Gamma\Phi\Theta C\sum_{j=0}^{i-1} A^{i-j-1} B\delta u_{k-1}(j) \qquad (28) \\ &= (I - \Gamma CB)\delta u_k(i) - \Gamma(CA + \Lambda C \\ &\quad +\Phi C)\sum_{j=0}^{i-1} A^{i-j-1} B\delta u_k(j) \\ &\quad +\Gamma\Phi\Theta C\sum_{j=0}^{i-1} A^{i-j-1} B\delta u_{k-1}(j) \qquad (29) \end{aligned}$$

Taking the norm $\|\cdot\|$ on both side of (29), we have

$$\begin{aligned} \|\delta u_{k+1}(i)\| &\le \|I - \Gamma CB\| \cdot \|\delta u_k(i)\| \\ &\quad +\|\Gamma(CA + \Lambda C + \Phi C)\| \\ &\quad \cdot\sum_{j=0}^{i-1} \|A^{i-j-1}\|\|B\|\|\delta u_k(j)\| \\ &\quad +\|\Gamma\Phi\Theta C\| \end{aligned}$$

$$\cdot\sum_{j=0}^{i-1}||A^{i-j-1}||||B||||\delta u_{k-1}(j)||$$

$$= \rho||\delta u_k(i)|| + h_0\sum_{j=0}^{i-1} a^{i-j-1}||\delta u_k(j)|| + h_1\sum_{j=0}^{i-1}||a^{i-j-1}||\delta u_{k-1}(j)|| \quad (30)$$

where $\rho \triangleq ||I - \Gamma CB||$, $h_0 \triangleq ||\Gamma(CA + \Lambda C + \Phi C)|| \cdot ||B||$, $h_1 \triangleq ||\Gamma\Phi\Theta C|| \cdot ||B||$, $a \triangleq ||A||$.

By multiplying both side of (30) by $a^{-\lambda i}$ and taking the norm $||\cdot||_{\lambda_d}$,

$$\begin{aligned} ||\delta u_{k+1}(i)||_{\lambda_d} &= \sup_{i\in[0,N]} a^{-\lambda i}||\delta u_{k+1}(i)|| \\ &\leq \rho||\delta u_k(i)||_{\lambda_d} + h_0 \sup_{i\in[0,N]} a^{-(\lambda-1)i} \\ &\quad \cdot\sum_{j=0}^{i-1} a^{(\lambda-1)j} \sup_{j\in[0,N]} a^{-\lambda j}||\delta u_k(j)|| \\ &\quad + h_1 \sup_{i\in[0,N]} a^{-(\lambda-1)i} \\ &\quad \cdot\sum_{j=0}^{i-1} a^{(\lambda-1)j} \sup_{j\in[0,N]} a^{-\lambda j}||\delta u_{k-1}(j)|| \\ &\leq (\rho + h_0\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})||\delta u_k(t)||_{\lambda_d} \\ &\quad +(h_1\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})||\delta u_{k-1}(t)||_{\lambda_d}. \end{aligned} \quad (31)$$

Now, we can easily show that $\lim_{k\to\infty}||\delta u_k(i)||_{\lambda_d} = 0$, if

$$[\rho + h_0\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}] + [h_1\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}] < 1. \quad (32)$$

Since $0 \leq \rho < 1$ by assumption, it is possible to choose λ sufficiently large so that

$$\rho + h_0\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1} + h_1\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1} < 1. \quad (33)$$

Thus,

$$\lim_{k\to\infty}||\delta u_k(i)||_{\lambda_d} = 0.$$

By definition of $||\cdot||_{\lambda_d}$, this implies

$$\lim_{k\to\infty} u_k(i) = u^*(i).$$

This completes the proof. ■

Remark 3 As case in the continuous time system, the proposed cantrol law (21) for the discrete-time system has similar effects. With $\Phi = 0$, the proposed method is the same as the learning law proposed by Hwang *et al.* [9] and with $\Phi = 0$ and $\Lambda = 0$, it is essentially the same as the learning law proposed by Saab [15]. Therefore the proposed method can be considered as a generalization of the previous works [9, 15]. Also, the convergence conditions in theorem 2 are similar to the previous works [9, 15].

4. Simulation Example

In the following, we shall consider linear continuous time-invariant dynamic system as that of Lee [16] :

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned}$$

Also, suppose the desired output trajectory is given, as in [16], by

$$y_d(t) = 12t(1-t) \qquad 0 \leq t \leq 1$$

and let

$$y_k(0) = y_0 = 0 \quad k = 0, 1, 2, \cdots.$$

Let us assume that we guess $CB = 1.3$, so Γ is chosen as 0.7. As shown in Figure 1, the output $y(t)$ approaches the desired output $y_d(t)$ as the control law (5) is repetitively applied. The result in Figure 2 shows $\sum_{k=1}^{10}\int_0^T e_k(t)dt$ according to the parameters, Φ and Θ. Figure 3 shows $\int_0^T e_k(t)dt$ according to the parameters, Φ and Θ, at $10th$ iteration. We can show that the tracking performance tends to heavily depend on the choice of Λ, Φ, Θ, and the proposed method is more free to adjust both convergence speed and tracking accuracy and can improve them by choosing the suitable Λ, Φ and Θ. Figure 4 and Figure 5 show that the proposed method is robust to the external disturbances such as state disturbances and measurement noise.

5. Conclusion

Robust ILC algorithms based on both time-domain performance and iteration-domain performance are presented and the convergence of the proposed algorithms is proved. Also, it is shown by a numerical example that the proposed method has robustness in the presence of external disturbances. The proposed algorithms give more freedom for adjustment of convergence speed and tracking accuracy and can be considered as a generalization of the previous works [1, 6, 8, 16, 9, 15]. Also, the convergence conditions are similar to the previous works [1, 6, 8, 16, 9, 15]. The proposed ILC will be useful when ILC is applied to real control systems in the presence of disturbance.

It should be noted that the present results are valid when the system is linear and time-invariant. Since the most of industrial processes are nonlinear and time-varying, the proposed learning algorithm should be extended to those systems. Now, we are studing this proplems.

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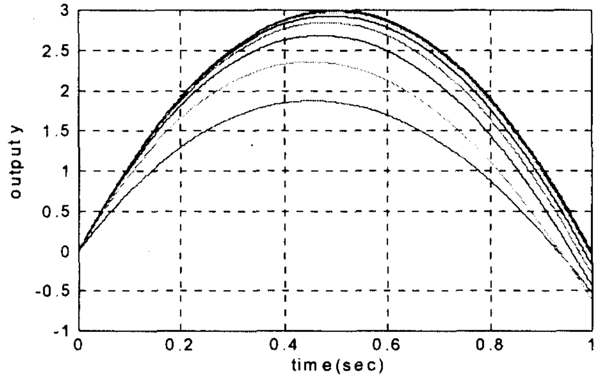
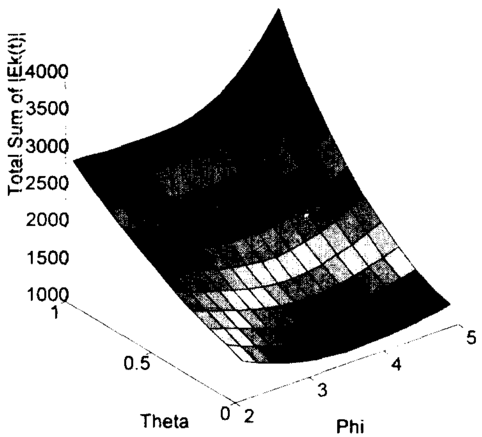


Fig. 1 Output Trajectories ($k \to \infty$) when $\Gamma = 0.7, \Lambda = 0.7, \Phi = 1.0$ and $\Theta = 0.7.$



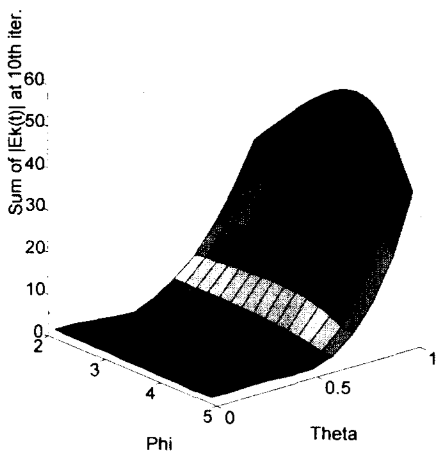
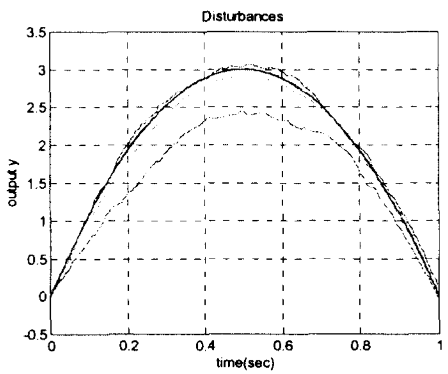


Fig. 2 $\sum_{k=1}^{10} \int_0^T |e_k(t)|\, dt$ according to parameters Change.

Fig. 3 $\int_0^T |e_{10}(t)|\, dt$ according to parameters Change



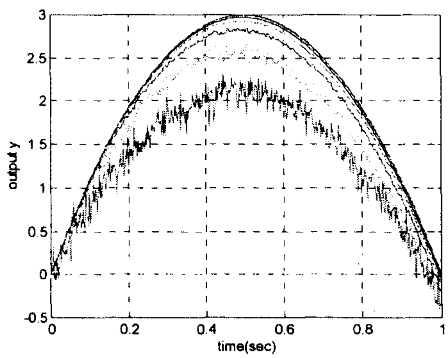


Fig. 4 Output Trajectories ($k \to \infty$)

under state disturbance

Fig. 5 Output Trajectories ($k \to \infty$)

under measurement noise