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외란 억제 및 분리기능을 갖는 고유공간 지정기법

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ABSTRACT

In this paper, in order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, a simultaneous assignment methodology of the left and right eigenstructure is proposed. The biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices are used to develop the methodology. The proposed concurrent eigenstructure assignment methodology guarantees that the desired eigenvalues are achieved exactly and the desired left and right eigenvectors are assigned to the best possible(achievable) sets of eigenvectors in the least square sense, respectively.

Key Words : disturbance decouplability, disturbance suppressibility, eigenstructure assignment

1. INTRODUCTION

The problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) is of great importance in control theory and applications because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system.[1] In general, the speed of response is determined by the assigned eigenvalues whereas the shape of the response is furnished by the assigned eigenvectors.

Eigenstructure assignment is an excellent method for incorporating classical specifications on damping, settling time, and mode decoupling into a modern multivariable control framework,[2] and has been shown to be a useful tool for flight control design.[3] The eigenstructure assignment technique is used to design flight control laws for aircraft with many control effectors, and the technique together with suitable feedforward design can achieve static decoupling with internal stability, which is an important requirement in many flight control systems.[4]

The eigenstructure assignment algorithm can be divided into two groups; the right eigenstructure (eigenvalues/right eigenv-ectors) assignment and the left eigenstructure (eigenvalues/left eigenvectors) assignment, and their roles in a system are different.[5] The right eigenstructure assignment is widely used to solve mode decoupling problems,[6,7,8] to design a controller for the vibration suppression of flexible structures,[9,10] and can be applied to disturbance decoupling problems.[11] On the other hand, the left eigenstructure is used to define the controllability measure[12] and also can be used to design an effective and disturbance suppressible controller.[13,14]

Thus, in order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, the appropriate assignment of a concurrent eigenstructure (that is, simultaneous assignment of the left and right eigenstructure) is required. In this paper, a concurrent eigenstructure assignment methodology, which is the generalized form of the previously proposed one in Ref. 14, is suggested by using the biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices. The whole procedure of the proposed methodology is simple and provides more insight into the concurrent eigenstructure assignment.

2. PROBLEM FORMULATION

Consider a linear time invariant multivariable controllable system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \tag{1}$$
$$= Ax(t) + \sum_{k=1}^{m} b_k u_k(t) + \sum_{l=1}^{n} e_l f_l(t),$$
$$u(t) = Kx(t), \tag{2}$$
$$z_j(t) = D_j x(t), \text{ for } j = 1,2 \tag{3}$$

where i) $x \in R^N, u \in R^m, f \in R^n, z_1 \in R^{r_1}$, and $z_2 \in R^{r_2}$, ($m \le N$, and $r_1 + r_2 \le N$) denote the state, control, disturbance, and controlled output vectors, respectively. And b_k and e_l are the k-th and l-th column vectors of the control input matrix B and disturbance input matrix E, respectively; (ii) A, B, E, K, and D_j are real constant matrices of appropriate dimensions; and (iii) rank $B = m \neq 0$.

The responses of the state and the controlled output of the given system due to control input $u(t)$ and disturbance $f(t)$ with zero initial conditions are represented using the modal matrices of the system by[15]

$$x(t) = \Phi \int_0^t e^{\Lambda(t-\tau)} \{ \Psi^T B u(\tau) + \Psi^T E f(\tau) \} d\tau \tag{4}$$

$$= \sum_{i=1}^{N} \phi_i e^{\lambda_i t} \left\{ \sum_{k=1}^{m} (\psi_i^T b_k) \int_0^t e^{-\lambda_i \tau} u_k(\tau) d\tau + \sum_{l=1}^{n} (\psi_i^T e_l) \int_0^t e^{-\lambda_i \tau} f_l(\tau) d\tau \right\},$$

$$z_j(t) = D_j \Phi \int_0^t e^{\Lambda(t-\tau)} \Psi^T B u(\tau) + \Psi^T E f(\tau) d\tau, \quad j = 1, 2 \tag{5}$$

where (i) $\Phi(\Psi)$ is the right(left) modal matrix of the closed-loop system, and Λ is the diagonal matrix of the desired closed-loop eigenvalues; (ii) λ_i, ϕ_i and ψ_i are the i-th eigenvalue, right and left eigenvectors of the closed-loop system, respectively, and $u_k(t)$ is the k-th control input; and (iii) D_1 in $R^{r_1 \times N}$ and D_2 in $R^{r_2 \times N}$ matrices are chosen such that the controlled outputs $z_1(t)$ and $z_2(t)$ may be composed of hopefully disturbance decoupled states and composed of (at most) the remaining states, respectively, by a designer depending on the system considered, and are called the orthogonal and parallel states selection matrices, respectively.

Note, from *Eqs.* (4), (5), that the response to the disturbance $f(t)$ can be eliminated if the columns(ψ_i) of Ψ are orthogonal to the columns(e_l) of E. Thus, for suppressing undesired disturbances, it is required that the left eigenvectors of the system lie in the space orthogonal to the columns of E. Note also that the control efforts are effectively transferred (that is, the desired maneuver is achieved with small control efforts), if the left eigenvectors are parallel to the columns(b_k) of B. Therefore, for both effective control and disturbance suppression, it is required that the left eigenvectors of the system lie simultaneously in the space orthogonal to the columns of E and parallel to the columns of B, at least, in the least square sense. Then, the corresponding system can be manipulated with small control efforts without being disturbed by the disturbance input.

On the other hand, the system is said to be *disturbance decoupled* relative to the pair $f(\cdot)$, $z_1(\cdot)$ if, for each initial state, the controlled output $z_1(t)$, $t \geq 0$, is the same for every $f(\cdot)$. Thus, disturbance decoupling simply means that the forced response

$$z_1(t) = D_1 \Phi \int_0^t e^{\Lambda(t-\tau)} \Psi^T B u(\tau) + \Psi^T E f(\tau) d\tau = 0 \tag{6}$$

for all $f(\cdot)$ and $t \geq 0$.[11] That is, from *Eq.*(6), if the right modal matrix Φ resides in the subspace of the kernel of the orthogonal states selection matrix D_1, the system (*Eqs.*(1),(6)) is disturbance decoupled. Thus, for solving *disturbance decoupling* problems, the appropriate assignment of the *right eigenstructure* of a system is required.

Meanwhile, for the r_2-states of $z_2(t)$ determined by D_2, the columns of the right modal matrix Φ are required to be parallel to the rows of the parallel states selection matrix D_2 in order to *preserve the control effectiveness and disturbance suppressibility* of the controller obtained by the appropriate assignment of the *left eigenstructure* of the system. Otherwise, that is, if the columns of Φ are designed not to be parallel to the rows of D_2, the control efforts may not be effectively transferred to the controlled output $z_2(t)$, even though the columns of Ψ are designed to be parallel to the columns of B for maximum control efforts transferring.

In order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, the left and right eigenstructure should be assigned to the appropriate ones simultaneously. Thus, the objective of this paper is to find a simultaneous eigenstructure assignment scheme to obtain such a controller.

3. EIGENSTRUCTURE ASSIGNMENT BY STATE FEEDBACK

Consider *Eq.*(1) in the previous section. If state feedback(*Eq.*(2)) is applied to *Eq.*(1), the closed-loop system becomes

$$\dot{x}(t) = (A + BK)x(t) + Ef(t). \tag{7}$$

Let $\Lambda = \{\lambda_1, \cdots, \lambda_N\}$ be a self-conjugate set of distinct complex numbers. Then, the right and left eigenvalue problems for the above closed-loop system can be defined by

$$(A + BK - \lambda_i I_N)\phi_i = 0 \tag{8}$$

$$(A + BK - \lambda_i I_N)^T \psi_i = 0 \tag{9}$$

where I_N is an $(N \times N)$ identity matrix. For the case that the system has repeated eigenvalues, the eigenvalue problem can be easily generalized.

Each problem of the right and left eigenstructure assignment is then to choose the feedback gain matrix K such that the required conditions for the eigenvalues and eigenvectors are satisfied, and therefore may be considered as inverse eigenvalue problem.

The right(Φ) and left(Ψ) modal matrices are defined as follows:

$$\Phi = [\phi_1, \phi_2, \cdots, \phi_i, \cdots, \phi_N], \quad \Psi = [\psi_1, \psi_2, \cdots, \psi_i, \cdots, \psi_N].$$

In the following, the superscript $(\cdot)^*$ denotes the conjugate of a given complex vector or scalar $(\cdot)$.

As mentioned in the previous sections, for solving disturbance or mode decoupling problems using eigenstructure assignment scheme, the appropriate assignment of the right eigenstructure of a system is required.

To present the well-known right eigenstructure assignment scheme, we define

$$S_{\lambda_i} \equiv [\lambda_i I_N - A \mid B], \quad R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ -- \\ M_{\lambda_i} \end{bmatrix}$$

where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}. For the case that rank $B = m$, it can be shown that the columns of N_{λ_i} are linearly independent.[6]

The following theorem gives necessary and sufficient conditions for the existence of K which yields the prescribed right eigenstructure.

Theorem 3.1[6]

Let $\{\lambda_1, \lambda_2, \cdots, \lambda_N\}$ be a self-conjugate set of distinct complex numbers. There exists a real $(m \times N)$ matrix K such that $(A+BK)\phi_i = \lambda_i \phi_i$, $i = 1, 2, \cdots, N$ if and only if, for each i, 1) $\phi_1, \phi_2, \cdots, \phi_N$ are a linearly independent set in C^N, the space of complex N-vectors, 2) $\phi_i = \phi_j^*$ when $\lambda_i = \lambda_j^*$, 3) $\phi_i =$ span N_{λ_i}. Also, if K exists and rank $B = m$, then K is unique, and is computed by using the obtained submatrices N_{λ_i} and M_{λ_i}.

The left eigenstructure assignment scheme plays an important role in designing an effective and disturbance suppressible controller. Choi *et al.*[14] found that a left eigenstructure assignment scheme by state feedback using Theorem 3.1 cannot be directly applied to get the desired left eigenstructure, and thus proposed a novel left eigenstructure assignment scheme using the biorthogonality property between the right and left modal matrices of a system.

The proposed left eigenstructure assignment scheme makes it possible to achieve the desired closed-loop left eigenstructure exactly, provided the desired left eigenvectors reside in the achievable subspace. In case the desired left eigenvectors do not reside in the achievable subspace, the closed-loop eigenvalues are achieved exactly and the left eigenvectors are assigned to the best possible set of eigenvectors in the least square sense. The details of the scheme are reported in Ref. 14.

4. RIGHT/LEFT EIGENSTRUCTURE ASSIGNMENT METHODOLOGY

The objective of this section is to find a solution for the simultaneous eigenstructure assignment in the least square sense to overcome the inherent conflicting nature of each eigenstructure. If the following three conditions are satisfied simultaneously, the desired left and right modal matrices are achieved in the least square sense guaranteeing the exact assignment of the desired eigenvalues.

$$q_1[(\Psi^d)^{\mathrm{T}} \cdot \Phi^a_{\mathrm{aug}} P - I_N] = 0, \qquad (10)$$

$$q_2 D_1 \cdot \Phi^a_{\mathrm{aug}} P = 0, \qquad (11)$$

$$q_3(D_2^{\mathrm{T}} \hat{K} - \Phi^a_{\mathrm{aug}} P \cdot S) = 0 \qquad (12)$$

where q_i are weighting factors corresponding to each condition and $0 \le q_i \le 1$ $(i = 1, 2, 3)$, $\sum_{i=1}^{3} q_i = 1$, and $P \in C^{mN \times N}$ is the coefficient matrix to be determined.

The first condition(*Eq.*(10)) denotes the biorthogonality property between the desired left modal matrix $(\Psi^d)^{\mathrm{T}}$ and the achievable right modal matrix $\Phi^a_{\mathrm{aug}} P$, and can be used to design the left eigenstructure of a system. The second condition(*Eq.*(11)) denotes the orthogonality property between D_1 and $\Phi^a_{\mathrm{aug}} P$ for disturbance decoupling. By adding the second condition to the first one, the corrupted disturbances in the states selected by $D_1(i.e., z_1(t))$ are decoupled in the least square sense. The third condition(*Eq.*(12)) denotes the parallel condition between D_2 and $\Phi^a_{\mathrm{aug}} P$, where the matrices $\hat{K}$, and $S = [s_1, s_2, \cdots, s_{r_2}]$ are a linear combination coefficient matrix and a state selection matrix, respectively. The element vector s_i is adopted to select any column vector of the matrix $\Phi^a_{\mathrm{aug}} P$, and is equal to the transpose of the i-th row vector of the parallel states selection matrix D_2.

Now, our objective is to find the feedback gain matrix K which yields Ψ^a and Φ^a with exact desired eigenvalues, satisfying the imposed three conditions simultaneously in the least square sense, through choosing P and $\hat{k}_{ij}$, which are the elements of the matrix $\hat{K}$.

For convenience, we vectorize the elements of the coefficient matrix P in *Eq.*(10)(See Ref. 14) as

$$\hat{p} = [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}, \cdots, p_i^{\mathrm{T}}, \cdots, p_N^{\mathrm{T}}]^{\mathrm{T}}. \qquad (13)$$

Assume, as an illustrative example, that all the three conditions are imposed only on the i-th achievable right eigenvector simultaneously, and only the first two conditions are imposed for the remaining achievable right eigenvectors. Then the stacked augmented coefficient vector $\hat{p}_{\mathrm{aug}}$ including the linear combination coefficients $\hat{k}_{ij}$ is formed as

$$\hat{p}_{\mathrm{aug}} = [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}, \cdots, \underbrace{p_i^{\mathrm{T}}, \hat{k}_{i1}, \hat{k}_{i2}, \cdots, \hat{k}_{ir_2}}_{\equiv \hat{p}^i_{\mathrm{aug}}}, p_{i+1}^{\mathrm{T}}, \cdots, p_N^{\mathrm{T}}]^{\mathrm{T}}. \qquad (14)$$

The elements of the i-th augmented coefficient vector $\hat{p}^i_{\mathrm{aug}}$ are used for determining the i-th achievable right eigenvector satisfying all the imposed conditions. The dimension and elements of the vector $\hat{p}_{\mathrm{aug}}$ are determined by the imposed conditions on each achievable right eigenvector.

Now, the imposed three conditions (*Eqs.*(10)-(12)) can be represented by the following compact form.

$$T\hat{p}_{\mathrm{aug}} = \eta \qquad (15)$$

where the vectors $\hat{p}_{\mathrm{aug}} \in C^{mN+r_2}$, $\eta \in R^{N(N+r_1)+r_2}$, and $T \in C^{(N(N+r_1)+r_2) \times (mN+r_2)}$ are given in this special case, respectively, by

$$\hat{p}_{\mathrm{aug}} = [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}, \cdots, p_i^{\mathrm{T}}, \hat{k}_{i1}, \hat{k}_{i2}, \cdots, \hat{k}_{ir_2}, p_{i+1}^{\mathrm{T}}, \cdots, p_N^{\mathrm{T}}]^{\mathrm{T}}, \qquad (16)$$

$$\eta = [\overbrace{\underset{1st}{\underline{q_1}}0\cdots0}^{N+r_1}\ \overbrace{0\,\underset{2nd}{\underline{q_1}}0\cdots0}^{N+r_1}\cdots\overbrace{0\cdots0\,\underset{ith}{\underline{q_1}}0\cdots0}^{N+r_1+r_2}\cdots\overbrace{0\cdots0\,\underset{Nth}{\underline{q_1}}}^{N+r_1}]^{\mathrm{T}}, \qquad (17)$$

$$T=\begin{bmatrix} q_1\Omega_l \\ q_2D_1N_{\lambda_l} \end{bmatrix}, \qquad T=\begin{bmatrix} q_1\Omega_l & | & 0_{N\times r_2} \\ q_2D_1N_{\lambda_l} & | & 0_{r_1\times r_2} \\ q_3N_{\lambda_l} & | & X \end{bmatrix},$$

for $l=1,\cdots,N,\ l\neq i,$ for $l=i.$

where X denotes the following matrix

$$\begin{bmatrix} -q_3I_{r_2} \\ 0_{(N-r_2)\times r_2} \end{bmatrix}_{N\times r_2}.$$

From *Eq.*(15), the stacked augmented coefficient vector $\hat{p}_{\text{aug}}$ is given by $\hat{p}_{\text{aug}}=T^{+}\eta$, and from *Eqs.*(10)-(12), the achievable right and left eigenvectors are given in the least square sense.

In general cases, all the three conditions are imposed on the achievable $(N-1)$ right eigenvectors of a system with repeated eigenvalues, and the dimensions of the vectors $\hat{p}_{\text{aug}}$, η, and the matrix T are extended appropriately.

Remember that the objective of this study is to find the state feedback gain matrix K satisfying the imposed three conditions in the least square sense. The following algorithm gives such a gain matrix. The algorithm guarantees that the desired eigenvalues are achieved exactly and the desired right and left eigenvectors are achieved in the least square sense.

Algorithm:

- Step 1: Determine the desired eigenvalues (λ_i), corresponding mode's desired left eigenvectors (ψ_i^d), the orthogonal states selection matrix D_1, the parallel states selection matrix D_2, and weighting factors q_i. $(i=1,2,3)$
- Step 2: Find the following matrices

$$S_{\lambda_i}\equiv[\lambda_iI_N-A \mid B],\quad R_{\lambda_i}\equiv\begin{bmatrix} N_{\lambda_i} \\ -- \\ M_{\lambda_i} \end{bmatrix}$$

  where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}.
- Step 3: Construct the augmented achievable right modal matrix Φ^a_{aug}.
- Step 4: Calculate the stacked augmented coefficient vector $\hat{p}_{\text{aug}}$ satisfying the three conditions described in this section in the least square sense.
- Step 5: Form the achievable right eigenvectors

$$\phi_i^a=N_{\lambda_i}p_i,$$

  and construct the achievable right modal matrix Φ^a.
- Step 6: Construct the achievable left modal matrix Ψ^a using the biorthogonality condition $((\Psi^a)^T\Phi^a)=I_N)$ between the left and right modal matrices of the given system.
- Step 7: Calculate vector chains and construct the matrix W as follows:

$$w_i=-M_{\lambda_i}p_i,\quad W=[w_1,w_2,\cdots,w_i,\cdots,w_N].$$

- Step 8: Calculate the state feedback gain matrix which yields the achievable right(Φ^a) and left(Ψ^a) matrices with exact desired eigenvalues satisfying the imposed three conditions simultaneously in the least square sense

$$K=W(\Phi^a)^{-1}.$$

5. CONCLUSIONS

In this paper, a concurrent eigenstructure assignment methodology for linear systems has been proposed to obtain an effective, disturbance suppressible and also decouplable controller by using the biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices. The proposed simultaneous right/left eigenstructure assignment methodology guarantees that the desired eigenvalues are achieved exactly and the desired left and right eigenvectors are assigned to the best possible(achievable) set of eigenvectors in the least square sense.

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