韓國精密工學會 '95年度 春季學術大會論文集 pp.342~347

Robust H_∞ Control of High-Speed Positioning Systems

(고속 위치제어계의 강인 H_∞ 제어)

Chin Tae Choi*(RIST), Jong Shik Kim (Pusan National University)
최진태*(산업과학기술연구소), 김종식(부산대학교)

ABSTRACT

Loop shaping H_∞ control with normalized coprime factorization was applied to a servo-motor driven high-speed positioning system. The high-gain controller was designed to attenuate the position errors caused by friction effects and external disturbances. The non-existence of limit cycle was analyzed, though there is actuator saturation. The designed H_∞ control system was experimently tested in a rotary index table. Results showed its effectiveness to improve position accuracy without any compensation scheme for friction, and robustness to model perturbation and external disturbances.

1. Introduction

High-speed and high-accuracy position control systems are indispensible to increase the productivity and manufacture precision products. While commonly neglected, because it is difficult to model and poorly understood, friction is present to some degree in all mechanical systems, and continues to impose limits on performance. The discontinuous nature of Coulomb-type friction near zero velocity causes stick-slip behavior which limits the fidelity of position and force control. As performance criteria, such as cycle time, are made more stringent, the dynamic load dependency of friction becomes more important. Stiction may cause steady state error, or limit cycles near the reference position in the linear control of positioning systems. Failure to account for the effects of friction can lead to tracking errors and oscillation.

An adaptive pulse width control scheme was proposed to achieve precise positioning in the presence of stiction and Coulomb friction by Yang and Tomizuka(1). It is supposed to be used near the reference position. Any conventional control law can be applied, until the system sticks near the reference position . When motion stops and error remains, a functional relationship between the pulse width and error is updated by an adaptive algorithm and a new pulse is applied. This sequence continues until the motion comes to a stop with zero error. The adaptive friction compensation at low velocities was dealt(2) and a simple model, linear in parameter, that captures the downward bends at velocities close to zero was proposed.

Ruina(3) worked on the experimental and theoretical development of constitutive frictions, and suggested a state variable friction models through their experiments on rock samples with and without lubricants. The ramifications of transient friction behavior as expressed by a state variable friction law for the control of low velocity motion was explored(4).

A repetitive control(5), a kind of learning control, was applied to compensate for stiction-induced errors which was a source of contouring errors in machine tool system.

Ohnishi(6) proposed two degrees-of-freedom controller using a disturbance observer which estimates the disturbance, and cancels the actual disturbance with the estimate. This method has a similiar control structure as time delay control(7) and time delayed uncertainty cancellation(8), in that the internal loop gain has a infinite loop gain, as delay time goes infinitely small. Lee and Tomizuka(9) designed a control scheme for perfect tracking of a positioning system. The excellent performance achieved for certain parameteric variations and external disturbances is due to infintely large bandwidth. Since the large bandwidth is inherent from the characteristics of the structure of the disturbance observer, this kind of controller is very sensitive to measurement noise and isn't robust to dynamic model uncertainties(10).

Beginning with Sampson et al.(11) and Rabinowitz(12) , it was noted that friction is not determined by current velocity alone; it depends on the history of motion and instantaneous velocity changes. In environments with the dynamic normal force variations and the changing velocity direction, it is very difficult to obtain the accurate model to completely cancel the friction effects in the servo systems. Therefore, it is more realistic to sufficiently attenuate the bounded friction to the desired magnitude level than resort to model-based compensation.

In this paper we propose a H_∞ control scheme to compensate the external disturbances including friction through a high-gain controller to accurately control the desired positions of servomechanisms, since positional errors are inversely proportional to the gain of the feedback controller. The high-gain controller results that a loop compensator with a large proportional gain is selected in loop shaping for a sensitivity function to have a small magnitude in the low frequency region. The designed high-gain controller is experimentally tested in a rotary index table. To check the robust performance when parametric modelling errors exist, tests are repeated with load variations.

2. Robust Controller Design Using Loop Shaping

Any other pertubated system of the same input/output dimensions by nornalized right coprime factorization (NRCF) can be written in the form

$$G_{\Delta} = (N + \Delta_N)(M + \Delta_M)^{-1} \tag{1}$$

where $\Delta_N, \Delta_M \in H_\infty$ are stable transfer functions.

The robust stabilization problem is to stabilize the nominal system, G, with NRCF(N,M) and the family of systems G_ε defined

$$G_\varepsilon = \{(N + \Delta_N)(M + \Delta_M)^{-1} : \left\| \begin{matrix} \Delta_N \\ \Delta_M \end{matrix} \right\|_\infty < \varepsilon\} \tag{2}$$

using a proper feedback control K[13].

Definition (NRCF Robust Stabilization): Let (N,M) be a NRCF of G, then, 1) Find the largest positive number ε_{max} called the maximum stability margin, such that (N, M,ε_{max}) is robustly stable.

2) For a particular value $\varepsilon \le \varepsilon_{max}$, synthesize a feedback controller, K, such that (N,M,K,ε) is robustly stable.

We can define the lower linear fractional transformation for this problem.

$$F_l(P,K) := M^{-1}(I - KG)^{-1}[K \quad I] \tag{3}$$

By multiplying the expression for $F_l(P,K)$ on the left by the inner transfer function $\begin{bmatrix} N \\ M \end{bmatrix}$, a two-block H_∞ problem is written as a four-block H_∞ problem. Rephrasing this, the solution of the the problem is given by minimizing

$$\varepsilon_{max}^{-1} = \gamma_{min} = \inf_K \left\| \begin{bmatrix} G \\ I \end{bmatrix} (I - KG)^{-1} [K \quad I] \right\|_\infty \tag{4}$$

where ε_{max} is the maximum stability margin such that (N,M,ε_{max}) is robustly stable. The maximun stability margin is also calculated from the condition satisfying the standard H_∞ solution in Glover and Doyle[14], and rewritten as

$$\varepsilon_{max} = (I + \lambda_{max}(XY))^{-\frac{I}{2}} \tag{5}$$

and given $0 < \varepsilon < \varepsilon_{max}$, then a state-space realization of the central controller satisfying the stability margin is

$$K = \left[\begin{array}{c|c} A + BB^T + \varepsilon^{-2} W_I^{-T} YC^T C & \varepsilon^{-2} W_I^{-T} YC^T \\ \hline B^T X & 0 \end{array} \right] \tag{6}$$

where $W_I = (I - \varepsilon^{-2})I + YX$.

This method has a remarkable characteristics to synthesize the optimal control directly from the Nehari extension of the normalized right coprime factorization of the nominal system, while other H_∞ syntheses require iterative procedures to obtain optimal or suboptimal solutions to satisfy the H_∞ norm.

The Loop-Shaping Design Procedure

(1) Loop Shaping : using a loop compensator, W, the magnitude of the nominal system G, is shaped to give a desired target loop which determines the open-loop shape of the closed-loop system. The selection of target loop uses familiar SISO loop-shaping guidelines. Shaping a crossover roll-off rate close to -20dB/dec. is consistent with Bode's observation that roll-off rate determines phase, and that a rate of -20dB/dec. corresponds to 90 degrees phase. The actual system can be unstable, if roll-off rate is -40db/dec. The nominal system and the loop compensator, W are combined to form the shaped system, G_S, where $G_S = GW$.

(2) Robust Stabilisation : a feedback controller, K_∞, is synthesized using the NRCF stabilization procedure which robustly stabilizes the NRCF of G_S, with stability margin ε.

(3) The final feedback controller, K, is thenconstructed by combining the H_∞ controller , K_∞, with the loop compensator W such that $K = WK_\infty$.

3. Analysis of the High-gain Controller

We deal with the position control system forced by friction forces illustrared in Fig. 1

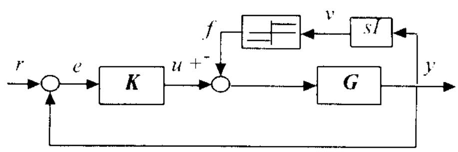

Fig. 1 The standard feedback system with friction

The friction forces, f, have bounded energy in low frequency region, y are the outputs equivalent to the position signal, v are velocity signals, and s is a Laplacian operator.

Obtaining the the error signals to the friction force,

$$e = (I + GK)^{-1} Gf \tag{7}$$

The norm of errors is

$$\|e\| = \|(I + GK)^{-1} Gf\| \le \|(I + GK)^{-1} G\| \|f\| \tag{8}$$

At low frequencies such that $\|GK\| >> 1$, Eq. (8) can be written as

$$\|e\| \le (1 / K(j\omega)) \|f\| \tag{9}$$

Therefore, tracking errors by the friction forces are inversely proportional to the magnitude of frequency response in low frequency region, and we can sufficiently attenuate the errors by the feedback controller having the high gain in this frequency. A loop compensator with large proportional gain is selected to obtain the high gain controller when we design a H_∞ controller using loop shaping approch.

Since the electrical time constant is much smaller than the mechanical time constant in the servomotor-driven position control system, we can neglect the armature inductance effect. Therefore, the dynamic model of the system is, in general, characterized by the 2nd order transfer function, and has no finite zeros. For the target loop to have -20db/dec. in the crossover frequency, inserting a finite zero as a loop compensator, which guarantees a good phase margin and robust stability is necessary. We determine the proportional gain and the locations of the zero by the desired error magnitude, the magnitude of the friction force and a roll-off rate of the unshaped open-loop Bode plot.

This gain is slightly changed in the final controller by inclusion of H_∞ controller, which causes deterioration in the open-loop shape, but the degradation in the loop shape caused by the H_∞ controller, K_∞, is limited at frequencies where the specified loop shape is sufficiently large or small. The high-gain scheme immediately saturates actuators. The saturation doesn't bring about a severe stability problem in the SISO system which doesn't have unstable poles and integral compensators. Robust stability is examined by experimental results in the section 5. Actuator saturation primarily affects the transient performance, and makes worse the speed of response. The describing function, $N(M)$, for the saturation is given as

$$N(M)=\frac{2k}{\pi}[\sin^{-1}\frac{a}{M}+\frac{a}{M}\sqrt{1-\frac{a^2}{M^2}}] \qquad (10)$$

with M, a, and k denoting the control input to the actuator, the range and the slope of the linearity. If $M \le a$, then $N(M)=1$, and the input remains in the linear range. The magnitude of the control input for cancelling friction force, in most cases, is within the linear range, not saturation range, and therefore, saturation doesn't degrade the capabilities of error attenuation. We must also discover the existence of limit cycles caused by the saturation.

Describing function is mainly used in predicting the limit cycles in nonlinear systems. Obtaining the relationship between a linear part and the describing function from the characteristic equation of the closed-loop system with actuator saturation,

$$G(j\omega)K(j\omega)=-\frac{1}{N(M)} \qquad (11)$$

The positioning system as mentioned above is stable 2nd order system without finite zeros and we assume that $K(j\omega)$ is a stabilizing controller for the closed-loop system, and has no finite zeros in the left s-plane. Then, the locus of $G(j\omega)K(j\omega)$ do not encircle the -1 point of the real axis. As M varies from 0 to ∞, the locus of $-1/N(M)$ starts from the -1 point on the real axis and extends to $-\infty$. We can plot the frequency response function $G(j\omega)K(j\omega)$ and the negative inverse describing function $-1/N(M)$ in the complex plane, as shown in Fig. 2. The locus of $-1/N(M)$ and the locus of $G(j\omega)K(j\omega)$ do not intersect each other in this figure, which means that limit cycle by the actuator saturation will not exist. Therefore, the high-gain controller does not introduce stable oscillation.

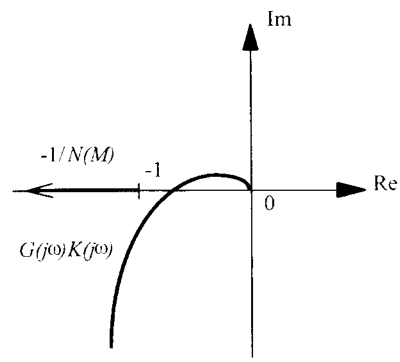

Fig.2 Plot of $-1/N(M)$ and $G(j\omega)K(j\omega)$

Robust stability is, in general, interpreted in terms of a combination of the robustness for the multiplicative and additive uncertainties. If the the gain of the controller, K, is becomes larger, the allowable additive uncertainties will be decreased. Therefore, the magnitude of the controller gain have to be selected by considering the magnitude of the position error and the robust stability.

4. Design of the Proposed Controller

Fig. 3 is a testbed for high-speed position control. The system consists of two main elements: the servomotor-driven index table and VME computer with interface. The resolution of the position sensor which is an incremental encoder is rad. The motor used is DC servo motor, and the controller is implemented using MVME143 computer that has VME bus and 32 bit microprocessor, MC68030, running at 25MHz. The computer is interfaced to a power amp via a DAC board and a encoder through a counter board. We generated the digital codes written in C for actual control, and they are excuted every 2 msec. under assistance of real time operating system, VMEexec.

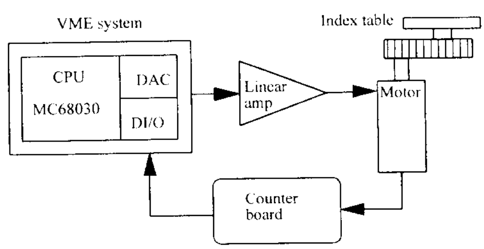

Fig.3 Experimental setup of the servomechanism

We assume that the transfer function of the index table is 2nd order system as stated previously. The transfer function for the index table is experimentally obtained using frequency response and aproximately given as follows:

$$G(s) = \frac{k}{s(Ts+1)} \tag{12}$$

where k is 1.52 and T is 0.009 sec.

The roll-off rate in the crossover frequency is -20 db/dec. But if a large proportional gain is used in the feedback controller, the shape goes up necessarily and the roll-off rate in the crossover frequency will be -40 db/dec. So the insertion of a finite zero near the crossover frequency guarantees for the target loop to have -20db/dec. The shaped plant $G_s(s)$ is obtained by inserting a loop compensator, $k_p(s/\omega_z+1)$, where we assign 105 rad/sec. to ω_z. The k_p is chosen to be 70 to sufficiently attenuate the position errors. Saturation level to the linear amp is set to 0.76 volt. to test the characteristics of the high-gain controller, and this k_p corresponds to the great high gain, compared to the linear range of the linear amp. The shaped transfer function is

$$G_s(s) = \frac{kk_p(s/\omega_z+1)}{s(Ts+1)} \tag{13}$$

Sufficiently large gain margin and phase margin are necessary for the system to have good relative stability which can be, to some degree, robustness measure in SISO LTI system. The stability margin, ε =0.53, appears to give good loop recovery, because the designed loop has slightly lower gain than the target loop. The gain and phase margin of the compensated system are about 168 db and 78 degree, which are still excellent margins. The smaller phase margin than the target loop is obtained, because the complete recovery is not achieved by adopting the lower ε than ε_{max}. The designed loop shows that it gently crosses the 0 db with -20db/dec., and it is thought that it will have a nice transient response. The maximum stability margin calculated from the Eq. (5) is 0.72. Here, the H_∞ controller, K_r, is designed with stability margin, ε =0.53 to prevent the fast poles from being introduced to the H_∞ controller. The final feedback controller K is obtained by combining the loop compensator and K_r.

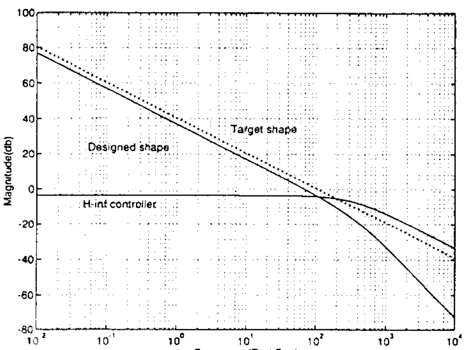

Fig. 4 Frequency responses of the designed system

Frequency responses of the target loop, the designed loop shape and the controller K_r are shown in Fig. 4. The designed loop shape shows higher roll-off rate in the higher frequency region and therefore robustness to the measurement noise. This shape well satisfies the requirements that the open loop shape in the low and high frequency region need to meet the closed-loop performance.

5. Experimental Results

This section details actual experimental results obtained using the previously described system. A step response experiments was carried out to demonstrate the effectiveness of the proposed controller. Step responses are useful information to examine the disturbance attenuation of the controller. The index table was given 0.4 radian step command, which corresponds a fairly large command, to check the characteristics of the high-speed positioning system.

Robust performance of the high gain controller to be examined is the ability to reject load uncertainty which causes a parametric modelling error, and results in shift of poles locations and a plant gain change. The friction force is also changed to a larger level. To demonstrate this, we repeated the experiments with a 6 Kg load attached to the index table. The angle errors with/without the load illustrate a good performance to the load perturbation as seen in Fig.5. It shows the error responses after 99 % settlement for an angle command of 0.4 radian. The settling time for the 99 % response is 330 msec. in this system. The steady-state errors are 10^{-5} radian without any mass and -9.2×10^{-5} with the mass, respectively. These are equivelent to 0.0025 % and 0.023 % errors for the command reference, respectively. It is shown that the steady-state errors for the friction are sufficiently attenuated by the high gain controller as expected.

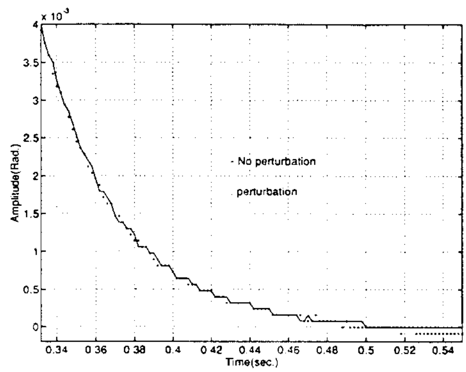

Fig. 5 The responses of errors with H_∞ controller

One of the major concern for the high gain controller is its performance under actuator saturation. To address this concern, the output of the controller and the saturated control signal are shown in Fig. 6 when the index table was given 0.4 radian step command. The motor is saturated for a control signal magnitude beyond 0.76 volt., which is indicated by the dached line on the plot, because the linear characteristics of the power amp for the control input ranges from -0.76 volt to +0.76 volt. Saturation occurs in the initial stage, and it only hampers the rapid rise to the desired reference.

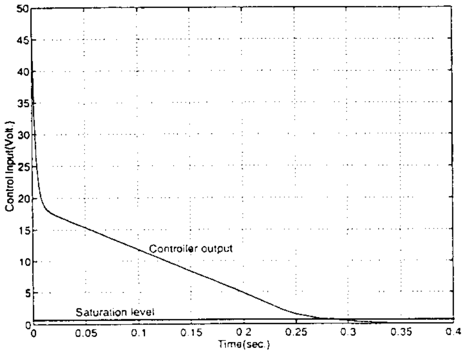

Fig. 6 The controller outputs with actuator saturation

The objective of this paper is to design the controller that precisely controls the servomechanism with the friction effect, but the ability to reject the external disturbance is also investigated to test the real system applicability. External disturbances are tested by an elastic spring that was physically attached to the index table as shown in Fig. 7.
Two experiments are repeated with different spring forces. While the index table rotates 0.4 radian, the spring forces vary linearly from 1.5 kg_f to 2.5 kg_f in the first experiment and from 2.6kg_f to 3.5kg_f in the second, respectively. The final opposed torques by the attached spring are 16 $kg_f cm$ and 23.5 $kg_f cm$ in the above two experiments, respectively. These torques, in general, deteriorate transient responses and give rise to large steady state errors.

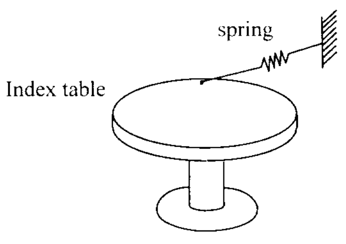

Fig. 7 The index table pulled with a coil spring

The steady state errors are increased to 4.2×10^{-4} and 5×10^{-4} radian as seen in Fig. 8. Though the steady state errors are larger than no disturbance test, but the designed controller still shows satisfactory servo responses despite the external spring torque.

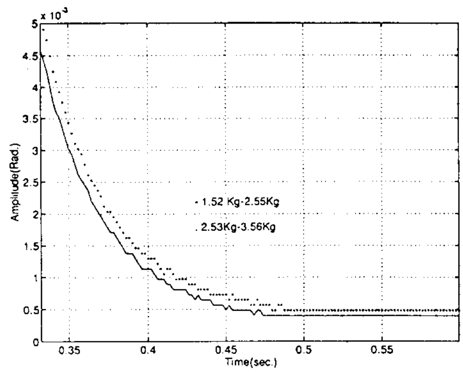

Fig. 8 The error responses with external disturbances

6. Conclusions

Friction causes the steady state position errors or tracking lags in the servomechani-sms. It is necessary to precisely control the postions without any stable oscillation near the reference position in the existence of friction and disturbances. A design method for the robust position controller based on the high-gain loop shaping H_∞ synthesis was presented. It was applied to the index table to require high-speed/high-accuracy position control. The loop shaping method with H_∞ synthesis provides us with the ease and the flexibility in designing the controller which meets our specifications. The high-gain loop shapng scheme shows that the designed H_∞ controller is effective to attenuate the friction forces to the desired magnitude.

The performance of the controller has been examined through experiments under various load uncertainties and disturbances including friction. The position error due to the effect of the friction in the index table was greatly reduced using a suboptimal H_∞ controller without any feedforward friction compensation. The experimental results verified that the saturation by the high gain contoller doesn't make a problem of nominal stability and robust stability to the parametric modelling perturbation. The high-gain controller can't completely eliminate positioning errors by friction, but it is able to sufficiently reduce the errors. Moreover, it has robustness to model perturbation, noise immumity and disturbance insensitivity which real world controller must possess.

References

[1] Yang,S. and Tomizuka, M, "Adaptive Pulse Width Control for Precise Positioning Under the Influence of Stiction and Coulomb Friction," *ASME, Journal ofDynamic Systems,Measurement, and Control*, Vol.110,pp.221-227,1988

[2] Canudas de Wit et al., "Adaptive Friction Compensation in Robot Manipulators:low Velocity,"*The International*

Journal of Robotics Research,Vol.10.No.3. pp.189-199, June, 1991

[3] Ruina, "Friction Laws and Instabilities: A Quasistatic Analysis of Some Dry Frictional Behavior," Ph.D. Dissertation, Brown University, 1980

[4] Dupont.P.E.,"Avoiding Stick-Slip in Position and Force Control Through Feedback," *Proceedings of the 1991 IEEE Int. Robotics and Automation Conf.*,Sacramento, pp.1470-1475. April,1991

[5] Tung.E.D. "Low Velocity Friction Compensation for Machine Tool Feed Drives," *Proceedings of the ACC*, San Francisco, pp1932-1936, 1993

[6] Ohnishi, K., "A New Servo Method in Mechatronics", *Trans. JSEE*, Vol. 107-D, pp.83-86, 1987

[7] Youcef-Toumi, K. and Ito, O., "A Time Delay Controller for Systems With Unknown Dynamics," *ASME, Journal of Dynamic Systems,Measurement, and Control*, Vol.112, pp.133-142,1988

[8] Hsia, T.C., Lasky, T.A. and Guo,Z., "Robbust Independent Joint Controller Design for Industrial Robot Manipulators, "*IEEE Trans. Indus. Electro.*, Vol.38, No. 1, pp. 21-25, 1991

[9] Lee.H.S. and Tomizuka.M., "Design of Friction-Compensated and Robust Digital Tracking Controllers for High-Accuracy Positioning System, "*Proceedings of the 1994 Japan-USA Symposium on Flexible Automation*, 1994

[10] Liang,Y. and Looze, D.P., "Evaluation of Time-delayed Uncertainty Cancellation Systems", *Proc. of the ACC*, , pp.1950-1954, 1992

[11] Sampson,J. et al., "Friction behivior During the Slip Portion of the Stick-Slip Process," *J. of Applied Physics*,Vol.14,No.12,pp.689-700,1943

[12] Rabinowitz,E., "A Study of the Stick-Slip Process," Friction and wear,Ed. Robert Davies, Elsevier Publishing Co.,New York, 1959

[13] McFarlane, D.C. and Glover, K., "Robust Controller Design Using Normalized Coprime Factor Plant Descriptions, "*Lecture Notes in Control and Information Sciences*. New York: Springer-Verlag, 1990

[14] Doyle, J.C., Glover, K., Khargonekar, P.P., and Francis, B.A., "State-Spce Solutions to Standard H_2 and H_∞ Control Problems," *IEEE Trans. on Automat. Contr.*, Vol.34, No.8, pp.831-847, 1989