# A STUDY ON CENTER THINNING IN ROTARY FORGING OF CIRCULAR PLATE

### SEOGOU CHOI\* AND HUNG-KUK OH\*\*

\* Metal forming Team, Korea Academy of Industrial Tecnology \*\* Dept. of Manufacturing and Automation, Ajou University

### **ABSTRACT**

The rotary forging process has a potential for producing high-precision parts because of smaller forging forces and incrementally controlled deformation, especially in cold forging of intricate parts to net shape. But while thin circular plate are made by rotary forging, center thinning and fracture will occur under given conditions. The trouble has seriously influenced the quality of products and the spreading of this technique. This paper intends to explain the phenomenon of center thinning and gives a criterion of it. In order to confirm the validity of the proposed criterion, experiments have been carried out by using the rotary forging press which has been designed and constructed in our laboratory.

### **NOTATIONS**

ratio  $(d\varepsilon_r/d\varepsilon_\theta)_{r=r_i}$  $C_{i}$ initial diameter of workpiece  $D_{\alpha}$ feeding rate of rocking die per revolution  $H_{\alpha}$ initial height of workpiece h current height of workpiece yield shear stress direction cosines  $I_{rv}, I_{\theta v}$ bending moment for the plastic hinge mechanism  $M_{o}$ normal stress on the entrance and exit boundary  $S_1, S_2$  $P_1, P_2$ force resultant acting on the boundary  $S_1, S_2$  $\mathbf{F}_{i}$ normal and tangential components of force resultant in the neutral direction  $F_{N_i}, F_{T_i}$ P., die pressure moment arm  $r_{Ni}$ R current radius of workpiece  $S_1 \sim S_3$ boundary of deformated zone I radial and circumferential displacements  $v_r, v_\theta$ 

inclination angle of upper die α angle of contact profile between the upper die and the workpiece β contact area ratio ( (area of zone I)/ $\pi R^2$ ) λ flow stress  $\overline{\sigma}_{c}$ tensile stress created by bending moment for the plastic hinge mechanism  $\sigma_{p}$ yield strength  $\sigma_v$ frictional stress on the interface between die and workpiece  $\tau_r, \tau_{\Theta}$ tangential stress on the entrance and exit boundary  $S_1$ ,  $S_2$  $\tau_1, \tau$ positioning angle of the entrance and exit boundary S<sub>1</sub>, S<sub>2</sub>  $\theta_1, \theta_2$ positioning angle of the neutral plane  $\theta_{N}$ 

### 1. INTRODUCTION

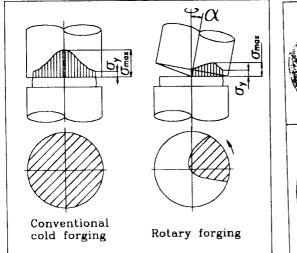
The force necessary to produce any plastic deformation is directly proportional to the contact area and the material flow stress. Therefore by applying incremental deformation process, bulk material can be forged by using only a fraction of the force required for total deformation. Rotary forging is an incremental deformation process in which the deformation is the continuous local deformation instead of the deformation as a whole in conventional forging, so that it has many advantages that are not possessed in conventional forging. The economizing in force is more prominent among them. By limiting the contact area of the upper die and workpiece to a small portion of the whole surface (Fig.1), the load advantage offered by rotary forging over that of conventional forging has been estimated as being between 10:1 and 30:1. Therefore the rotary forging is mainly applied in the production of thin circular plate, which are usually difficult to be made by conventional forging.

Nevertheless, while thin circular plates are made by rotary forging, center thinning and fracture will occur under any given conditions (Fig.2). The trouble has seriously influenced the qualities of products and the spreading of this technique.

In recent years, several attempts have been made in order to analyze this problem in theory or practice. Lu<sup>(1)</sup> suggested the experimental criterion for center thinning on the basis of the simulated experiments by pure lead samples. But this criterion is only suitable for pure lead material and to the ideal hot rotary forging for some material. It can not be directly applied for other materials and cold forging process. Hawkyard<sup>(2)</sup> and Zhou<sup>(3)</sup> explained the center thinning phenomenon by using plastic hinge model but any criterion has not been suggested.

In the present work, the theoretical criterion on center thinning in rotary forging is derived by applying the plastic hinge model on the basis of the stress analysis. In order to confirm the validity of the proposed criterion, experiments are carried out by using

the rotary forging press which has been designed and constructed in our laboratory.



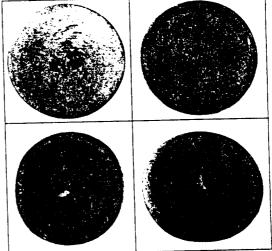


Fig. 1. Principle of rotary forging.

Fig.2. Samples thinned and fractured at center

### 2. ANALYSIS OF STRESS

Considering an element in the deformated zone shown in Fig.3, the equilibrium eqs. can be obtained in terms of an  $r, \theta, z$  cylindrical system.

circumferential direction: 
$$\frac{\partial (h\tau_{,\theta})}{\partial r} + \frac{\partial (h\sigma_{,\theta})}{r\partial \theta} + \frac{2h\tau_{,\theta}}{r} - \sigma_{,\theta} - \frac{\partial h}{r\partial \theta} - 2\tau_{,\theta} = 0$$
 (1)

radial direction: 
$$\frac{\partial (h\sigma_r)}{\partial r} + \frac{\partial (h\tau_{r\theta})}{r\partial \theta} + \frac{h}{r}(\sigma_r - \sigma_\theta) - \sigma_z \frac{\partial h}{\partial r} - 2\tau_r = 0$$
 (2)

Referring to three-dimensional analysis for strip rolling<sup>(4)</sup>, it is assumed that the ratio of the radial strain to the circumferential strain is constant at the arbitrary radius of deformated zone.

$$(d\varepsilon_r/d\varepsilon_\theta)_{r=r_i} = (\varepsilon_r/\varepsilon_\theta)_{r=r_i} = C_i$$
(3)

Then, when von Mises yield criterion, Levy-von Mises eq. and eq.(3) are used, the stress components in the deformated zone I can be obtained as follows:

$$\sigma_z = (\sigma_\theta - \sqrt{k^2 - \tau_{r\theta}^2} (C_i + 2)) / \sqrt{C_i^2 + C_i + 1}$$
(4)

$$\sigma_r = ((2C_i + 1)\sigma_0 + (1 - C_i)\sigma_z)/(2 + C_i)$$
(5)

$$\sigma_r = \frac{(2C_i + 1)\sigma_{\theta} + (1 - C_i)\sigma_z}{(2 + C_i)}$$

$$\tau_{r\theta} = k / \sqrt{1 + 4(d\varepsilon_z / d\gamma_{r\theta})^2 (C_i^2 + C_i + 1) / (C_i + 1)^2}$$
(6)

Hence, if the circumferential stress  $\sigma_{\theta}$  and shear strain  $\gamma_{\theta}$  are evaluated, eqs. (4), (5) and (6) could be solved.

Considering Su's analysis<sup>(5)</sup> of strain on rotary forging, the axial strains are expressed as follow:

$$d\varepsilon_z = d\varepsilon_{zo} \left(\varepsilon_z / \sum_{\theta=\theta_1}^{\theta_2} d\varepsilon_{zo}\right) \tag{7}$$

where,

$$d\varepsilon_{zo} = \ln\left[\frac{h - f \, d\theta/2\pi}{h}\right] + \ln\left[\frac{h - dh}{h}\right] + \ln\left[\frac{h - (\theta_1 - \theta)\left(\left(\frac{dh}{d\theta}\right)_{\theta = -\theta_2} \frac{d\theta}{2\pi}/(\theta_1 - \theta_2)\right)}{h}\right]$$

$$\varepsilon_z = \ln((h-f))/h$$

Next, from eq. (3) together with eq.(7) and the volume constant eq.  $\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0$ , the others of the normal strain are determined:

$$d\varepsilon_{\theta} = -d\varepsilon_z/(1 + C_i) \tag{8}$$

$$d\varepsilon_r = -C_i d\varepsilon_z / (1 + C_i) \tag{9}$$

Furthermore, we can solve the displacement as follows:

$$v_r = \int_0^r d\mathbf{\varepsilon}_r dr \tag{10}$$

$$v_{\theta} = \int_{\theta_N}^{\theta} (d\varepsilon_{\theta} r - v_r) d\theta \tag{11}$$

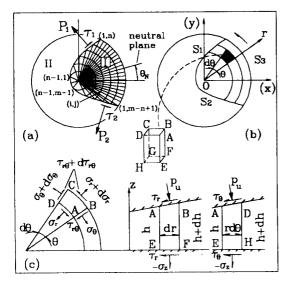
Thus, shearing strain $\gamma_{r\theta}$  and friction stress components between dies and workpiece can be expressed as follows:

$$\gamma_{r\theta} = \frac{\partial v_r}{r \partial \theta} - \frac{v_{\theta}}{r} + \frac{\partial v_{\theta}}{\partial r} \tag{12}$$

$$\tau_r = \frac{v_r}{\sqrt{v_r^2 + v_\rho^2}} k \tag{13}$$

$$\tau_{\theta} = \frac{v_{\theta}}{\sqrt{v_r^2 + v_{\theta}^2}} k \tag{14}$$

The boundary conditions in the  $S_1, S_2$  of the deformated zone I are indistinct. But the force equilibrium must be satisfied in the undeformated zone II shown in fig.3(a). Therefore boundary conditions can be derived as follows:



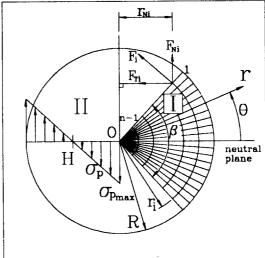


Fig.3. Schematic view of contact area between tools and blank and notations used inanalysis.

Fig.4. Analysis of the tensile stress in a rotary forged workpiece.

$$\sum F_{y} = \int_{S_{1}} \{ P_{1} \cos \theta_{1} + \tau_{1} \sin \theta_{1} \} h_{1} ds_{1} - \int_{S_{2}} \{ P_{2} \cos \theta_{2} + \tau_{2} \sin(-\theta_{2}) \} h_{2} ds_{2} = 0$$
 (15)

$$\sum F_{x} = \int_{s_{1}}^{s_{1}} \{ P_{1} \sin \theta_{1} - \tau_{1} \cos \theta_{1} \} h_{1} ds_{1} + \int_{s_{2}}^{s_{2}} \{ P_{2} \sin(-\theta_{2}) - \tau_{2} \cos \theta_{2} \} h_{2} ds_{2} = 0$$
 (16)

Now, the value of  $C_i$  in eq(3), the positioning angle  $\theta_N$  of neutral plane shown in Fig.3 and initial value of circumferential stress  $\sigma_{\theta}$  in the entrance boundary  $S_1(Fig.3)$  can be found satisfying the force equilibrium condition shown in eqs(15) and (16) by using iteration method and by FDM. Consequently, all components of stress in the deformated zone I shown in Fig.3 can be evaluated.

## 3. CRITERION ON CENTER THINNING AND PLASTIC HINGE MODEL

When the upper die is pushed down on a workpiece, if rotary rolling does not occur, the acting force of the deformated zone I on the opposite zone II is shown as Fig.4. A simple model is assumed for the plastic hinge mechanism that the bending moment occurs at the center of the workpiece and the tensile bending deformation takes place in the hinge point H by the effect of this bending moment. Therefore when  $\beta = 180\,^\circ$ ,  $\lambda = 0.5$ , the bending moment and the tensile bending deformation do not occur because the moment arm  $r_{Ni}$  is zero. According to this assumed hinge model, a tensile stress  $\sigma_p$  will appear in the central part of the workpiece because of the effect of force  $F_{Ni}$ . The maximum tensile stress in the central part of the workpiece is given by

$$M_{o} = \sum_{i=1}^{n} F_{Ni} r_{Ni}$$
 (17)

$$\sigma_{\text{p max}} = \frac{6M_{\text{o}}}{R^2h} \tag{18}$$

Therefore, overlapping the tensile stress which is given by eq.(18), the new stress field is derived in the central part of the workpiece (zone I). And so the flow stress of the defomated zone I can be given by

$$\bar{\sigma}_{c} = \sqrt{\frac{1}{2}((\sigma_{r}^{\circ} - \sigma_{\theta}^{\circ})^{2} + (\sigma_{\theta}^{\circ} - \sigma_{z}^{\circ})^{2} + (\sigma_{z} - \sigma_{r}^{\circ})^{2}) + 3(\tau_{r\theta}^{2} + \tau_{\theta z}^{2} + \tau_{zr}^{2})}$$
where,
$$\sigma_{r}^{\circ} = \sigma_{r} + \sigma_{p \max} l_{rp}, \qquad \sigma_{\theta}^{\circ} = \sigma_{\theta} + \sigma_{p \max} l_{\theta p}$$
(19)

Because the center thinning is a kind of tensile plastic instability, it is considered that center thinning occurs as soon as the flow stress in the central part of the workpiece achieves the yield  $stress(\sigma_y)$  of work hardening material at which material is going into the state of instability. If the center thinning were occurred, the center has the appearance of not being contacted by the upper die. Therefore the axial stress  $\sigma_z$  will be zero. From the eq.(19), the criterion on the center thinning in rotary forging can be stated as follow;

$$\bar{\sigma}_{c} = \sqrt{\frac{1}{2} \left[ \left( \sigma_{r}^{2} - \sigma_{\theta}^{2} \right)^{2} + \sigma_{\theta}^{2} + \sigma_{r}^{2} \right] + 3(\tau_{r\theta}^{2} + \tau_{\theta z}^{2} + \tau_{zr}^{2})} \ge \sigma_{y}$$

$$: occurring the center thinning (20)$$

Then so long as the stress state of the central part of the workpiece can be determined by using plastic hinge model and the analysis of stress, the theoretical criterion of center thinning will be obtained because the yield stress can be easily derived from material properties.

Consequently, if the state of stress in the central part of deformated zone I is found, the theoretical criterion of the center thinning in rotary forging process would be easily obtained.

### 4. EXPERIMENTS

Carbon steel(0.45%C) and aluminum(Al6061) were chosen and prepared shown in table 1 as the work material for the theory and the experiments. Experiments were conducted in the rotary forging press which has been designed and constructed in our laboratory. The prospective view of the machine is shown in Fig.5. The forming load is supported by the hemispherical bearing. Pressure-raised oil is supplied for the lubrication of the bearing surface. To reduce required load a slightly lager inclination

angle of upper die of 3 degrees is used. Therefore experiments have been carried out only in case of 3 degrees.

	Height Ho/mm	Diameter Do/mm	$\overline{\sigma} = A\overline{\varepsilon}^n (MPa)$
Steel	10.0	25.0	$\overline{\sigma} = 1029.7(\overline{\epsilon})^{0.2}$
(0.45%C)	15.0	25.0	$\sigma = 1029.7(\epsilon)$
Aluminun	10.0	25.0	$\overline{\sigma} = 543.3(\overline{\varepsilon})^{0.16}$
(Al6061)	15.0	25.0	0 = 343.3(E)

Table 1 Dimensions and stress-strain relationship of workpiece.

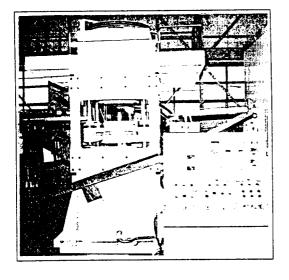


Fig.5. Prospective view of the rotary forging machine(RF100).

Fig.6. Theoretical criterion curve on center thinning in rotary forging.

### 5. ANALYSIS ON CENTER THINNING

The theoretical criterion on center thinning shown in table 2 is obtained by applying the plastic hinge model on the basis of the stress analysis where  $\alpha$  must be expressed by a radian value. But the exponents of (R/h) and the coefficients of criterion are fairly close for the cases in table 2. Hence the exponents and the coefficients can be unified as follow:

$$\frac{f}{h} = 0.435\alpha \left(\frac{R}{h}\right)^{1.51} \tag{21}$$

The relevant curves to formula (21) were drawn in Fig.6. The meaning of the criterion

is that no center thinning occurs when  $(f/h)>0.435\alpha(R/h)^{1.51}$ , that is, the right and lower part of the curves is the center thinning region.

	upper die angle(degree)	the theoretical criterion
steel	3	$(f/h) = 0.4068\alpha (R/h)^{1.53}$
(.45%C)	6	$(f/h) = 0.4383\alpha (R/h)^{1.52}$
aluminum	3	$(f/h) = 0.4526\alpha(R/h)^{1.50}$
(Al6061)	6	$(f/h) = 0.4412\alpha(R/h)^{1.49}$

Table 2 The theoretical criterion on center thinning.

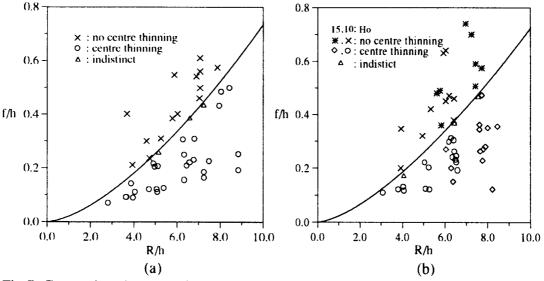


Fig. 7. Comparison between theoretical criterion on center thinning and experimental center thinning (a) steel (0.45%C) (b) aluminum(Al6061),  $\alpha = 3^{\circ}$ .

As the result of theoretical analysis about center thinning in rotary forging, the following facts are observed.

1) The height that center thinning begins(h) does not depend on the original height of workpiece and the rotational speed and the materials.

2) The height that center thinning begins(h) increases with the decreasing of the feeding rate of rocking die per revolution(f) and the increasing of the inclination angle of upper  $die(\alpha)$  and the radius of workpiece.

According to the above-mentioned results, it is possible that when the size of product is determined, the minimum feeding rate of rocking die per revolution( $f_{min}$ ) can be estimated to avoid the center thinning.

In Fig.7 the calculated criterion on center thinning is compared with the experimental center thinning. The calculated results agree fairly well with the experimental ones.

It is found that there is good agreement between the theoretical criterion of the center thinning and the experimental results.

### 6.CONCLUSIONS

- 1)The height that center thinning begins(h) does not depend on the original height of workpiece and the rotational speed and the materials.
- 2) The height that center thinning begins(h) increases with the decreasing of the feeding rate of rocking die per revolution(f) and the increasing of the inclination angle of upper  $die(\alpha)$  and the radius of workpiece.
- 3)The criterion of center thinning is derived as follow:

$$\frac{f}{h} = 0.435\alpha \left(\frac{R}{h}\right)^{1.51}$$

Therefore, it is possible that when the size of product is determined, the minimum feeding rate of rocking die per revolution( $f_{min}$ ) can be estimated to avoid the center thinning.

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