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점소성 유한요소법에 의한 이차원 절삭의 구성인선 해석

BUILT-UP EDGE ANALYSIS OF ORTHOGONAL CUTTING BY VISCO-PLASTIC FINITE ELEMENT METHOD

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Abstract

The behavior of the work materials in the chip-tool interface in extremely high strain rates and temperatures is more that of viscous liquids than that of normal solid metals. In these circumstances the principles of fluid mechanics can be invoked to describe the metal flow in the neighborhood of the cutting edge. In the present paper an Eulerian finite element model is presented that simulates metal flow in the vicinity of the cutting edge when machining a low carbon steel with carbide cutting tool. The work material is assumed to obey visco-plastic (Bingham solid) constitutive law and Von Mises criterion. Heat generation is included in the model, assuming adiabatic conditions within each element. The mechanical and thermal properties of the work material are accepted to vary with the temperature. The model is based on the virtual work-stream function formulation. Emphasis is given on analyzing the formation of the stagnant metal zone ahead of the cutting edge. The model predicts flow field characteristics such as material velocity, effective stress and strain-rate distributions as well as built-up layer configuration

NOTATION

A	matrix of constants
c	specific heat
D	rate-of-strain tensor
F	traction forces
J	mechanical equivalent of heat
N	trial function
K(ψ)	overall stiffness matrix
K	thermal conductivity
k	yield stress
k_o, μ_o	values of k and μ obtained in ambient temperature
n, m, m*	material constant
Q_p	equivalent heat energy per unit volume
T	deviatoric part of stress tensor
Δt	passing time
u, *v*	components of the velocity **u** of a point of space with coordinates *x*, *y*
u_p	plastic work
μ	macroscopic viscosity coefficient
η	frictional coefficient
ρ	density
$\bar{\sigma}$, $\dot{\bar{\varepsilon}}$	average effective stress and strain rate in an element
$\Theta^{(e)}$	adiabatic temperature in an element
Θ_o	ambient temperature
$\Delta\Theta^{(e)}$	temperature rise, due to plastic work or friction
τ_f	friction stress
ψ, **ψ**	stream function
II_D	second invariant of **D**

1. INTRODUCTION

The metal cutting is one of the basic operations in manufacturing industry. Some estimates show that about 10% of all the metal produced is turned into chips [1]. Numerous theoretical and experimental investigations have been carried out in this field hitherto, but some of the basic aspects of the cutting process are not clearly understood yet. The recent development of computer technology enables us to simulate the cutting process more precisely by using the finite element technique, and some analyses along these lines have been performed in the last twenty years. Usui, Maekawa & Shirakashi [2] have utilized the finite element method to simulate a steady state cutting, while the shear angle, chip geometry and the flow lines of the work material are assumed in advance. Iwata, Osakada & Terasaka [3] have developed a rigid-plastic FEM for orthogonal cutting in a steady state, wherein fracture of the chip is predicted by employing a criterion based on stress history. The chip formation process has been continuously simulated from the beginning of the cutting by the elastic-plastic FEM developed by Strenkowski & Caroll [4]. In this model an adiabatic heating model has been assumed to simulate the heat generation effects and no heat is assumed to be conducted between the chip and workpiece. The first application to metal cutting using a viscoplastic model has been reported by Carol & Strenkowski [5]. Authors have introduced two orthogonal models. One is based on the updated Lagrangian formulation and the other is based on Eulerian formulation. Three-dimensional simulation of the cutting process has been reported by Maekawa *et al* [6], however the three-dimensional cutting mechanism is not sufficiently clarified [7]. Strenkowski & Moon [8] have represented an Eulerian FEM of orthogonal cutting. The FEM analysis of chip flow reported by Childs & Maekawa [9] requires some experimental data to be obtained in advance. In their analysis Komvopoulos & Erpenbeck [10] have used an elastic-perfectly plastic and an elastic-plastic with isotropic strain hardening and strain-rate sensitivity models. Lin & Lin [11] have represented a coupled model of the thermo-elastic-plastic material under large deformation. A three-dimensional deformation field in the chip formation process has been analyzed by the rigid-plastic FEM developed by Ueda & Manabe [7]. A rigid-plastic FEM has been applied also by Moriwaki & Sigimura [12] to analyze the mechanics of steady-state orthogonal micromachining of copper. In the model represented by Lin & Pan [13] deformation has been analyzed by finite-element method and heat transfer by a finite difference method.

In this paper, a finite element model of orthogonal metal cutting is described in an Eulerian framework. The model is used to predict the material flow characteristics such as material velocity and stream lines as well as stress and strain-rate distribution in the vicinity of the cutting edge. Emphasis is given on analyzing the formation of stagnant metal zone ahead the cutting edge when cutting low carbon steel with carbide cutting tool. The work material is assumed to obey visco-plastic constitutive law and Von Mises criterion. An adiabatic heating model is included to simulate the heat generation due to plastic work and friction within each element. The mechanical and thermal properties of the work material are allowed to vary with the temperature. The model is based on the virtual work-stream function formulation.

2. THEORETICAL FOUNDATIONS

2.1. *Constitutive law*

In cutting, the work material is subjected to extremely large deformations in primary and secondary shear zones. In this conditions, the mechanism of plastic deformation is changed from dislocation movement and strain hardening to one in which recovery process accompanies dislocation movement, resulting in the formation of very small equi-axed grains and many new boundaries [14]. Grain boundary sliding plays an important role now, plastic deformation takes place without strain hardening and metals behave like a very viscous liquid [1]. Campbell & Ferguson [15] have found that in these circumstances the flow strength begins to increase substantially as a function of strain rate, material obeys visco-plastic constitutive law and the elastic deformations can be neglected. Constitutive law can be expressed in an Eulerian form relating the stresses and strain rates in the form of a viscous incompressible fluid flow equation:

$$\mathbf{T} = \mu_A \mathbf{AD} \tag{1}$$

For two-dimensional problems eqn.(1) is given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \mu_A \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot\varepsilon_x \\ \dot\varepsilon_y \\ \dot\gamma_{xy} \end{Bmatrix} \tag{2}$$

The viscosity coefficient is defined as

$$\mu_A = \mu + k / 2\sqrt{II_D} \tag{3}$$

Since $\mu_A \to \infty$ when $II_D \to 0$, to avoid some computational awkward using eqn.(3), a slightly modified equation proposed by Papanastasiou [16] is employed instead, given by

$$\mu_A = \mu + k[1 - \exp(-m^* \sqrt{II_D})] / \sqrt{2II_D} \tag{4}$$

This equation approximates well the Bingham fluid with $m^* > 100$.

In terms of rheology, eqn.(1) represents a non-Newtonian fluid, so-called Bingham fluid, which obeys Von Mises yield criterion and in a state of flow can be regarded as a fluid with variable viscosity, function of the strain rate tensor, involving two material constants, μ and k.

2.2. *Virtual work-stream function formulation*

Finite element modeling of forming processes by a Bingham fluid flow in stream function formulation has been introduced by Zienkiewicz & Godbole [17]. The first FEM application of Bingham fluid flow to metal cutting based on penalty function method has been represented by Carol&Strenkowski [5]. In the context of the present paper the use of stream function and virtual work principle is made following the work of Zienkiewicz & Godbole [17] and hence, the basic theory will only be outlined briefly here.

Let *u*, *v* describe the components of the velocity **u** of a point of space with coordinates *x*, *y*

$$\mathbf{u}^T = [u, v]^T \tag{5}$$

Stream function ψ is defined as

$$u = -\partial\psi / \partial y; \quad v = \partial\psi / \partial x \tag{6a}$$

or

$$\mathbf{u} = \bar{\mathbf{L}}\psi; \quad \bar{\mathbf{L}} = \begin{Bmatrix} -\partial/\partial y \\ \partial/\partial x \end{Bmatrix} \tag{6b}$$

With the trial function **N** defined, we can express stream function as

$$\psi = \mathbf{N}\Psi \tag{7}$$

and from eqn.(6b) it follows

$$\mathbf{u}^T = [\bar{\mathbf{L}}\mathbf{N}\psi]^T \tag{8}$$

Rate-of-strain tensor **D** is defined in terms of velocities as

$$\mathbf{D}^T = [\mathbf{Lu}]^T \tag{9}$$

where

$$\mathbf{L} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \tag{10}$$

Substituting (8) into (9),

$$\mathbf{D}^T = [\mathbf{L}\bar{\mathbf{L}}\mathbf{N}\psi]^T \tag{11}$$

Let *V* represents a region of flow under consideration and *S* is a boundary surface on which the traction forces **F** act. Then, by the virtual work principle the equilibrium statement is given by

$$\int_V \delta\mathbf{D}^T\mathbf{T}dV - \int_S \delta\mathbf{u}^T\mathbf{F}dS = 0 \tag{12}$$

Substituting eqns.(1),(8) and (11) into eqn.(12) it follows

$$\int_V \delta[\mathbf{L}\bar{\mathbf{L}}\mathbf{N}\psi]^T \mu_A \mathbf{A}[\mathbf{L}\bar{\mathbf{L}}\mathbf{N}\psi]dV - \int_S \delta[\bar{\mathbf{L}}\mathbf{N}\psi]^T \mathbf{F}dS = 0 \tag{13}$$

or

$$\delta\psi^T\{[\int_V [\mathbf{L}\bar{\mathbf{L}}\mathbf{N}]^T \mu_A \mathbf{A}[\mathbf{L}\bar{\mathbf{L}}\mathbf{N}]dV]\psi - \int_S [\bar{\mathbf{L}}\mathbf{N}]^T \mathbf{F}dS\} = 0 \tag{14}$$

As this equation is true for any $\delta\psi$, the expression in the braces gives a set of equations

$$[\mathbf{K}(\psi)][\psi] - [\mathbf{R}] = 0 \tag{15}$$

where $\mathbf{K}(\psi)$ is the overall stiffness matrix,

$$[\mathbf{K}(\psi)] = [\int_V [\mathbf{L}\bar{\mathbf{L}}\mathbf{N}]^T \mu_A \mathbf{A}[\mathbf{L}\bar{\mathbf{L}}\mathbf{N}]dV] \tag{15a}$$

and **R** is the vector of all applied forces,

$$[\mathbf{R}] = [\int_S [\bar{\mathbf{L}}\mathbf{N}]^T \mathbf{F}dS] \tag{15b}$$

2.3. *Shape function*

It is seen from eqn.(14) that second differentials of **N** occur and thus shape functions satisfying C^1 continuity are required. In this paper we have used two-dimensional rectangular elements with Hermite interpolation polynomials [17] and 4 degree of freedom for each node.

2.4. *Temperature and temperature effect on the material properties*

At very high deformation rates an adiabatic temperature $\Theta^{(e)}$, assuming no heat losses, can be calculated within each element and is given by

$$\Theta^{(e)} = \Theta_o + \Delta\Theta^{(e)} \tag{16}$$

A. Plastic work

Temperature rise due to the plastic work is calculated according to

$$\Delta\Theta^{(e)} = Q_p / c\rho \tag{17}$$

where $Q_p = u_p / J$ and u_p is defined as

$$u_p = \bar\sigma\dot{\bar\varepsilon}\Delta t \tag{18}$$

B. Friction heat

Friction occurs at work-tool boundaries and the generated heat is distributed into the work (chip) and tool by the ratio [13] α_w / α_t, where $\alpha = (K / \rho c)^{1/2}$ and subscripts *w* and *t* indicate the work (chip) and tool, respectively. Hence

$$\Theta^{(e)} = \Theta_o + \frac{\alpha_w}{\alpha_w + \alpha_t} \frac{\bar\sigma\dot{\bar\varepsilon}\Delta t}{Jc\rho} \tag{19}$$

C. Temperature effect on the work material properties

The material properties strongly depend on the temperature [15], that is

$$k = k_o(\Theta_o / \Theta)^n \quad \text{and} \quad \mu = \mu_o(\Theta_o / \Theta)^m \tag{20}$$

The thermal properties c and K are also assumed to vary with the temperature.

2.5. *Friction characteristics*

Friction conditions at chip (work)-tool interface vary from heavily loaded at the cutting edge to lightly loaded to the end of contact zone. The friction stress τ_f is empirically related to the shear flow stress k of the work material by [9]

$$\tau_f = k\{1 - \exp[-(\eta\sigma_n / k)]\} \tag{21}$$

3. RESULTS AND DISCUSSION

To solve eqn.(15) a simple iterative procedure of the form

$$[\psi]^n = [\mathbf{K}(\psi^{n-1})]^{-1}[\mathbf{R}] \qquad (22)$$

is required since the matrix **K** is dependent on ψ. Also, the material and thermal properties of the workpiece depend on the temperature, which is unknown initially.

The FEM was used to simulate the cutting process of 0.12% carbon steel for depth of cut 0.2 mm, width of cut 1 mm and cutting velocity 0.5 mm/s. The experimental results represented by Stevenson & Oxley [18] were used to determine the chip geometry, since the Eulerian approach requires the chip boundary to be known in advance. An initial tool flank wear length of 0.1 mm was considered. The rake angle was selected zero degree. Fig.1 shows the geometry and boundary conditions used to simulate the cutting process. Tractionless boundary conditions are imposed along the finished surface as well.

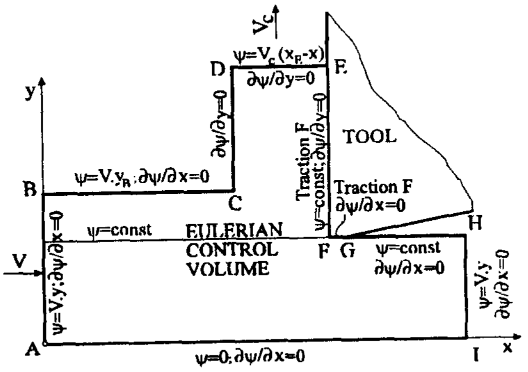

FIG. 1.Geometry and boundary conditions

Fig.2 shows the flowchart of the visco-plastic FEM of the orthogonal cutting process. The finite element grid is consisted of 524 four node rectangular elements, considerably refined in the vicinity of the cutting edge where the highest changes in metal flow are expected.

Friction along the tool face was modeled by introducing frictional forces to the surface nodes, opposing the direction of motion. Frictional forces are estimated as

$$F_f = \tau_f l w \qquad (23)$$

where l is the length of the infinite thin frictional element on the boundary, w is the width of cut and τ_f is defined from eqn.(21). Although the boundary frictional elements are very small, element distortion is not a problem with Eulerian model because the grid remains spatially fixed. Fig.3a shows stream lines' pattern for entire control volume while Fig.3b shows the area that includes both the primary and secondary shear zones.

The pattern pictures indicate a presence of a stagnant work material ahead of the tool rake face, which actually is the commonly known built-up layer. The shape of the stagnation zone is approximately triangular with a length of ~ 0.7 mm and maximum height of ~ 0.1 mm in a point about 0.05 mm along the rake face. This implies that the approach described in the present work could be useful in predicting the built-up edge/built-up layer formation, since the above results coincide fairly well with the common experimental observation in metal cutting.

Fig.4 shows the predicted effective strain rate contours in the entire volume (a) and in the vicinity of the cutting edge (b). It is evident the high concentration of the strain rates in the primary and secondary shear zone with values up to 1.10^4 s^{-1} and 1.10^5 s^{-1} respectively.

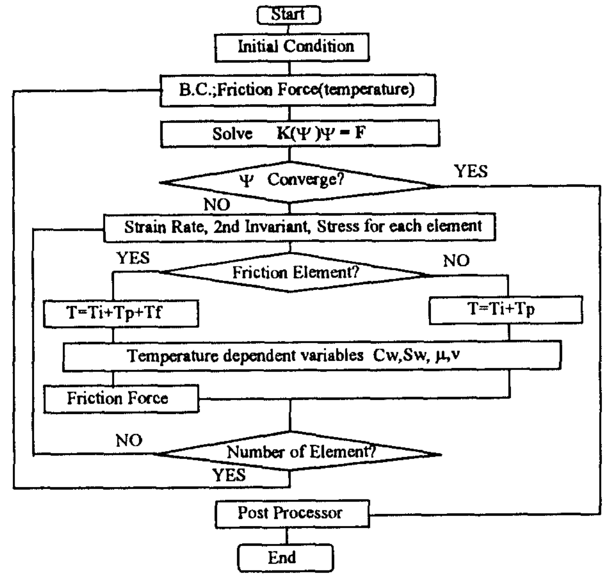

Fig.2 Flowchart of the Visco-Plastic FEM of Cutting Process

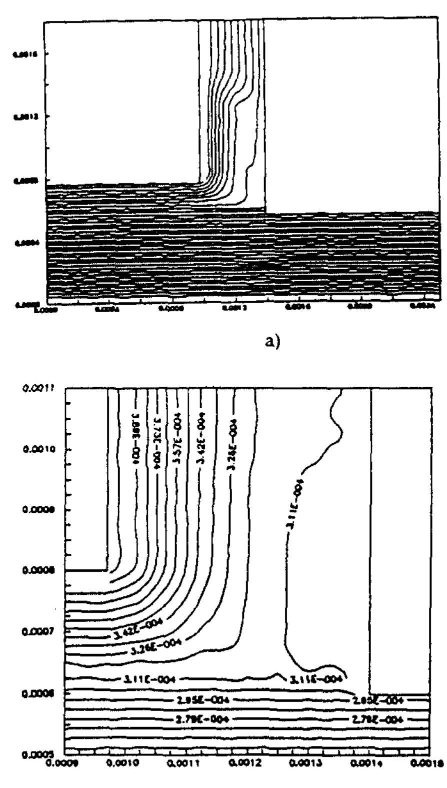

Fig.3. Plots of predicted stream lines

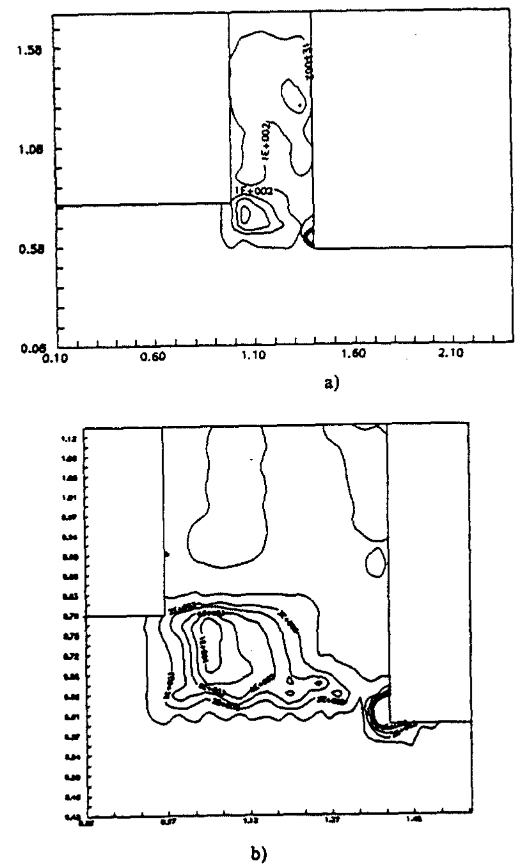

Fig.4. Plots of predicted effective strain rates

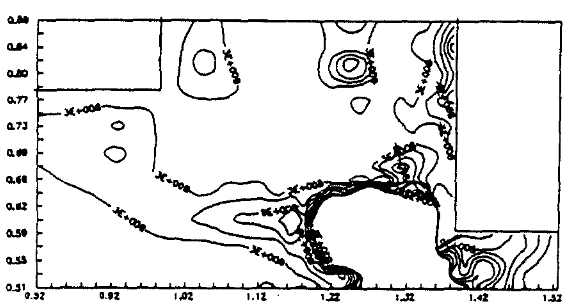

Fig.5 Plot of predicted effective stresses

The strain rate concentration is extremely high close to the cutting edge where the work material flows around the stagnant metal zone. Fig.5 shows the predicted effective stress contours in the vicinity of the cutting edge. Again a high stress concentration is clearly evident ahead the cutting edge with values up to 2000 MPa at the very cutting edge and to 30~50 MPa in the primary shear zone.

4. CONCLUSIONS

An Eulerian finite element model of orthogonal metal cutting has been presented. The model is based on virtual work-stream function formulation and was used to stimulate metal flow in the vicinity of the cutting edge. It is shown that the model could be useful in predicting such a phenomenon as built-up edge/built up layer formation.

The model might be improved additionally by introducing a coupled FEM to estimate cutting temperature since the approach adopted above is easy applicable but gives only an approximate solution. Also it is possible an iterative approach to be applied [19] to the chip boundary aiming determination of the chip shape.

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