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# LOCAL MOTION PLANNER FOR NONHOLONOMIC MOBILE ROBOTS

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**Abstract** This paper deals with the problem of motion planning for a unicycle-like robot. We present a simple local planner for unicycle model, based on an approximation of the desired configuration generated by local holonomic planner that ignores motion constraints. To guarantee a collision avoidance, we propose an inequality constraint, based on the motion analysis with the constant control input and time interval. Consequently, we formulate our problem as the constrained optimization problem and a feedback scheme based on local sensor information is established by simply solving this problem. Through simulations, we confirm the validity and effectiveness of our algorithm.

**Keywords** Local motion planner, Nonholonomic mobile robot, Collision avoidance

## 1. INTRODUCTION

In this paper, we address the problem of planning constrained motions where the constraints are nonholonomic in nature. Specifically, we focus on the motion planning of the unicycle-like robot.

Generally, for the nonholonomic robots, the control scheme of generating the trajectory between the initial and final configuration can be grouped into two categories: open-loop control and feedback control. Open-loop control scheme has attracted a significant amount of interest during the last few years. Tools from the differential geometric control theory have been utilized to derive this open-loop control input [1]. On the other hand, the feedback control schemes are more robust to their performance, however the restriction, imposed by R. W. Brockett [2], that nonholonomic system cannot be stabilized to a given configuration based on the continuous and smooth feedback law makes the design of control law difficult. This motivated some complex feedback laws, for example, the time-periodic function based law [3] and so on. A common property most of both the open-loop and feedback control scheme have is that they do not take into account the presence of the unknown obstacles and require a sequence of feasible targets to complete a point-to-point motion such as a car parking problem.

In contrast to these nonholonomic planners, the local holonomic planners furnish the simple and powerful feedback scheme for the mobile robot navigating through the partially known environment. Using these merits, A. D. Luca and his collegues [4] have proposed a feasible projection strategy to modify the output of local holonomic planner. Their scheme furnishes a very simple and powerful feedback motion planner in a local sense. Instead, their method has a serious drawback that it cannot guarantee the collision avoidance with obstacles in spite of the collision avoidance being a basic and intrinsic demand for motion planning.

The basic idea of this paper is on the extension of A. D. Luca *et al.*'s work. To overcome their serious drawback of no guarantee for the collision avoidance, we propose the inequality constraints after analyzing the robot's motion with the constant control inputs during the time interval for the command updating and combining this result with the local sensor information. We formulate our problem as the constrained optimization problem under the consideration of the lower-bounded curvature due to the mechanical restrictions. Through the computer simulations, we will show the validity and effectiveness of our algorithm.

## 2. MOTION PLANNER FOR NONHOLONOMIC MOBILE ROBOTS

Consider a unicycle-like robot positioned on the plane $\Re^2$ with respect to the base frame $< b >$, whose motion is governed by the combined action of both the angular velocity $w$ and the linear velocity vector $v$. A linear velocity vector $v$ is assumed to be always directed as $x$ axis of its attached frame $< a >$ where its origin is located at the center of robot, as depicted in Fig. 1. Then, the kinematic model of the unicycle-like robot, which involves the vehicle's Cartesian position $x$, $y$ and its own orientation $\theta$, is known as follows [1,4]:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \tag{1}$$

where $v$ is the linear velocity and $w$ is the angular velocity. The values of $x$, $y$, $\theta$ are all measured with respect to the base frame $< b >$. This model takes into account the nonholonomic constraint

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{2}$$

that specifies the tangent direction along any feasible path for the robot. Note that the unicycle model can be used as the simplified version of the car-like robot as shown in Fig. 2, by simply imposing some constraints on the control input vector $\mathbf{u} = (v, w)^T$: as depicted in Fig. 2, the constraint on the turning radius is expressed by $|\phi| \leq \phi_0$ where $\phi$ is the angle between the front wheels and the main direction of the car and $\phi_0$ is strictly positive real. Let us fix $\phi_0 = \frac{\pi}{4}$. In addition, we assume that the velocity of vehicle is upper-bounded by $v_{max}$. Then, the following constraint on the controls enables the unicycle model to be used as the alternative instead of the car-like robot when concerning the collision avoidance problem [1].

$$|w| \leq |v| \leq v_{max} \tag{3}$$

Through this paper, we assume that the equation (3) holds for all $t$ in the domain of definition. The equation (3) involves the steering and velocity limit due to the mechanical restrictions.

As stated earlier, A. D. Luca *et al.* [4] have presented a direct projection strategy to modify the output of local holonomic planner in the on-line manner. Their approach generates the velocity level control inputs that realize the desired motion, which is updated by a local holonomic planner, in the least-square sense. For any desired motion $\dot{\mathbf{x}}_{\mathbf{d}}$, their solution is given by the pseudo-inversion, as follows:

$$\begin{aligned} \mathbf{u} &= \mathbf{G}^{\#}(\mathbf{x})\dot{\mathbf{x}}_{\mathbf{d}} \\ &= [\mathbf{G}^T(\mathbf{x})\mathbf{G}(\mathbf{x})]^{-1}\mathbf{G}^T(\mathbf{x})\dot{\mathbf{x}}_{\mathbf{d}} \end{aligned} \tag{4}$$

where $\mathbf{x} \in \Re^n$ is a configuration vector of the generalized coordinate, $\mathbf{u} \in \Re^{n-m}$ is an admissible control input vector, and $\mathbf{G}(\mathbf{x})$ is a uncycle model. Although their simple and efficient local planner furnishes a feasible motion in most situations where including even the unknown environment, no guarantee for the collision avoidance can cause some serious problems. This is because the least-square error $[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]^T[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]$ can break the safety of mobile robot when the desired motion $\dot{\mathbf{x}}_{\mathbf{d}}$ does not hold the nonholonomic constraints.

Thus, now we analyze the robot's motion with the constant control input vector $\mathbf{u} = (v, w)^T$ which satisfies the equation (3), during the time interval of the command updating. This will be helpful to derive the constraints for guaranteeing the collision avoidance. For simplicity, we fix the attached frame $< a >$ shown in Fig. 1 at the command updating instant and with respect to this frame, we can obtain the robot's motion according to the analysis of the following equation. Note that we denote this fixed frame as $< f, a >$ through this paper.

$$\begin{pmatrix} \dot{x}_{f,a} \\ \dot{y}_{f,a} \\ \dot{\theta}_{f,a} \end{pmatrix} = \begin{pmatrix} \cos\theta_{f,a} & 0 \\ \sin\theta_{f,a} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_{f,a} \\ w_{f,a} \end{pmatrix}$$
$$(x_{f,a}(0), y_{f,a}(0), \theta_{f,a}(0))^T = (0,0,0)^T, 0 \leq t \leq \delta t_c \tag{5}$$

where $v_{f,a}$ and $w_{f,a}$ have the constant values respectively and $\delta t_c$ is the time interval for the control input updating. The subscript "$f, a$" denotes that the values of $x_{f,a}, y_{f,a}, \theta_{f,a}$ are all measured with respect to the attached frame which is fixed at the instant for the control input updating (that is, $t = 0$ in a local view). As shown in Fig. 3, a constant control input $\mathbf{u}_{\mathbf{f,a}} = (v_{f,a}, w_{f,a})^T$ yields a circular path with a turning radius $\rho$ during $\delta t_c$. Note that the value of $v_{f,a}$ is not negative at all because a linear velocity vector $v_{f,a}$ is assumed to be always directed as $x$ axis of the attached frame.

Integrating the equation for the main direction of vehicle yields

$$\int_0^t \dot{\theta}_{f,a} d\tau = \int_0^t w_{f,a} d\tau \tag{6}$$

$$\theta_{f,a}(t) = w_{f,a} t \qquad (\theta_{f,a}(0) = 0) \tag{7}$$

where $0 \leq t \leq \delta t_c$. Then, the value of turning radius $\rho$ can be computed as follows:

$$\rho|\theta_{f,a}(t)| = \int_0^t \sqrt{\left(\frac{dx_{f,a}}{d\tau}\right)^2 + \left(\frac{dy_{f,a}}{d\tau}\right)^2} d\tau \tag{8}$$

$$\rho = \left|\frac{v_{f,a}}{w_{f,a}}\right| \tag{9}$$

Thus, for the given $v_{f,a}$ and $w_{f,a}$, the robot tracks the following trajectory during $\delta t_c$.

$$\begin{aligned} x_{f,a}(t) &= sgn(w_{f,a})\rho\sin\theta_{f,a}(t) \\ y_{f,a}(t) &= sgn(w_{f,a})\rho(1 - \cos\theta_{f,a}(t)) \end{aligned} \tag{10}$$

where the function of $sgn(w_{f,a})$ returns the sign value.

To guarantee the collision avoidance, we must choose the control input $\mathbf{u}_{\mathbf{f,a}} = (v_{f,a}, w_{f,a})^T$ so that the predicted locus in the equation (10) will not intersect any edges of obstacles. For this purpose, we assume that the function of $d(\psi_{f,a})$ represents the model of robot's surroundings and it returns a proximity measure in the direction of $\psi_{f,a}$ with respect to the fixed frame $< f, a >$. This model is updated per every $\delta t_c$. During $\delta t_c$, the following constraint should be satisfied to guarantee the collsion avoidance for the given constant control input $\mathbf{u}_{\mathbf{f,a}} = (v_{f,a}, w_{f,a})^T$.

$$d(\psi_{f,a}(t)) > \sqrt{x_{f,a}^2(t) + y_{f,a}^2(t)} \tag{11}$$

$$= 2\rho|\sin(\frac{\theta_{f,a}(t)}{2})| \tag{12}$$

$$= 2\rho|\sin(\psi_{f,a}(t))| \tag{13}$$

where $\psi_{f,a}(t) = \frac{w_{f,a}t}{2}$, $0 \leq t \leq \delta t_c$ and $\rho = |\frac{v_{f,a}}{w_{f,a}}|$. Fig. 3 will be helpful for the comprehensive understanding of the derivation for the above equations.

Based on the derived results so far, we can formulate our algorithm as follows:

$$\text{Minimize } [\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]^T[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}] \quad \text{subject to} \quad d(\psi_{f,a}(t)) > 2\rho|\sin(\psi_{f,a}(t))| \tag{14}$$

$$\rho = \left|\frac{v_a}{w_a}\right| = \left|\frac{v}{w}\right| \tag{15}$$

$$\psi_{f,a}(t) = \frac{w_{f,a}t}{2} = \frac{wt}{2} \tag{16}$$

$$0 \leq t \leq \delta t_c, \quad |w| \leq |v| \leq v_{max}$$

The solution of the equation (14) can be found based on the nonlinear programming technique. The relationship between the base frame $< b >$ and the fixed frame $< f, a >$ can be easily implemented through a simple transformation matrix.

## 3. SIMULATION RESULTS

We demonstrated the proposed motion planner for a unicycle model in the presence with the circular obstacles, based on the potential field approach as the local holonomic motion planner.

Based on the potential field approach, we can easily generate the desired velocity at every sampling instant. The growth of obstacle range by the robot size can make the holonomic path planning for only the center point of robot possible. Then the desired position vector $\dot{\mathbf{q}}_{\mathbf{d}} = (\dot{x}_d, \dot{y}_d)$ can be obtained from the following equation.

$$\dot{\mathbf{q}}_{\mathbf{d}} = -\nabla_{\mathbf{q}}(U_a(\mathbf{q}) + U_r(\mathbf{q})) \tag{17}$$

where $U_a(\cdot)$ is an attractive potential function and $U_r(\cdot)$ is a repulsive potential function [5]. To complete finding the desired configuration, we require to assign the desired rotation of $\dot{\theta}_d$. For simplicity, we choose the desired steering input as follows:

$$\dot{\theta}_d = tan^{-1}\frac{\dot{y}_d}{\dot{x}_d} - \theta \tag{18}$$

where the function of $tan^{-1}(\cdot)$ returns the radian value from 0 to $2\pi$. Therefore, we can compute the desired configuration $\dot{\mathbf{x}}_{\mathbf{d}}$ for the equation (14).

For the uncertain environment cluttered with circular osbtacles, we perform the simulation for the proposed motion planner. All the parameter values used in the simulation are given in Table 1. To solve the problem in equation (14), we use the exhaustive search technique because the serach domain is relatively small and its method is very simple for the computer analysis. We believe that another nonlinear programming techniques can also solve this problem in a short time.

In Fig. 5, we can show the successful result of the local holonomic planner. The nonholonomic motions obtained for the initial $\theta_0 = 0$ are shown in Fig. 6, with the associated desired configuration. In Fig. 6, we can see that the proposed planner constructs the approximated path to the holonomic one and the position error goes to zero in the terminal phase. We have applied our method to several other situations, and the satisfactory results, which guarantee the collision avoidance, were always obtained.

## 4. CONCLUSIONS

We have presented a efficient motion planner for a unicycle-like robot in the presence of unknown obstacles. Our feedback scheme can be utilized in a real time, to guarantee the collision avoidance. This makes our method be useful in a real application.

The proposed scheme has been applied to the unicycle-like robot. The potential field approach has been used as the local holonomic motion planner. The simulation results confirmed that the obtained configuration approximates the desired motion in a least-square sense and the proposed planner provides the satisfactory results for the most cases. Since the proposed planner is based on the local strategy, even in the unknown situation the robot system can reach the target successfully.

Future research directions include a real application of our algorithm in a real world.

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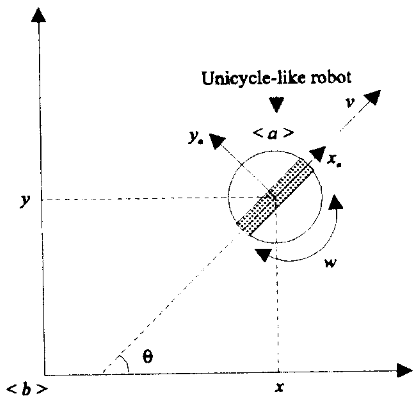


Fig. 1. A unicycle-like robot *w.r.t* the base frame

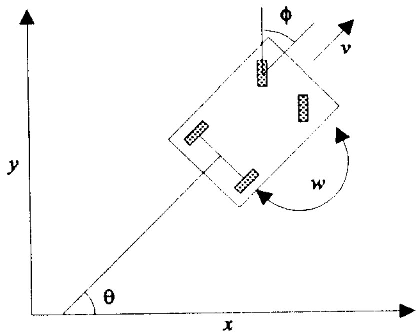


Fig. 2. A car-like robot

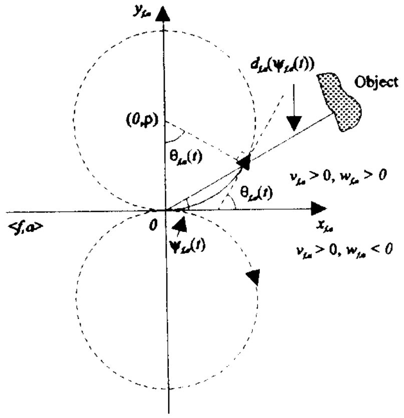


Fig. 3. A motion analysis during the time interval $\delta t_c$

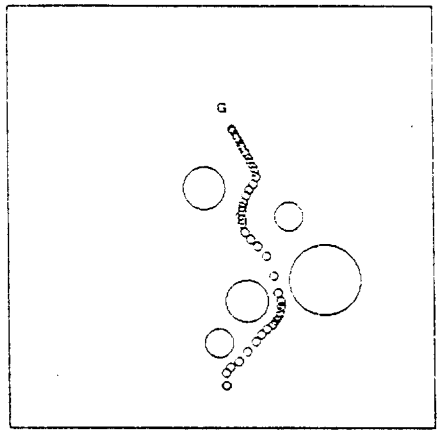


Fig. 4. A holonomic motion with potential field approach

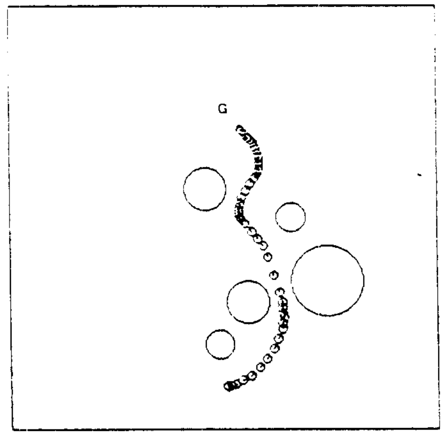


Fig. 5. A nonholonomic motion with potential field approach

Table 1. All the parameter values used in the simulations

| | |
|---|---|
| Environment size | $300m \times 300m$ |
| Radius of circular robot | $3m$ |
| The number of sensors | 36 |
| The time interval for the command updating $\delta t_c$ | $0.5sec$ |
| Maximum speed $v_{max}$ | $2.0m/sec$ |
| Sensor range | $20.0m$ |