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Observers for Linear Descriptor Systems with Unmeasurable Disturbances

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Abstract A simple method to design observers for linear descriptor systems with unmeasurable disturbances is presented, using the generalized matrix inverse. The unknown disturbance is represented by the response of a linear free system. The sufficient conditions for the existence of the observer are given. The design procedures of an identity and a minimal order observers are shown, respectively.

Keywords Observers, Descriptor systems, Unmeasurable disturbances, Observability

1. Introduction

The problem of observing the state of regular systems, subjected to unknown inputs, has been extensively studied in the last two decades, see *e.g.*, [1]–[3]. On the other hand, descriptor (singular or generalized state space) systems have been a more convenient description since it arises more naturally in many applications [4, 5], and the design of unknown–input observer for descriptor system has been studied in [6]–[8]. In their methods, the observers are constructed such that the unknown inputs don't affect the observer outputs.

Another and the most interesting approach to the design of such observers is to model the unknown disturbances as the response of a dynamical system so as to characterize and exploit all the significant disturbance modes [9]. The original system is then augmented with the disturbance model and an observer is designed for the augmented system.

The aim of this paper is to present an observer for linear descriptor systems for a class of unmeasurable disturbance which can be modeled by a dynamical free system. The existence conditions and design procedure for the observers are given.

2. Problem statement

Consider the following linear time–invariant descriptor system

$$E\dot{x} = Ax + Bu + Hd \tag{1a}$$

$$y = Cx \tag{1b}$$

where $x \in \Re^n$ is the descriptor variables, $u \in \Re^m$ is the known input, $d \in \Re^r$ is the unmeasurable disturbance, and $y \in \Re^p$ is the output. E, A, B, H and C are constant matrices with appropriate dimensions, and rank $C = p$. The matrix E is not necessarily nonsingular. And we assume that the system (1) is solvable, *i.e.*, there exists a scalar μ such that

$$\det(A - \mu E) \neq 0$$

The disturbance vector $d(t)$ is further assumed to be modeled by the linear dynamical system

$$\dot{v} = Mv \tag{2a}$$

$$d = Nv \tag{2b}$$

where $v \in \Re^q$ is the state vector. Since the dynamical modes of $v(t)$ which don't appear in (2b) are of no interest, without loss of generality the pair (N, M) is observable. For more details about the model (2) and its advantages, see [9].

By augmenting the system (1) with the disturbance model (2), the overall system may be described by

$$\bar{E}\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \tag{3a}$$

$$y = \bar{C}\bar{x} \tag{3b}$$

where

$$\bar{x} = \begin{bmatrix} x \\ v \end{bmatrix},\ \bar{E} = \begin{bmatrix} E & 0 \\ 0 & I_q \end{bmatrix},\ \bar{A} = \begin{bmatrix} A & HN \\ 0 & M \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$

Thus the problem of reconstructing the state of the descriptor system (1) is reduced to that of reconstructing the state of the augmented system (3). So, we will consider the existence conditions and construction method of the observer.

3. Existence Conditions for Observer

According to [10], an observer for the descriptor system (1) can be realized if the system (1) is observable in the sense of Rosenbrock [11]. Further the observability condition is relaxed by eliminating the purely static parts of the system, *i.e.*, an observer can be realized if the system is observable in the sence of Verghese [12]. Then, the observability of the descriptor system (3) will be considered in the following.

Theorem 1 :

The descriptor system (3) is observable in the sense of Rosenbrock if and only if the following conditions are satisfied.

$$\text{(i)}\ \ \text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n \tag{4}$$

$$\text{(ii)}\ \ \text{rank}\begin{bmatrix} sE-A \\ C \end{bmatrix} = n,\ \forall s \in \mathbf{C} \tag{5}$$

$$\text{(iii)}\ \ \text{rank}\begin{bmatrix} sE-A & -HN \\ 0 & sI-M \\ C & 0 \end{bmatrix} = n+q,\ \forall s \in \mathbf{C} \tag{6}$$

Proof : Using the well known observability conditions in the sense of Rosenbrock for the descriptor system (3), we have

$$\text{rank}\begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix} = n+q = \text{rank}\begin{bmatrix} E & 0 \\ 0 & I_q \\ C & 0 \end{bmatrix} = q + \text{rank}\begin{bmatrix} E \\ C \end{bmatrix} \tag{7}$$

which becomes the condition (i).

$$\text{rank}\begin{bmatrix} s\bar{E}-\bar{A} \\ \bar{C} \end{bmatrix} = n+q = \text{rank}\begin{bmatrix} sE-A & -HN \\ 0 & sI-M \\ C & 0 \end{bmatrix},\ \forall s \in \mathbf{C} \tag{8}$$

Hence, if (5) holds, the condition (8) simplifies to the condition (6), thereby establishing the proof of the sufficiency of the results. Since it is obvious that (5) is also a necessary condition that must hold, the proof of the necessity of the results immediately follows. Q.E.D.

Remark 1 : The conditions (i) and (ii) mean that the observability of the original system (1).

Remark 2 : Eq. (8) is equivalent to

$$\text{rank}\begin{bmatrix} sE-A & -H & 0 \\ C & 0 & 0 \\ 0 & 0 & I_q \end{bmatrix}\begin{bmatrix} I_n & 0 \\ 0 & N \\ 0 & sI-M \end{bmatrix} = n+q$$

from which A.-M. Mohamed [13] concluded that

$$\text{rank}\begin{bmatrix} N \\ sI-M \end{bmatrix} = q,\ \forall s \in \mathbf{C}$$

$$\text{rank}\begin{bmatrix} sE-A & -H \\ C & 0 \end{bmatrix} = n+q,\ \forall s \in \mathbf{C}$$

But this conclusion is not correct obviously.

4. Observer Synthesis

The observer for the system (3) may be constructed by using the procedure which was proposed in [10] under the assumption that the system (3) is obervable in the sense of Rosenbrock, *i.e.*,

$$\text{rank}\begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix} = n+q \tag{7}$$

$$\text{rank}\begin{bmatrix} s\bar{E}-\bar{A} \\ \bar{C} \end{bmatrix} = n+q,\ \forall s \in \mathbf{C} \tag{8}$$

Let's design an observer for the system (3) in the following form (Luenberger type)

$$\dot{z} = \hat{A}z + \hat{B}y + \hat{J}u \tag{9a}$$

$$\hat{\bar{x}} = \hat{C}z + \hat{D}y \tag{9b}$$

such that $\hat{\bar{x}} \to \bar{x}$ as $t \to \infty$ for any initial conditions, where $z \in \Re^l$ and $\hat{\bar{x}} \in \Re^{n+q}$.

Lemma 1 [10] **:**

The system (9) is an observer for the system (3) if

$$\text{Re}\ \lambda_i[\hat{A}] < 0,\quad i = 1, 2, \ldots, l \tag{10}$$

and if there exists a matrix $\bar{U} \in \Re^{l\times(n+q)}$ such that

$$\hat{A}\bar{U}\bar{E} + \hat{B}\bar{C} = \bar{U}\bar{A} \tag{11}$$

$$\hat{J} = \bar{U}\bar{B} \tag{12}$$

$$\hat{C}\bar{U}\bar{E} + \hat{D}\bar{C} = I_{n+q} \tag{13}$$

We will find the matrices $\hat{A}$, $\hat{B}$, $\hat{J}$, $\hat{C}$, $\hat{D}$ and $\bar{U}$ satisfying the conditions (10)–(13). First, we can have two matrices $\bar{E}^{\#}$ and $\bar{C}^{\#}$ such that

$$\bar{E}^{\#}\bar{E} + \bar{C}^{\#}\bar{C} = I_{n+q},\quad \det \bar{E}^{\#} \neq 0 \tag{14}$$

The existence of such matrices is guaranteed from the condition (7).

Remark 3 : It is remarked that two matrices $\bar{E}^\#$ and $\bar{C}^\#$ are obtained as

$$\bar{E}^\# = \begin{bmatrix} E^\# & 0 \\ 0 & I_q \end{bmatrix}, \quad \bar{C}^\# = \begin{bmatrix} C^\# \\ 0 \end{bmatrix} \tag{15}$$

where $E^\#$ and $C^\#$ satisfy

$$E^\# E + C^\# C = I_n, \quad \det E^\# \neq 0 \tag{16}$$

An algorithm to find $E^\#$ and $C^\#$ is given in Appendix.

Now we can apply the procedure in [10] directly to construct an identity observer and minimal order observer for the system (3) by using the matrices $\bar{E}^\#$ and $\bar{C}^\#$.

4.1. Identity observer

For the system (3), choose any matrices $\bar{E}^\#$ and $\bar{C}^\#$ satisfying (14). And letting

$$\hat{C} = I_{n+q}, \ \bar{U} = \bar{E}^\#, \ \hat{D} = \bar{C}^\# \tag{17}$$

we get

$$\hat{C}\bar{U}\bar{E} + \hat{D}\bar{C} = \bar{E}^\#\bar{E} + \bar{C}^\#\bar{C} = I_{n+q}$$

Condition (13) is thus satisfied, and $\operatorname{rank}\bar{U} = n+q$. Substituting (17) into (11) and use of (14) yields

$$\hat{A} + (\hat{B} - \hat{A}\bar{C}^\#)\bar{C} = \bar{E}^\#\bar{A} \tag{18}$$

Letting $\bar{K} = \hat{B} - \hat{A}\bar{C}^\#$, we have

$$\hat{A} = \bar{E}^\#\bar{A} - \bar{K}\bar{C} \tag{19}$$

$$\hat{B} = \hat{A}\bar{C}^\# + \bar{K} = \bar{E}^\#\bar{A}\bar{C}^\# + \bar{K} - \bar{K}\bar{C}\bar{C}^\# \tag{20}$$

Furthermore, from (12) and (17)

$$\hat{J} = \bar{E}^\#\bar{B} \tag{21}$$

Therefore, we obtained an identity observer as

$$\dot{z} = (\bar{E}^\#\bar{A} - \bar{K}\bar{C})z + (\bar{E}^\#\bar{A}\bar{C}^\# + \bar{K} - \bar{K}\bar{C}\bar{C}^\#)y + \bar{E}^\#\bar{B}u \tag{22a}$$

$$\hat{\bar{x}} = z + \bar{C}^\# y \tag{22b}$$

The remaining problem is the determination of the parameter $\bar{K}$ in such a way that a stable observer results. For this, in (19), the pair $(\bar{C}, \bar{E}^\#\bar{A})$ must be observable (the detectability is sufficient, strictly speaking. But an observer with freedom to choose all eigenvalues is frequently important.). That is,

$$\operatorname{rank}\begin{bmatrix} sI - \bar{E}^\#\bar{A} \\ \bar{C} \end{bmatrix} = n+q, \quad \forall s \in \mathbf{C} \tag{23}$$

is necessary. But, the following holds

$$\begin{bmatrix} sI - \bar{E}^\#\bar{A} \\ \bar{C} \end{bmatrix} = \begin{bmatrix} \bar{E}^\# & s\bar{C}^\# \\ 0 & I_p \end{bmatrix}\begin{bmatrix} s\bar{E} - \bar{A} \\ \bar{C} \end{bmatrix}$$

and $\det \bar{E}^\# = n+q$. Further from the assumption

$$\operatorname{rank}\begin{bmatrix} s\bar{E} - \bar{A} \\ \bar{C} \end{bmatrix} = n+q, \quad \forall s \in \mathbf{C}$$

Hence (23) is satisfied, and the system (22) can qualify as an observer.

Finally we can summarize the design procedure of identity observer as follows.

step 1. Find any $E^\#$ and $C^\#$ satisfying (16), see Appendix.

step 2. Let

$$\bar{E}^\# = \begin{bmatrix} E^\# & 0 \\ 0 & I_q \end{bmatrix}, \quad \bar{C}^\# = \begin{bmatrix} C^\# \\ 0 \end{bmatrix} \tag{24}$$

step 3. Find the gain $\bar{K}$ such that $\hat{A}$ in (19) is stable, and let

$$\bar{K} = \begin{bmatrix} \bar{K}_1 \\ \bar{K}_2 \end{bmatrix}$$

step 4. Observer parameters are determined as

$$\begin{aligned}
\hat{A} &= \bar{E}^\#\bar{A} - \bar{K}\bar{C} = \begin{bmatrix} E^\# A - \bar{K}_1 C & E^\# HN \\ -\bar{K}_2 C & M \end{bmatrix} \\
\hat{B} &= \hat{A}\bar{C}^\# + \bar{K} = \begin{bmatrix} (E^\# A - \bar{K}_1 C)C^\# + \bar{K}_1 \\ -\bar{K}_2 C C^\# + \bar{K}_2 \end{bmatrix} \\
\hat{J} &= \bar{E}^\#\bar{B} = \begin{bmatrix} E^\# B \\ 0 \end{bmatrix} \\
\hat{C} &= I_{n+q} = \begin{bmatrix} I_n & 0 \\ 0 & I_q \end{bmatrix} \\
\hat{D} &= \bar{C}^\# = \begin{bmatrix} C^\# \\ 0 \end{bmatrix}
\end{aligned} \tag{25}$$

4.2. Minimal order observer

We will show only the design procedure of minimal order observer in the following.

step 1. Find any $E^\#$ and $C^\#$ satisfying (16), and letting $\bar{E}^\#$ and $\bar{C}^\#$ as (24).

step 2. Choose a matrix $\widetilde{C} \in \Re^{(n+q-p)\times(n+q)}$ such that

$$\begin{bmatrix} \widetilde{C} \\ \bar{C} \end{bmatrix} \begin{matrix} \}n+q-p \\ \}p \end{matrix} \quad (\triangleq S^{-1}) \tag{26}$$

is nonsingular.

step 3. Decompose the matrix as

$$S^{-1}\bar{E}^{\#}\bar{A}S = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{matrix} \}n+q-p \\ \}p \end{matrix} \qquad (27)$$

$$\underbrace{\quad}_{n+q-p} \quad \underbrace{\quad}_{p}$$

step 4. Find a matrix $\bar{L}$ which makes

$$\hat{A} = \bar{A}_{11} + \bar{L}\bar{A}_{21} \qquad (28)$$

stable.

step 5. Observer parameters are determined as

$$\begin{aligned} \hat{A} &= \bar{A}_{11} + \bar{K}\bar{A}_{21} \\ \hat{B} &= \bar{A}_{12} + \bar{K}\bar{A}_{22} - \hat{A}(\bar{K} - \bar{U}^{\#}\bar{C}^{\#}) \\ \hat{J} &= \bar{U}^{\#}\bar{E}^{\#}\bar{B} \\ \hat{C} &= S\begin{bmatrix} I_{n+q-p} \\ 0 \end{bmatrix} \\ \hat{D} &= \bar{C}^{\#} + S\begin{bmatrix} -\bar{K} \\ I_p \end{bmatrix}(I_p - \bar{C}\bar{C}^{\#}) \end{aligned} \qquad (29)$$

where

$$\bar{U}^{\#} = \begin{bmatrix} I_{n+q-p} & \bar{K} \end{bmatrix} S^{-1}$$

5. Conclusions

In this paper, we have presented a method for designing the observers for linear descriptor systems with unmeasurable disturbance which can be modeled by a dynamical free system. The problem has more significance for descriptor than for regular systems, and the method is based only on simple matrix calculations.

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Appendix

< Algorithm for $(E^{\#}, C^{\#})$ >

Note that the condition

$$\text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n$$

implies that the pair (C, E) is observable, and then we can find any matrix G such that

$$E + GC \triangleq F$$

is nonsingular. Therefore we have

$$\begin{aligned} E^{\#} &= F^{-1} \\ C^{\#} &= F^{-1}G \end{aligned}$$