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Analysis and Control of The Falling Cat Phenomenon

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Abstract In this paper, we investigate so-called the falling cat problem. It is well known that a cat, when released from an upside down configuration starting from rest, is able to land on her feet without violating angular momentum conservation. This has being an interesting problem for engineers for a long time. We consider a model of a falling cat as connected two rigid columns, which is a nonholonomic system. We design the controller for it, using time-state control form of the model and exact linearization technique. Finally, we test the controller thorough simulation on the model of a falling cat.

Keywords Falling cat problem, Non-holonomic, Time-state control form

1 Introduction

It is well known that a cat, when released from an upside down configuration starting from rest, usually land on her feet. As shown in Fig 2, at glance, it seems that the angular momentum conservation is violated. How does a falling cat make it without violating angular momentum conservation? This phenomenon give rise to questions to control engineer, mechanical engineer, mathematician, and robotics.

Classical study[1-3], the main point was to explain the movement. Recently in [4], Kawamura showed that make a robot, which has two rigid body. But, the configuration, when the robot cat land on, was neglected.

In this paper we analy the falling cat phenomenon, design the controller which makes it land in right configuration, and verify it through simulation.

2 Control of a Falling Cat

2.1 Model

The model (as shown Fig.1) consists of two links which are rigid and identical. Each link corresponds to a limb. The model has no head, no feet and no tail. We assume that links are column and rotate around its centroidal principal axis.

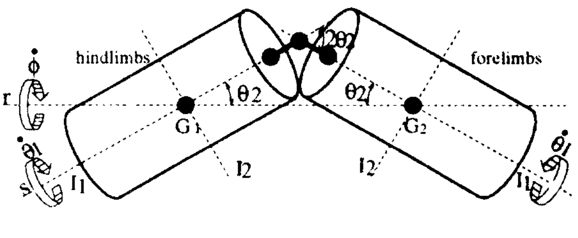

Figure 1: Two Link Model

The definition of the in Fig.1 as follows:

s	:	the centroidal principal axis of column
r	:	the axis through the center of column's gravity
θ_1	:	the angle around s (twisting angle)
$2\theta_2$	:	the angle between principal axes of inertia of columns (bending angle)
ϕ	:	the angle around the axis r
I_1	:	the inertial moment around s
I_2	:	the inertial moment around perpendicular of s

introduce frame s which is centroidal principal axe of inertia of body, frame r which passes through the center of either mass, the angle between centroidal principal axes of inertia of bode keeps $2\theta_2$. I_1 means the inertial moment in frame s, and I_r does the inertial moment in frame r.

2.2 State Equation

The law of momentum conservation yields the equation:

$$2I_r\dot{\phi} + 2I_1\dot{\theta_1}\cos\theta_2 = 0$$

Suppose

$$I_r = I_1\cos^2\theta_2 + I_2\sin^2\theta_2$$

where I_2 is the inertial moment in frame r. combining these two equation leads to

$$\frac{d}{dt}\begin{pmatrix}\phi\\ \theta_1\\ \theta_2\end{pmatrix} = \begin{pmatrix}A\\ 1\\ 0\end{pmatrix}u_1 + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u_2 \qquad (1)$$

,and

$$A = -\frac{I_1\cos\theta_2}{I_1\cos^2\theta_2 + I_2\sin^2\theta_2}$$

$$u_1 = \frac{d\theta_1}{dt}$$

$$u_2 = \frac{d\theta_2}{dt}$$

where $(\phi, \theta_1, \theta_2)^T$ is a state vector. u_1 and u_2 are control inputs.

2.3 Time-State control from

The system (1) can not be stabilized with any continuous static state feedbacks. i.e. there do not exist any continuous functions. Time-state control form is proposed of this system in order to overcome this problem. First, we transform the state equation (1) into a time-state control form.

Difine the difference between angles and their desired ones as follows.

$$\overline{\phi} = \phi - \widehat{\phi}$$

$$\overline{\theta_1} = \theta_1 - \widehat{\theta_1}$$

$$\overline{\theta_2} = \theta_2 - \widehat{\theta_2}$$

where $\widehat{\phi}, \widehat{\theta_1}, \widehat{\theta_2}$ are desired angles. With the coordinate transformation:

$$\xi_1 = \overline{\phi} + \frac{I_1 \cos\widehat{\theta_2}}{I_1 \cos^2\widehat{\theta_2} + I_2 \sin^2\widehat{\theta_2}} \cdot \overline{\theta_1}$$

$$\xi_2 = \overline{\theta_2}$$

$$\tau = \overline{\theta_1}$$

the state equation is transformed into

$$\frac{d}{d\tau}\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} B \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\mu_1 \qquad \text{(2-a)}$$

$$\frac{d\tau}{dt} = \frac{d\theta_1}{dt} = \mu_2 \qquad \text{(2-b)}$$

where

$$B = -\frac{I_1 \cos\left(\xi_2 + \widehat{\theta_2}\right)}{I_1 \cos^2\left(\xi_2 + \widehat{\theta_2}\right) + I_2 \sin^2\left(\xi_2 + \widehat{\theta_2}\right)} + \frac{I_1 \cos\widehat{\theta_2}}{I_1 \cos^2\widehat{\theta_2} + I_2 \sin^2\widehat{\theta_2}}$$

$$\mu_1 = \frac{u_2}{u_1}$$

$$\mu_2 = u_1$$

The state equation (2-a) is called as the state control part. Its state is $(\xi_1, \xi_2)^T$ and the new time scale is τ. The equation (2-b) is called the time control part. It is controlled by single input μ_2. In other words, while monotonally increasing θ_1, which corresponds to the twisting angle, we control the angle θ_2 which corresponds to the bending angle.

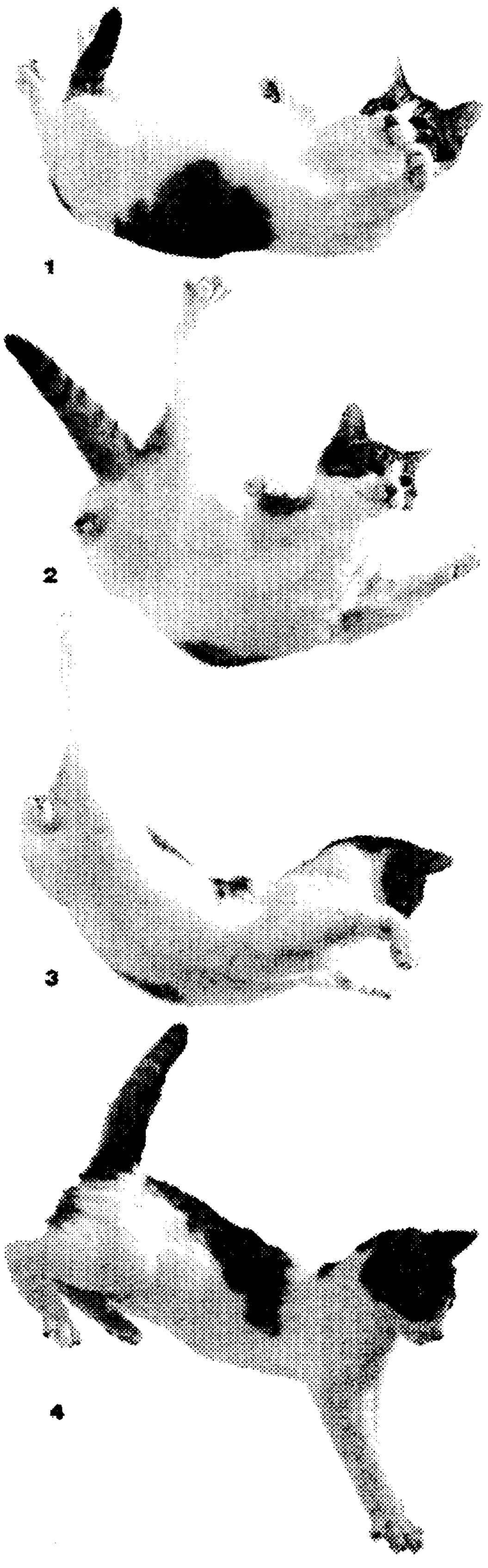

Figure 2: Falling Cat Phenomenon
Kodansha Panorama Pictorial Book 'CAT'[1]

2.4 Exactly Linearization

Since the state control part (2-a) is a controllable second order nonlinear system, it may be exactly linearized. we define some veriable to simplify the statement.

$$\eta_1 = \xi_1$$

$$\eta_2 = -\frac{I_1 \cos\left(\xi_2 + \widehat{\theta}_2\right)}{I_1 \cos^2\left(\xi_2 + \widehat{\theta}_2\right) + I_2 \sin^2\left(\xi_2 + \widehat{\theta}_2\right)} + \frac{I_1 \cos\widehat{\theta}_2}{I_1 \cos^2\widehat{\theta}_2 + I_2 \sin^2\widehat{\theta}_2}$$

$$\nu = \frac{C}{2\left(I_1 \cos^2\widehat{\theta}_2 + I_2 \sin^2\widehat{\theta}_2\right)^2}\mu_1 \quad (3)$$

where

$$C = I_1(3I_2 - 2I_1)\sin\left(\xi_2 + \widehat{\theta}_2\right)\cos^2\left(\xi_2 + \widehat{\theta}_2\right) + I_1 I_2 \sin^3\left(\xi_2 + \widehat{\theta}_2\right)$$

and ν is a new input. Then the state control part (2-a) will be transformed into the following linear state equation without approximation.

$$\frac{d}{d\tau}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\nu \quad (4)$$

Thus we can readily design a stabilizing controller for system (4). For example, a linear static state feedback

$$\nu = F\eta$$

Finally may be chosen, we get the original input μ_1 from equation (3).

3 Simulation

3.1 Limitation of Input

The results of the observation are the maximum of θ_3 is about 100[deg] and a falling cat needs about 50[cm] height. From the second observation, we see that $\dot{\theta}_2$ is 20[rad/s] and assume the maximum of $\dot{\theta}_3$ is 20[rad/s].

Fig.3 and Fig.4 show the results of simulation. When a falling cat land, the deviation between angles and their desired values are follows.

A falling cat, when released from initial position between -10[deg] and 10[deg], suffer from the limitation of input, it was stabilized with feedback mevertje:es.

4 Conclusion

In this paper, we studied how to control a falling cat, especially its configuration at landing time. The system can not be stabilized by any continuous static state feedbacks, because it is a nonholonomic system. We can design the controller for the system using time-state control form and the exact linearization technique. both its rotation and configuration.

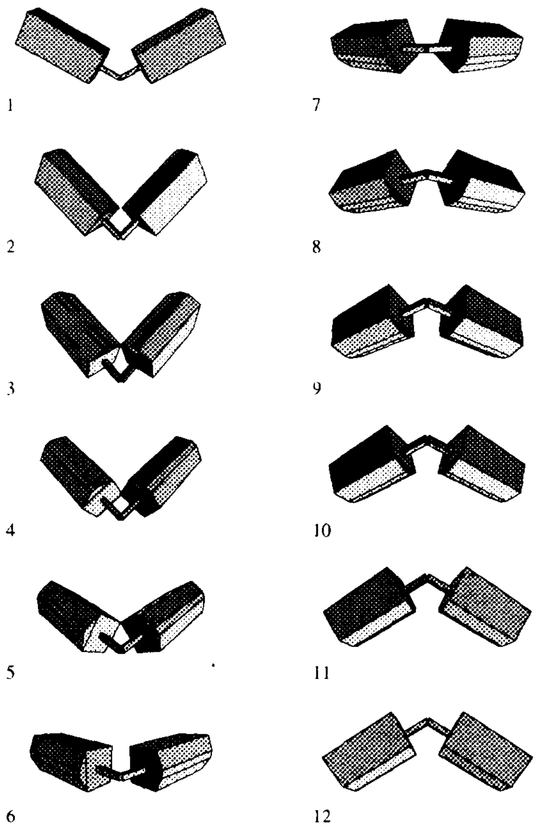

Figure 3: Initial Position = 0.0[deg]

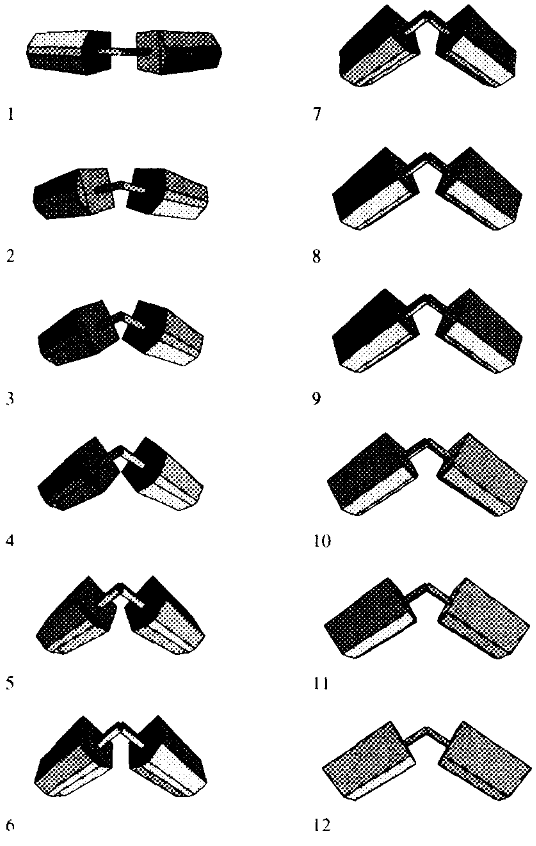

Figure 4: Initial Position = 90.0[deg]

Table 1: Deviation of angles

Initial Position[deg]	$\phi - \hat{\phi}$[rad]	$\theta_3 - \hat{\theta_3}$[rad]
-10	-0.000321	-0.0104
0	0.00246	-0.0138
10	0.00465	-0.0173
30	-0.0150	-0.0295
60	-0.0137	-0.128
90	0.0223	0.00901
120	-0.0137	0.128
150	-0.0612	0.149

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