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# Attitude Control of Space Robots with a Manipulator using Time-State Control Form

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**Abstracts** In this paper, we propose a new strategy for a space robot to control its attitude. A space robot is an example of a class of non-holonomic systems, a system of which cannot be stabilized into its equilibria with continuous static state feedbacks even in the case that the system is, in some sense, controllable. Thus, we cannot design stabilizing controllers for space robots using conventional control theories.

The strategy presented here transforms the non-holonomic system into a time-state control form, and allows us to make the state of the original system any desired one. In the stabilization, any conventional control theory can be applied.

For simplicity, a space robot with a two-link manipulator is considered, and a simulated motion of the controlled system is shown.

**Keywords** Space Robot, Non-holonomic system, Time-State Control Form, Manipulator, Exact linearization

## 1. INTRODUCTION

A space robot is an example of a class of non-holonomic systems, a system of which cannot be stabilized into its equilibria with continuous static state feedbacks even in the case that the system is, in a nonlinear systems theory, controllable. Thus, we cannot design stabilizing controllers for space robots using conventional control theories.

On the other hand, for a multi-trailer system which is the other example of a class of non-holonomic systems, some control strategies such as the chained form[4, 5] and the *time-state control form*[3] were proposed and applied.

In this paper, we will show a new strategy for a space robot to control its attitude using the time-state control form.

As for a space robot, the law of conservation of the angular momentum makes the system non-holonomic, while, when we consider a space robot with a manipulator, using the properties of the non-holonomic constraints, we can control not only the positions of the joints but also the attitude with respect to the inertia coordinates. In these cases, we do not have to consume the fuel to gain reaction of jet.

In section 2, a class of non-holonomic systems in question is defined.

In section 3, the time-state control form and a control strategy are introduced, which allow us to control positions of systems in a class of non-holonomic systems.

In section 4, a space robot with a two-link manipulator, for simplicity, is considered. At first, we get a state space representation of the system from the conservation of the angular momentum, and transform the equation into a time-state control form using a coordinate transformation and an input transformation. The *time* scale of the form is a function of the state of the original equation, and the behavior of a state of the form are stabilizable by conventional control theories along the advance of *time*. A stabilizing controller is given as a feedback, when we use exact linearization method. An attitude control of the original system accomplishes as the *time* goes back to zero.

Finally, a simulated motion of the controlled space robot is shown.

## 2. A CLASS OF NON-HOLONOMIC SYSTEMS

Many non-holonomic systems such as multi-trailer systems and space robots are modeled in the following $n$-state $m$-input nonlinear system

$$\frac{dx}{dt} = g_1(x)\,u_1 + g_2(x)\,u_2 + \cdots + g_m(x)\,u_m\,, \tag{1}$$

where $x \in R^n$ is a state, $u = (u_1, \cdots, u_m)^T \in R^m$ is an input with $m < n$, and $\{g_i(x)\}$ are $n$-dimensional vector valued functions with $\{g_1(0), g_2(0), \cdots, g_m(0)\}$ linearly independent. As Brockett[1] showed, the system (1) can not be stabilized with any continuous static state feedback, i.e. there do not exist any continuous functions $\{\gamma_i(x)\}$ with respect to $x$ such that a feedback $u_i = \gamma_i(x)\,(i = 1, \cdots, m)$ will stabilize the system. Thus, the controller design for non-holonomic systems is quite hard.

We proposed in a previous paper[3] a control strategy for the system (1) by defining the time-state control form. This strategy allows us to control a class of non-holonomic systems using conventional controller design methods.

## 3. TIME-STATE CONTROL FORM[3]

### *3.1 Transformation into the Time-State Control Form*

**The time-state control form** is the following

$$\frac{d\xi}{d\tau} = f_0(\xi) + f_1(\xi)\,\mu_1 + \cdots + f_{m-1}(\xi)\,\mu_{m-1}\,, \tag{2}$$

$$\frac{d\tau}{dt} = h(\xi, \tau)\,\mu_m\,. \tag{3}$$

This system consists of two state equations.

The former equation (2) is called **the state control part**, where $\xi \in R^{n-1}$ is a state, $\tau$ is a time scale rather than the actual time scale $t$, and $\mu_1, \cdots, \mu_{m-1}$ are $m-1$ control inputs. We assume $\xi = 0$ is an equilibrium point, i.e. $f_0(0) = 0$. We also assume that the first order approximation of this part is controllable, i.e. the system

$$\frac{d\xi}{d\tau} = \left.\frac{\partial f_0}{\partial \xi}\right|_{\xi=0} \xi + f_1(0)\,\mu_1 + \cdots + f_{m-1}(0)\,\mu_{m-1} \tag{4}$$

$$= A\,\xi + b_1\,\mu_1 + \cdots + b_{m-1}\,\mu_{m-1}\,. \tag{5}$$

is a controllable linear system.

The latter equation (3) is called **the time control form.** Its state is $\tau \in R^1$, i.e. the time scale of the state control part (2). It is controlled by single input $\mu_m$. In other words, this part controls the time flow of the state control part.

There is a class of non-holonomic systems (1) each of which can be transformed into the time-state control form by a coordinate transformation

$$\begin{pmatrix} \xi \\ \tau \end{pmatrix} = T(x) \quad (T(0) = 0), \tag{6}$$

and an input transformation

$$\mu_i = V_i(x, u_1, \cdots, u_m), \quad i = 1, 2, \cdots, m. \tag{7}$$

We will see in section 4 that a state equation of a space robot can be transformed into a time-state control form.

### *3.2 A Control Strategy for the Time-State Control Form*

In this subsection, we will show a control strategy which makes the state of the original non-holonomic system (1) approximately zero. We assume that the original system (1) is transformed into the time-state control form (2) (3) by the coordinate transformation (6) and the input transformation (7). Hence, all we have to do is to make $\xi$ and $\tau$ approximately zero.

Before showing the control strategy, we will consider two feedbacks $\{\alpha_i(\cdot)\}$ and $\{\beta_i(\cdot)\}$ which stabilize the state control part (2) in the following situations:

- A continuous static state feedback $\mu_i = \alpha_i(\xi)\,(i = 1, \cdots, m-1)$ with respect to $\xi$ stabilizes the state control part (2) as the time $\tau$ *increases* monotonically.
- A continuous static state feedback $\mu_i = \beta_i(\xi)\,(i = 1, \cdots, m-1)$ with respect to $\xi$ stabilizes the state control part (2) as the time $\tau$ *decreases* monotonically.

**Remarks:**
Since the first order approximation (5) of the state control part is controllable, we can easily design state feedbacks such as $\{\alpha_i(\cdot)\}$ and $\{\beta_i(\cdot)\}$ using conventional control theories. For example, at least we can design a linear state feedback $\mu = F\xi$ which stabilizes the approximated system (5), and it is well known that this feedback also stabilizes the original system (2) in a neighborhood of $\xi = 0$. Of course, we may use any control theory, such as nonlinear control theory geometric approach, to design stabilizing controllers for the state control part. We will use exact linearization to control a space robot in section 4. □

The following control strategy allows us to make the state $\xi$ and the time scale $\tau$ of the time-state control form approximately zero. Assume that $\mu_i = \alpha_i(\xi)$ and $\mu_i = \beta_i(\xi)$ are the above-mentioned feedbacks.

**Control Strategy**

**Step 1:**
Control the time control part (3) using the input $\mu_m$ so that $\tau$ monotonically increases, while we control the state control part using the inputs $\mu_i = \alpha_i(\xi)\,(i = 1, \cdots, m-1)$. At this step, $\xi$ converges to zero, but $\tau$ does not.

**Step 2:**
When $\xi$ becomes sufficiently close to zero with step 1, then we switch the controller. Control the time control part (3) using the input $\mu_m$ so that $\tau$ monotonically decreases, while we control the state control part using the inputs $\mu_i = \beta_i(\xi)\,(i = 1, \cdots, m-1)$. At this step, $\xi$ converges to zero, i.e. $\xi$ will not diverge, and $\tau$ decreases. We will stop $\tau$ decreasing when $\tau$ becomes zero.

**Step 3:**
If $\xi$ and $\tau$ is not sufficiently close to zero, repeat the steps 1 and 2. Otherwise, this is the end of this strategy.

**Remarks:**
From the property of the coordinate transformation (6), if the state $\xi$ and the time scale $\tau$ of the time-state control form are all zero, then the state of the original non-holonomic system (1) is also zero. If the coordinate transformation (6) and its inverse are both continuous mappings in a neighborhood of the origin, the state of the original non-holonomic system approaches zero as $\xi$ and $\tau$ of the time-state control form do so. □

## 4. CONTROL OF A SPACE ROBOT

### *4.1 A Space Robot with a Manipulator*

Only for simplicity, we will consider the space robot shown in Fig. 1.

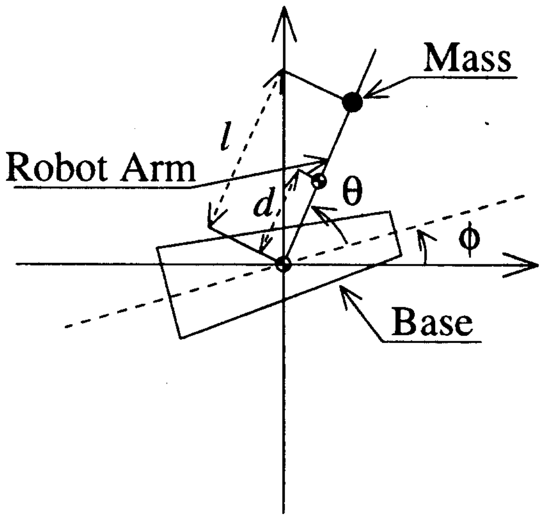


**Fig.1: A Space Robot with a Two-Link Manipulator**

The space robot consists of a base, an arm and a moving weight. We assume that the base and the arm are connected at the center of gravity of the base and an end point of the arm. We will use the following symbols.

$m_0$, $I_0$ : the mass and the moment of inertia of the base
$m_1$, $I_1$ : the mass and the moment of inertia of the arm
$m_2$, $I_2$ : the mass and the moment of inertia of the weight
$d$ : the distance between the center of gravity of the base and that of the arm
$\phi$ : the attitude of the base with respect to the inertia frame
$\theta$ : the angle between the base and the arm
$\ell$ : the distance between the center of gravity of the base and that of the weight

As control inputs, let us take the angular velocity $\dot{\theta}$ and the weight's velocity $\dot{\ell}$. This implies that the motion of the space robot is kinematic.

We would like to control the robot so that $\theta \to \theta_d$, $\ell \to \ell_d$ and $\phi \to \phi_d$ for any given $\theta_d$, $\ell_d$ and $\phi_d$.

### *4.2 Modeling of the Space Robot*

We assume that the space robot system above has zero total angular momentum at the initial time, and there exists no exogenous torque for the system.

From the basic equations of kinematics of space robots[2], we have the angular momentum conservation equation of the system

$$(c_0' + c_1\,\ell + c_2\,\ell^2)\,\dot\phi + (c_0 + c_1\,\ell + c_2\,\ell^2)\,\dot\theta = 0\,, \tag{8}$$

where

$$c_0 := I_1 + I_2 + \frac{m_1}{M}(m_0 + m_2)\,d^2, \tag{9}$$

$$c_0' := J + \frac{m_1}{M}(m_0 + m_2)\,d^2, \tag{10}$$

$$c_1 := -2\,\frac{m_1 m_2}{M}\,d, \tag{11}$$

$$c_2 := \frac{m_2}{M}(m_0 + m_1), \tag{12}$$

$$M := m_0 + m_1 + m_2,\ J := I_0 + I_1 + I_2. \tag{13}$$

Since the inputs of the system are $\dot\theta$ and $\dot\ell$, we can readily obtain the following state equation which represents the behavior of the position error of the space robot

$$\frac{d}{dt}\begin{pmatrix}\bar\theta\\ \bar\ell\\ \bar\phi\end{pmatrix} = \begin{pmatrix}1\\ 0\\ H(\bar\ell)\end{pmatrix} u_1 + \begin{pmatrix}0\\ 1\\ 0\end{pmatrix} u_2\,, \tag{14}$$

where

$$(\bar\theta, \bar\ell, \bar\phi)^{\mathrm T} := (\theta, \ell, \phi)^{\mathrm T} - (\theta_d, \ell_d, \phi_d)^{\mathrm T} \tag{15}$$

is a position error, $u_1 := \dot\theta$ and $u_2 := \dot\ell$ are inputs, and

$$H(\bar\ell) := -\frac{c_0 + c_1(\bar\ell + \ell_d) + c_2(\bar\ell + \ell_d)^2}{c_0' + c_1(\bar\ell + \ell_d) + c_2(\bar\ell + \ell_d)^2}. \tag{16}$$

Thus, our purpose is to make the state $(\bar\theta, \bar\ell, \bar\phi)^{\mathrm T}$ zero.

Obviously, the state equation (14) satisfies the conditions in the section 2, therefore this system can not be stabilized with any continuous static state feedback. In order to control this system, we will use the time-state control form.

### *4.3 Designing a Control System*

#### *4.3.1 Transformation into a Time-State Control Form*

We will transform the state equation (14) into a time-state control form using a coordinate transformation and an input transformation.

We define new state variables $\xi := (\xi_1, \xi_2)^{\mathrm T}$, $\tau$ as

$$\xi_1 := \bar\phi - H(0)\,\bar\theta\,, \tag{17}$$

$$\xi_2 := \bar\ell\,, \tag{18}$$

$$\tau := \bar\theta\,, \tag{19}$$

and new inputs

$$\mu_1 := \frac{u_2}{u_1},\quad \mu_2 := u_1\,. \tag{20}$$

From the above transformations, the state equation (14) will be transformed into the following time-state control form

$$\frac{d\xi}{d\tau} = \begin{pmatrix}H(\xi_2) - H(0)\\ 0\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\mu_1\,, \tag{21}$$

$$\frac{d\tau}{dt} = \mu_2\,, \tag{22}$$

where $H(\xi_2) = H(\bar\ell)$ for $\xi_2 = \bar\ell$. It is easily seen that $\xi = 0$ is an equilibrium point of the state control part (21), and the first order approximation of the part is controllable. As it will be shown in section 4.3.2, the state control part can be stabilized by exact linearization method.

Someone may wonder if there is any difficulty by using the input $\mu_1 = u_2/u_1$. When we control the state control part (21) with the input $\mu_1$, the time scale $\tau$ must be increasing (or decreasing), i.e. $\mu_2 = u_1 \neq 0$ from the time control part (22). Thus, $\mu_1$ will be finite if $u_2$ is finite. This implies that there is no difficulty by using $\mu_1 = u_2/u_1$.

#### *4.3.2 Exact Linearization of the State Control Part*

Since the state control part (21) is a controllable second order nonlinear system, it can be exactly linearized[6] unless $\ell_d = \frac{m_1}{m_0 + m_1}\,d$.

We define new state variables $\eta := (\eta_1, \eta_2)^{\mathrm T}$ as

$$\eta_1 := \xi_1\,, \tag{23}$$

$$\eta_2 := \frac{d}{d\tau}\xi_1 = -\frac{k_0 + k_1\,\xi_2 + k_2\,\xi_2^{\,2}}{k_0' + k_1\,\xi_2 + k_2\,\xi_2^{\,2}} + \frac{k_0}{k_0'}\,. \tag{24}$$

and a new input

$$\nu := -\frac{(k_1 + 2\,k_2\,\xi_2)\,I_0}{(k_0' + k_1\,\xi_2 + k_2\,\xi_2^{\,2})^2}\,\mu_1\,, \tag{25}$$

where

$$k_0 := c_0 + c_1\,\ell_d + c_2\,\ell_d^{\,2}\,,\quad k_0' := I_0 + k_0\,,\qquad k_1 := c_1 + 2\,c_2\,\ell_d\,,\quad k_2 := c_2\,. \tag{26}$$

From the above transformations, the state control part (21) will be transformed into the following linear state equation without approximation

$$\frac{d\eta}{d\tau} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\eta + \begin{pmatrix}0\\ 1\end{pmatrix}\nu\,. \tag{27}$$

Thus, we can readily design stabilizing controller for (27), for example, a linear static state feedback

$$\nu = F\,\eta \tag{28}$$

is available. In this case, the stabilizing controller for the original state control part (21) should be

$$\mu_1 = -\frac{(k_0' + k_1\,\xi_2 + k_2\,\xi_2^{\,2})^2}{(k_1 + 2\,k_2\,\xi_2)\,I_0}\cdot F \cdot \begin{pmatrix}\xi_1\\ -\frac{k_0 + k_1\,\xi_2 + k_2\,\xi_2^{\,2}}{k_0' + k_1\,\xi_2 + k_2\,\xi_2^{\,2}} + \frac{k_0}{k_0'}\end{pmatrix}. \tag{29}$$

From mechanical properties of the space robot, under many practical cases the coordinate transformation (23) (24) and the input transformation (25) will be valid globally so that controllers, such as (29), for the state control part (21) will be also valid globally.

In the sequel, let us use the controller (29). Hence, all we have to do is to design the linear static state feedback gain $F$ in (28).

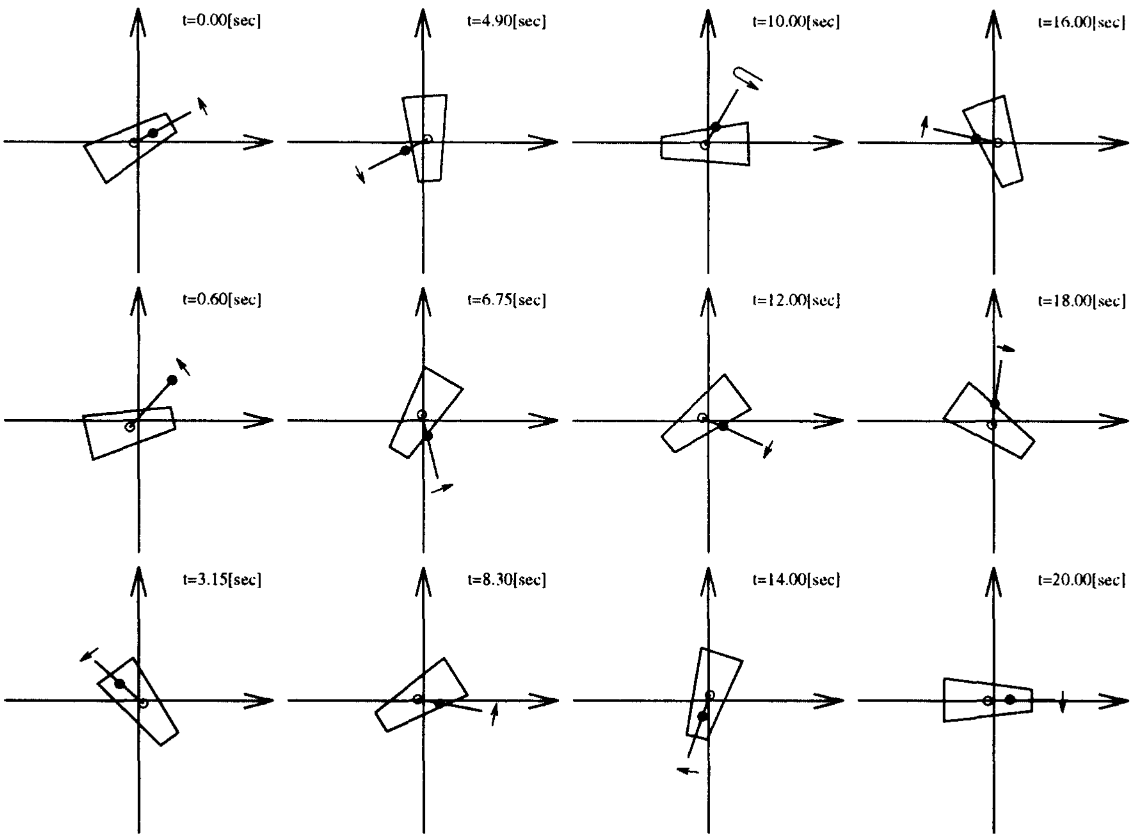


Fig.2: A Simulated Motion of the Space Robot

### *4.4 For the Control Strategy*

For the control strategy mentioned in section 3.2, two types of stabilizing controllers are required: one for the increasing time scale , another for the decreasing time scale. Since exact linearization of a system is independent of a direction of the time scale, we need only to design a stabilizing feedback gain $F = F_{\rm inc}$ for the system (27), and $F = F_{\rm dec}$ for the system

$$\frac{d\eta}{d\tau'} = -\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \eta - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \nu, \qquad (30)$$

where $\tau' := -\tau$.

Utilizing the linear static state feedback gains $F_{\rm inc}$ and $F_{\rm dec}$ with the control strategy, we can control the position of the robot.

### *4.5 Simulation*

Fig. 2 shows the simulated motion of the space robot, which is controlled with our strategy from the initial position $(\theta_0, \ell_0, \phi_0)^{\rm T}$ to the desired one $(\theta_d, \ell_d, \phi_d)^{\rm T} = (\theta_0, \ell_0, \phi_0 + \frac{\pi}{6})^{\rm T}$.

## 5. CONCLUSION

We applied the control strategy based on the time-state control form to the space robot with a two-link manipulator. An attitude control of the space robot was accomplished by a state feedback controller designed with exact linearization method. It is expected that using conventional control theories, we can also design other controllers robust for the disturbances and noises which do not conflict non-holonomic constraints of the system.

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