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AVERAGE PERFORMANCE OF RISK-SENSITIVE CONTROLLED ORBITING SATELLITE AND THREE-DEGREE-OF-FREEDOM STRUCTURE

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Abstract

The satellite in a circular orbit about a planet with disturbances and a three-degree-of-freedom (3DOF) structure under seismic excitations are modeled by the linear stochastic differential equations. Then the risk-sensitive optimal control method is applied to those equations. The mean and the variance of the cost function varies with respect to the risk-sensitivity parameter, γ_{RS}. For a particular risk-sensitivity parameter value, risk-sensitive control reduces to LQG control. Furthermore, the derivation of the mean square value of the state and control action are given for a finite-horizon full-state-feedback risk-sensitive control system. The risk-sensitive controller outperforms a classical LQG controller in the mean square sense of the state and the control action.

Keywords Risk-Sensitive, LQG, Performance, Satellite, Structure

1. FULL STATE FEEDBACK RISK-SENSITIVE CONTROL

In his paper "Optimal Stochastic Linear Systems with Exponential Performance Criteria and Their Relationship to Deterministic Differential Games," Jacobson defines and solves the linear exponential quadratic Gaussian control problem [3]. In this setting the LQG method is extended by replacing the quadratic criterion with the exponential of a quadratic function. Then the solution shows that the controller depends on the statistics of the noise. The system considered in the paper is given in equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + E(t)w(t) \tag{1}$$

The problem is to minimize equation

$$\begin{aligned} J_J^\sigma &= \sigma\phi E\left\{\exp\left(\frac{\sigma\phi}{2}\int_0^{t_F}[x'(t)Qx(t)+u'(t)Ru(t)]\,dt \right.\right. \\ &\left.\left. +\frac{1}{2}x'(t_F)Px(t_F)\right)\right\} \end{aligned} \tag{2}$$

with respect to $u(\cdot)$ and subject to (1), where w is a Gaussian white process with $E\{w(t)\} = 0$ and $E\{w(t)w'(\sigma)\} = S\delta(t-\sigma)$. Here σ denotes a sign operator which is equal to -1 for the risk seeking case and $+1$ for the risk aversive case, and ϕ denotes a real scalar parameter. Assuming perfect state observation, the solutions are derived using the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} -\frac{\partial J^\sigma(x,t)}{\partial t} &= \min_u\left\{\left(\frac{\partial J^\sigma(x,t)}{\partial x}\right)'(A(t)x+B(t)u)\right. \\ &+ tr\left(\frac{1}{2}C(t)SC'(t)\frac{\partial^2 J^\sigma(x,t)}{\partial x^2}\right) \\ &+ \left.\frac{\sigma\phi}{2}(x'Qx+u'Ru)J^\sigma(x,t)\right\}. \end{aligned} \tag{3}$$

Take derivative of both sides with respect to u to get

$$u^\sigma(x,t) = -\frac{R^{-1}B(t)\frac{\partial J^\sigma(x,t)}{\partial x}}{\sigma\phi J^\sigma(x,t)}. \tag{4}$$

If we assume an optimal cost of the form

$$J_J^\sigma = \sigma\phi F^\sigma(t)\exp\left(\frac{\sigma\phi}{2}x'M^\sigma(t)x\right) \tag{5}$$

and substitute the above equations into the Hamilton-Jacobi-Bellman equation (3), we get two differential equations similar to the results in the last section, namely

$$\begin{aligned} -\dot{M}^\sigma(t) &= Q - M^\sigma(t)(B(t)R^{-1}B'(t) \\ &\quad -\sigma\phi C(t)SC'(t))M^\sigma(t) \\ &\quad +M^\sigma(t)A(t)+A'(t)M^\sigma(t) \end{aligned} \tag{6}$$

$$-\dot{F}^\sigma(t) = \frac{\sigma\phi}{2}F^\sigma tr[C(t)SC'(t)M^\sigma(t)] \tag{7}$$

with the boundary conditions $M^\sigma(t_F) = P$ and $F^\sigma(t_F) = 1$. Finally, then, the minimum cost is

$$\min J_J^\sigma(x,0) = \sigma\phi F^\sigma(0)\exp\left(\frac{\sigma\phi}{2}x'(0)M(0)x(0)\right). \tag{8}$$

2. AVERAGE PERFORMANCE OF AN RS CONTROLLER

In LQG control, we are minimizing the average value of the cost function, $\hat{J}(t_F)$, which is closely related to the minimization of the covariance of the state, x. In RS control this average value of the cost function and the covariance of the state can be varied by choosing the appropriate risk-sensitivity parameter, γ_{RS}. Consider the system described by the Ito type stochastic differential equation,

$$dx(t) = (Ax(t)+Bu(t))\,dt + E\,dw(t), \quad x(0) = x_0, \tag{9}$$

where w is a Brownian motion, $x(t) \in \mathbb{R}^n$ is the state, and $u(t) \in R^m$ is the control action. Assume the following:

$$\begin{aligned} E\{w\} = 0, &\quad E\{dw(t)dw'(t)\} = W dt, \\ E\{x(0)\} = 0, &\quad E\{x(0)x'(0)\} = \Sigma_0. \end{aligned} \tag{10}$$

Furthermore assume that w and $x(0)$ are mutually independent. The cost function that is to be minimized is given by

$$J_{RS}(\gamma) = \log\left(E\left\{\exp\left(\frac{1}{2\gamma_{RS}}\hat{J}(t_F)\right)\right\}\right) \quad (11)$$

where

$$\hat{J}(t_F) = \int_0^{t_F} [x'(t)Qx(t) + u'(t)Ru(t)]\, dt, \quad (12)$$

$Q \geq 0$, and $R > 0$. Then the state feedback optimal controller is found to be [4]

$$u(t) = k(x(t)) = -R^{-1}B'S(t)x(t) = -K(t)x(t) \quad (13)$$

where $S(t)$ is obtained from

$$\dot{S}(t) + A'S(t) + S(t)A + Q - S(t)\left(BR^{-1}B' - \frac{1}{\gamma_{RS}}EWE'\right)S(t) = 0. \quad (14)$$

Proposition 2.1 *For a full-state-feedback RS control with the above assumptions, the covariace of the state, $E\{x(t)x'(t)\} = \Sigma(t)$, is given by*

$$\begin{aligned}\dot{\Sigma}(t) &= (A - BR^{-1}B'S(t))\Sigma(t)\\ &+ \Sigma(t)(A - BR^{-1}B'S(t))' + EWE', \quad (15)\end{aligned}$$

with $\Sigma(0) = \Sigma_0$; the covariance of the control force is obtained from

$$E\{u(t)u'(t)\} = R^{-1}B'S(t)\Sigma(t)S(t)BR^{-1}; \quad (16)$$

and the average value of the cost function, $\hat{J}(t_F)$ is given by

$$E\{\hat{J}(t_F)\} = tr\left[S(0)\Sigma(0) + \int_0^{t_F}\left(S(t)EWE' - \frac{1}{\gamma_{RS}}S(t)EWE'S(t)\Sigma(t)\right)dt\right]. \quad (17)$$

Thus, we note that by varying the risk-sensitivity parameter, γ_{RS}, we can change the average performance. And as γ_{RS} approaches ∞ we get the classical linear quadratic Gaussian (LQG) average performance.

2.1. A Simple Satellite Example of an RS Controller

A linear perturbation model of a satellite in orbit about a planet with an ideal inverse-square gravity field are given by (see [1, p. 569])

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 2R\omega \\ 0 & 0 & -\frac{2\omega}{R} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} w(t)$$

where $x = [r \;\; \theta \;\; \dot{r} \;\; \dot{\theta}]'$, r and θ describe the position and the angle of the satellite in the polar coordinate. The constants, $R = 7063$km is the orbit radius, $\omega = \frac{\mu}{R^3} = 1.064 \times 10^{-3}$rad/s, and $w(t)$ is the white noise with zero mean with $E\{w(t)w(\tau)\} = W\delta(t-\tau)$. Here $w(t)$ is viewed as some perturbation in the space. The terminal time t_F is taken to be 10 seconds.

2.2. Structure Example of an RS Controller

A 3DOF, single-bay structure with an active tendon controller as shown in Figure 5 is considered here. The structure is subject to an one-dimensional earthquake excitation. Let the state x consists of the displacements of first, second, and third floor, augmented with the velocities of first, second, and third floor respectively. If we assume a simple shear frame model for the structure, then we can write the governing equations of motion in state space form

$$\begin{aligned} dx(t) &= \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix} x(t)\, dt \\ &+ \begin{bmatrix} 0 \\ M_s^{-1}B_s \end{bmatrix} u(t)\, dt + \begin{bmatrix} 0 \\ -\Gamma_s \end{bmatrix} dw(t) \end{aligned}$$

where

$$M_s = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \quad B_s = \begin{bmatrix} -4k_c \cos\alpha \\ 0 \\ 0 \end{bmatrix}$$

$$C_s = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \quad \Gamma_s = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$K_s = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

and m_i, c_i, k_i are the mass, damping, and stiffness, respectively, associated with the i-th floor of the building. We assume the mass, m, stiffness k, and damping c to be independent normal random variables. The Brownian motion $w(t)$ with $E\{dw(t)\} = 0$ and $E\{dw(t)dw(t)'\} = Wdt$; in this example, $W = 1.00$ in^2/sec^3. The terminal time, t_F, is taken to be 10 seconds.

We designed the controller for this system assuming nominal parameter values for the cost function (11). The parameters, mass, damping, and stiffness are chosen to match modal frequencies and dampings of the experimental structure in [2].

3. CONCLUSIONS

This paper derived the average performance of an RS controller. As applicaitons, a satellite in orbit about a planet and a 3DOF structure were considered. In both cases the RMS value of the states were reduced by decreasing the risk-sensitivity parameter, γ_{RS}.

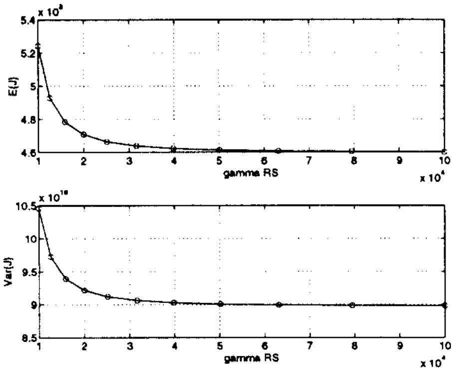

Figure 1: Full-State-Feedback; Optimal Mean and Variance

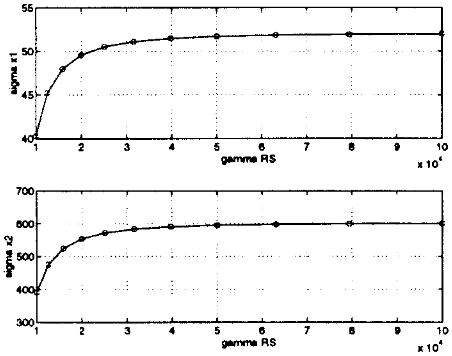

Figure 2: Full-State-Feedback; RMS of r and θ

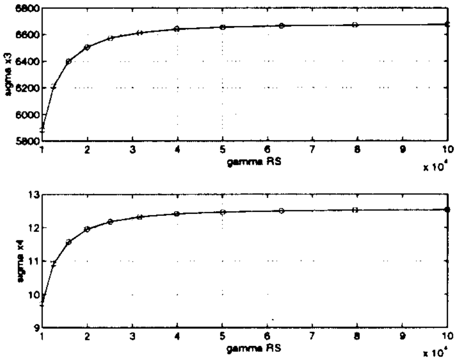

Figure 3: Full-State-Feedback; RMS of $\dot{r}$ and $\dot{\theta}$

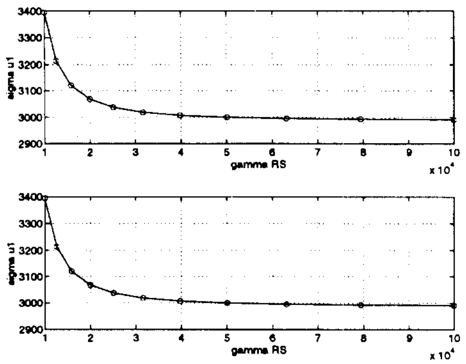

Figure 4: Full-State-Feedback; RMS of the inputs, u_1 and u_2

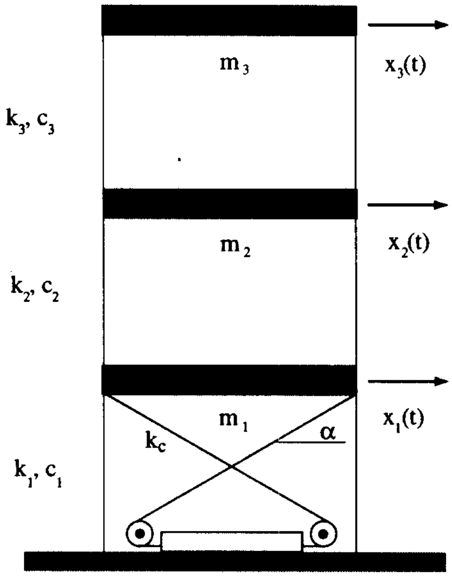

Figure 5: Schematic Diagram for Three Degree-of-Freedom Structure

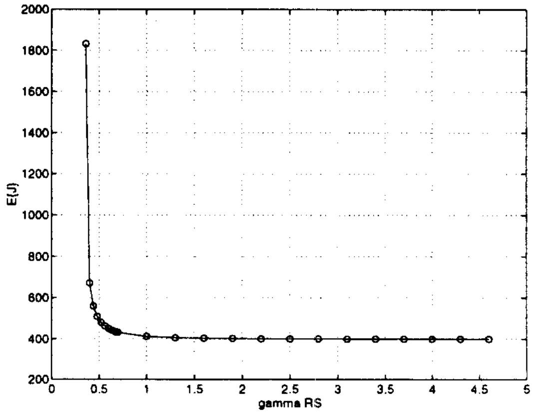

Figure 6: Optimal Mean; Full-State-Feedback, RS, 3DOF

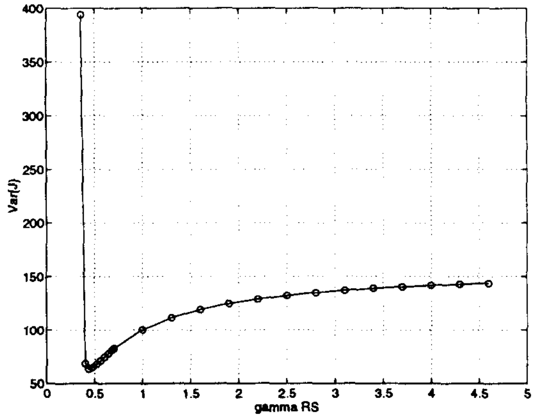

Figure 7: Optimal Variance; Full-State-Feedback, RS, 3DOF

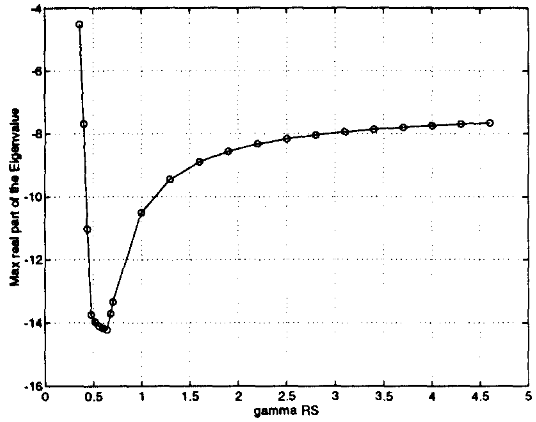

Figure 8: Stability; Full-State-Feedback, RS, 3DOF

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