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H_∞ CONTROL OF NONLINEAR SYSTEMS WITH NORM BOUNDED UNCERTAINTIES

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Abstract Previously obtained results of L_2-gain and H_∞ control via state feedback of nonlinear systems are extended to a class of nonlinear system with uncertainties. The required information about the uncertainties is that the uncertainties are bounded in Euclidian norm by known functions of the system state. The conditions are characterized in terms of the corresponding Hamilton-Jacobi equations or inequalities(HJEI). An algorithm for finding an approximate local solution of Hamilon-Jacobi equation is given. This results and algorithm are illustrated on a numerical example.

Keywords L_2 gain, H_∞ control, Nonlinear systems, Norm bounded uncertainties

1. INTRODUCTION

The robust control problem is to decide under what conditions there are feedback control laws for the uncertain systems such that the closed-loop uncertain systems have required stability and/or performance for all admissible uncertainties, and then design the control law. There have been a lot of research activities in robust control problem for uncertain nonlinear systems, see for example, [3], [2], [9]. There it is assumed that a known Lyapunov function guarantees stability of a nominal system. The goal is to construct a state feedback law which guarantees stability for all admissible uncertainties. The uncertainties are deterministic and the goal is to guarantee stability rather than performance.

Recently, the extension of linear H_∞ theory to nonlinear control systems has attracted many researchers' attention, see for example, [1], [5], [7], [12] [13]. Van der Schaft in [13] showed that a sufficient condition for the H_∞ control problem via state feedback can be given in terms of the related HJEI. In the case of measurement feedback case, Isidori and Astolfi in [7] provided a sufficient condition, which can be reduced to the solutions of two Hamilton-Jacobi equations.

In this paper, we propose an alternative approach to the H_∞ control problem via state feedback of a class of nonlinear system with uncertainties. The required information about the uncertainties is that the uncertainties are bounded in Euclidian norm by known functions of the system state. Sufficient conditions for the H_∞ control problem via state feedback of nonliner systems with structured or unstructured uncertainties are given in terms of HJEI respectively.

In the state space H_∞ control problem of nonlinear systems with uncertainties or not, the conditions are basically characterized in terms of the corresponding HJEI. Therefore, one of the major concerns is how to solve these HJEI. Due to the nonlinear nature, it is rarely possible to find a closed form solution of the HJEI. Van der Schaft in [13] proposed an approach to approximate a local solution of HJEI. A approximate polynominal solution and a special radial solution of HJEI have been addressed Isidori and Kang in [10] and Ji and Gao in [8] respectively.

In [11], an approximation approach via Taylor series was suggested and provided an algorithm and data structure in [4]. The results described in these paper were developed for a specific case of the Hamilton-Jacobi equations. Whence, we give an alternative approach via Taylor series for finding approximate local solution of a general form of Hamilon-Jacobi equation ,which is related with the H_∞ control problem for nonlinear systems with uncertainties or not. To the end, we examine this methods with a numerical example.

For vector z and matrix Z, $||z||$ denotes the Euclidian norm and $||Z||$ denotes the matrix norm induced by the Euclidian vector norm, i.e., $||Z|| = [\lambda_{\max}(Z^TZ)]^{1/2}$ where $\lambda_{\max}$ denotes the maximum eigenvalue. For differentiable function $V(x) : R^n \to R$ we denote by $\frac{\partial V}{\partial x}(x)$ the row vector of partial derivatives. For vector-valued function $z(t) : [0,T] \to R^k$, we say that $z(t)$ is in $L_2[0,T]$ if $\int_0^T ||z(t)||dt < \infty$. The solution at time t of the system with initial condition $x(t_0) = x_0$ and input u will be denoted by $x(t) = \phi(t, t_0, x_0, u)$.

2. PRELIMENARY

In this section, the existing results of L_2-gain and H_∞ control problem via state feedback for nonlinear systems are stated.

Consider a smooth nonlinear system described by

$$\dot{x} = f(x) + G(x)u, \quad y = h(x) \tag{1}$$

where, throughout this paper, $x \in R^n$ is the state defined on a neighborhood X of the origin, $u \in R^m$ is the input and $y \in R^p$ is the output. We assume the existence of an equilibrium $x = 0$ when $u = 0$, i.e., $f(0) = 0$. Without loss of generality we assume that $h(0) = 0$.

Definition 1 The system (1) is called zero-state observable if for all $x \in X$

$$h(\phi(t,0,x,0)) = 0 \Rightarrow \phi(t,0,x,0) = 0, \quad \forall t \geq 0 \tag{2}$$

Definition 2 Let $\gamma \geq 0$. The system (1) is said to have L_2 gain less than or equal to γ if for all $u \in L_2[0,T]$

$$\int_0^T ||y(t)||^2 dt \leq \gamma^2 \int_0^T ||u(t)||^2 dt \tag{3}$$

for all $T \geq 0$, with $y(t) = h(\phi(t,0,0,u))$.

Theorem 1 [13] The system (1) has L_2 gain less than or equal to γ if there exists a smooth solution $V(x) \geq 0$ ($V(0) = 0$) , defined in a neighborhood of $x = 0$, of the HJEI

$$\frac{\partial V}{\partial x}(x)f(x) + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}(x)G(x)G^T(x)\frac{\partial^T V}{\partial x}(x) + h^T(x)h(x) = 0 (\text{or } \leq 0) \tag{4}$$

Moreover, suppose that the system (1) is zero-state observable, then the free system $\dot{x} = f(x)$ is asymptotically stable.

Now we consider the H_∞ control problem via state feedback of a smooth nonlinear system described by

$$\dot{x} = f(x) + G(x)u + K(x)d, \quad y = h(x) + J(x)u \tag{5}$$

where $d \in R^s$ is the exogenous input or disturbance. We assume that $f(0) = 0$ and $h(0) = 0$.

The purpose of the control is to find a state feedback law such that asymptotic stability of the closed-loop system with $d = 0$ is guaranteed and the closed-loop system has L_2 gain less than or equal to a specific constant γ.

Assumption 1 $J^T(x)J(x) > 0$ for all $x \in X$.

Theorem 2 [6] Consider the system (5) with Assumption 1 and let $\gamma \geq 0$. Assume that there exists a smooth solution $V(x) \geq 0$ ($V(0) = 0$), defined in a neighborhood of $x = 0$, of the HJEI

$$\frac{\partial V}{\partial x}(x)[f(x) - G(x)M^{-1}(x)J^T(x)h(x)] + \frac{1}{4}\frac{\partial V}{\partial x}(x) \cdot \left[\frac{1}{\gamma^2}K(x)K^T(x) - G(x)M^{-1}(x)G^T(x)\right]\frac{\partial^T V}{\partial x}(x) + h^T(x)[I - J(x)M^{-1}(x)J^T(x)]h(x) = 0 \text{ (or } \leq 0) \quad (6)$$

Then, the closed-loop system with control input

$$u = -\frac{1}{2}M^{-1}(x)[G^T(x)\frac{\partial^T V}{\partial x}(x) + 2J^T(x)h(x)] \quad (7)$$

has L_2 gain less than or equal to γ, where $M(x) = J^T(x)J(x)$. Moreover, if the closed-loop system is zero-state observable, then the closed-loop system with $d = 0$ is asymptotically stable.

3. L_2 GAIN AND H_∞ CONTROL OF NONLINEAR SYSTEMS WITH UNCERTAINTIES

Consider a smooth nonlinear system described by

$$\dot{x} = f(x) + \Delta f(x) + G(x)u, \quad y = h(x) + \Delta h(x) \quad (8)$$

where the existence of an equilibrium $x = 0$ when $u = 0$, i.e., $f(0) = 0$ and $\Delta f(0) = 0$, and we assume that $h(0) = 0$ and $\Delta h(0) = 0$.

Assumption 2 The uncertainties $\Delta f(\cdot)$ and $\Delta h(\cdot)$ are bounded in Euclidian norm by known functions;that is, there are non-negative functions $\rho_f(x)$ and $\rho_h(x)$ such that

$$||\Delta f(x)|| \leq \rho_f(x), \quad ||\Delta h(x)|| \leq \rho_h(x)$$

for all $x \in X$, with $\rho_f(0) = 0, \ \rho_h(0) = 0$.

We refer the uncertainties satisfying the assumption 2 as the admissible uncertainties.

Theorem 3 Consider the system (8) with Assumption 2. We assume that there exist positive constants δ, θ and ϵ such that the HJEI

$$\frac{\partial V}{\partial x}(x)f(x) + \frac{1}{4}\frac{\partial V}{\partial x}(x)\left[\frac{1}{\gamma^2}G(x)G^T(x) + \frac{1}{\delta}I\right]\frac{\partial^T V}{\partial x}(x) + \delta\rho_f^2(x) + (1+\epsilon)h^T(x)h(x) + (1+\frac{1}{\epsilon})\rho_h^2(x) = 0 (\text{or} \leq 0) \quad (9)$$

has a smooth solution $V(x) \geq 0$ ($V(0) = 0$), defined in a neighborhood of $x = 0$, then for all admissible uncertainties the system has L_2 gain less than equal to γ. Moreover, for all admissible uncertainty, the system is zero-state observable then the free system $\dot{x} = f(x) + \Delta f(x)$ is asymptotically stable.

Remark Consider the linear system described by

$$\dot{x} = (A + \Delta A(\cdot))x + Bu, \quad y = (C + \Delta C(\cdot))x \quad (10)$$

with $||\Delta A(\cdot)|| \leq \epsilon_A$ and $||\Delta C(\cdot)|| \leq \epsilon_C$. If $V(x)$ take the form $V(x) = x^T P x$ where P is symmetric semi-positive definite matrix, then the HJEI reduce to the following Riccati equation or inequality(REI)

$$PA + A^T P + P\left[\frac{1}{\gamma^2}BB^T + \frac{1}{\delta}I\right]P + \delta\epsilon_A^2 I + (1+\epsilon)C^T C + (1+\frac{1}{\epsilon})\epsilon_C^2 I = 0 \text{ (or } \leq 0) \quad (11)$$

Now, consider the H_∞ control problem of a smooth nonlinear system described by

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x) + \{(G(x) + \Delta G(x)\}u + K(x)d \\ y &= h(x) + \Delta h(x) + \{J(x) + \Delta J(x)\}u \end{aligned} \quad (12)$$

Assumption 3 There are non-negative functions $\rho_G(x)$ and $\rho_J(x)$ such that for all $x \in X$

$$||\Delta G(x)|| \leq \rho_G(x), \quad ||\Delta J(x)|| \leq \rho_J(x)$$

Theorem 4 Consider system the (12) with Assumption 1, 2 and 3 and let $\gamma \geq 0$. We assume that there exist positive constants $\delta, \theta, \epsilon, \beta$ such that the HJEI

$$\frac{\partial V}{\partial x}(x)[f(x) - (1+\epsilon)G(x)\tilde{M}^{-1}(x)J^T(x)h(x)] + \frac{1}{4}\frac{\partial V}{\partial x}(x)[\frac{1}{\gamma^2}K(x)K^T(x) + \frac{1}{\delta}I - G(x)\tilde{M}^{-1}(x) \cdot G^T(x)]\frac{\partial^T V}{\partial x}(x) + (1+\epsilon)h^T(x)[I - (1+\epsilon)J(x) \cdot \tilde{M}^{-1}(x)J^T(x)]h(x) + \delta(1+\theta)\rho_f^2(x) + (1+\frac{1}{\epsilon})(1+\beta)\rho_h^2(x) = 0 \text{ (or } \leq 0) \quad (13)$$

has a smooth solution $V(x) \geq 0$ ($V(0) = 0$), defined in a neighborhood of $x = 0$, then for all admissible uncertainties the closed-loop system with control input

$$u = -\frac{1}{2}\tilde{M}^{-1}(x)[G^T(x)\frac{\partial^T V}{\partial x}(x) + 2(1+\epsilon)J^T(x)h(x)] \quad (14)$$

has L_2 gain less than or equal to γ where $\tilde{M}(x) = (1+\epsilon)J^T(x)J(x) + \delta(1+\frac{1}{\theta})\rho_G^2(x)I + (1+\frac{1}{\epsilon})(1+\frac{1}{\beta})\rho_J^2(x)I$. Moreover, for all admissible uncertainties, the closed-loop system is zero-state observable, then the closed-loop system (12), (14) with $d = 0$ is asymptotically stable.

Remark Consider the linear system described by

$$\begin{aligned} \dot{x} &= (A + \Delta A(\cdot))x + (B + \Delta B(\cdot))u + Kd \\ y &= (C + \Delta C(\cdot))x + (D + \Delta D(\cdot))u \end{aligned} \quad (15)$$

with $||\Delta A(\cdot)|| \leq \epsilon_A$, $||\Delta B(\cdot)|| \leq \epsilon_B$, $||\Delta C(\cdot)|| \leq \epsilon_C$ and $||\Delta D(\cdot)|| \leq \epsilon_D$. If $V(x)$ takes the form $V(x) = x^T P x$ where P is symmetric semi-positive definite matrix, then the HJEI reduces to the following REI

$$P\tilde{A} + \tilde{A}^T P + P\left[\frac{1}{\gamma^2}KK^T + \frac{1}{\delta}I - BM^{-1}B^T\right]P + (1+\epsilon)C^T[I - (1+\epsilon)DM^{-1}D^T]C + \delta(1+\theta)\epsilon_A^2 I + (1+\frac{1}{\epsilon})(1+\beta)\epsilon_C^2 I = 0 \text{ (or} \leq 0) \quad (16)$$

where $\tilde{A} = A - (1+\epsilon)B\tilde{M}^{-1}D^T C$, $\tilde{M} = (1+\epsilon)D^T D + \delta(1 + 1/\theta)\epsilon_B^2 I + (1 + 1/\epsilon)(1 + 1/\beta)\epsilon_D^2 I$. A control input such that asymptotic stability of the closed-loop system with $d = 0$ is guaranteed and L_2 gain of the closed-loop system is less than and equal to γ, for all admissible uncertainties, is

$$u = -\tilde{M}^{-1}(B^T P + (1+\epsilon)D^T C)x \quad (17)$$

Next, we assume that the uncertainties satisfy the following structure.

Assumption 4 The uncertainties $\Delta h(\cdot) = 0$ and $\Delta J(\cdot) = 0$, and the uncertainties $\Delta f(\cdot)$ and $\Delta G(\cdot)$ have the following structure

$$\Delta f(x) = D(x)\Delta F(x)e_f(x), \quad \Delta G(x) = D(x)\Delta F(x)E_G(x)$$

where $D(x), E_G(x)$ and $e_f(x)$ are respectively known matrices and a vector of appropriate dimensions which characterize the structure of the uncertainties. $\Delta F(\cdot)$ is bounded as follows;for all $x \in X$, $||\Delta F(x)|| \leq 1$.

Theorem 5 Consider the system (12) with Assumption 1 and 4, and let $\gamma \geq 0$. We assume that there exists positive constant δ such that the following HJEI

$$\begin{aligned}&\frac{\partial V}{\partial x}(x)[f(x) - G(x)R^{-1}(x)\{\delta E_G^T(x)e_f(x) + J^T(x)\\ &\cdot h(x)\}] + h^T(x)h(x) + \delta e_f^T(x)e_f(x) + \frac{1}{4}\frac{\partial V}{\partial x}(x)\\ &\cdot[\frac{1}{\gamma^2}K(x)K^T(x) + \frac{1}{\delta}D(x)D^T(x) - G(x)R^{-1}(x)\\ &\cdot G^T(x)]\frac{\partial^T V}{\partial x}(x) - [J^T(x)h(x) + \delta E_G^T(x)e_f(x)]^T\\ &\cdot R^{-1}(x)[J^T(x)h(x) + \delta E_G^T(x)e_f(x)] = 0 (\text{or} \leq 0) \quad (18)\end{aligned}$$

has a smooth solution $V(x) \geq 0 (V(0) = 0)$, defined in a neighborhood of $x = 0$, then, for all admissible uncertainties, the closed-loop system with control input

$$\begin{aligned}u &= -\frac{1}{2}R^{-1}(x)[G^T(x)\frac{\partial^T V}{\partial x}(x) + 2J^T(x)h(x)\\ &+2\delta E_G^T(x)e_f(x)] \quad (19)\end{aligned}$$

has L_2 gain less than or equal to γ where $R(x) = J^T(x)J(x) + \delta E_G^T(x)E_G(x)$. Moreover, for all admissible uncertainties, the closed-loop system is zero-state observable, then the closed-loop system (12), (19) with $d = 0$ is asymptotically stable.

Remark Consider the linear system described by

$$\begin{aligned}\dot{x} &= (A + D\Delta F(\cdot)E_A)x + (B + D\Delta F(\cdot)E_B)u + Kd\\ y &= Cx + Ju\end{aligned} \quad (20)$$

with $||\Delta F(\cdot)|| \leq 1$. If $V(x)$ takes the form $V(x) = x^T Px$ where P is symmetric semi-positive definite matrix, then the HJEI reduces to the following REI

$$\begin{aligned}&P\tilde{A} + \tilde{A}^T P + P\left[\frac{1}{\gamma^2}KK^T + \frac{1}{\delta}DD^T - BR^{-1}B^T\right]P\\ &+\delta E_A^T E_A + C^T C - [J^T C + \delta E_B^T E_A]^T R^{-1}\\ &\cdot[J^T C + \delta E_B^T E_A] = 0(\text{or} \leq 0) \quad (21)\end{aligned}$$

where $R = J^T J + \delta E_B^T E_B$, $\tilde{A} = A - BR^{-1}(\delta E_B^T E_A + J^T C)$. And a control input such that asymptotic stability of the closed-loop system with $d = 0$ is guaranteed and L_2 gain of the closed-loop system is less than or equal to γ, for all admissible uncertainties, is

$$u = -R^{-1}(B^T P + J^T C + \delta E_B^T E_A)x \quad (22)$$

Remark The REI (21) has been already appeared in [14] for the robust H_∞ control problem of the linear system (20) with $J^T[C \;\; J] = [0 \;\; I]$.

4. AN APPROXIMATE SOLUTION OF HAMILTON-JACOBI EQUATION

Consider the Hamilton-Jacobi equation of a general form described by

$$\frac{\partial V}{\partial x}(x)\tilde{f}(x) + \frac{1}{4}\frac{\partial V}{\partial x}(x)\tilde{G}(x)\frac{\partial^T V}{\partial x}(x) + \tilde{h}(x) = 0 \quad (23)$$

where $\tilde{f}(0) = 0, \tilde{h}(0) = 0, \frac{\partial \tilde{h}}{\partial x}(0) = 0$ and $\tilde{G}(x)$ is symmetric.

First, we have the following definition[4],[11]. For matrix Z, define $Z^{(0)} = 1$, $Z^{(1)} = Z$ and

$$Z^{(i)} = \underbrace{Z \otimes Z \otimes \cdots \otimes Z}_{i-1 \text{ times}}, \quad i = 2, 3, \cdots \quad (24)$$

where $\otimes$ stands for Kroneker product, and for vector $x \in R^n$, define $x^{[0]} = 1$, $x^{[1]} = x$ and

$$\begin{aligned}x^{[k]} = [x_1^k \;\; x_1^{k-1}x_2 \;\; \cdots \;\; x_1^{k-1}x_n \;\; x_1^{k-2}x_2^2 \;\; x_1^{k-2}x_2x_3\\ \cdots \;\; x_1^{k-2}x_2x_n \;\; x_1^{k-2}x_3^2 \cdots\cdots \;\; x_n^k \;]^T, \; k = 3, 4, \cdots \quad (25)\end{aligned}$$

Now, we consider that $V(x)$ of a solution of (23) takes the special form

$$V(x) = x^T Px + \sum_{k=3}^{\infty} p_k x^{[k]} \quad (26)$$

where P is a symmetric positive definite matrix.

The following algorithm is to generate all row vectors $p_k (k = 3, 4, \cdots)$.

step 1 Let $k = 3$.

step 2 Find P satisfying the Riccati equation

$$PA_1 + A_1 P + P\tilde{G}(0)P + \tilde{C} = 0 \quad (27)$$

where

$$A_1 = \frac{\partial \tilde{f}}{\partial x}(0), \; \tilde{C} = \frac{1}{2}\begin{bmatrix} \frac{\partial^2 \tilde{h}}{\partial x_1^2}(0) & \cdots & \frac{\partial^2 \tilde{h}}{\partial x_1 x_n}(0)\\ \frac{\partial^2 \tilde{h}}{\partial x_2 x_1}(0) & \cdots & \frac{\partial^2 \tilde{h}}{\partial x_2 x_n}(0)\\ \vdots & & \vdots\\ \frac{\partial^2 \tilde{h}}{\partial x_n x_1}(0) & \cdots & \frac{\partial^2 \tilde{h}}{\partial x_n^2}(0)\end{bmatrix}$$

step 3 If for all $i_1, i_2, \cdots, i_k \in \{1, 2, \cdots, n\}$ the equation

$$\lambda_{i_1} + \lambda_{i_2} + \cdots + \lambda_{i_k} \neq 0 \quad (28)$$

is satisfied, then go to the next step, else if go to the previous step, where $\lambda_1, \lambda_2, \cdots, \lambda_n$ are eigenvalues of $A_1 + G(0)P$.

step 4 Compute U_k as follows

$$U_k = M_k\left(\sum_{i=1}^{k}(I_n^{(i-1)} \otimes (\tilde{A}_1 + \tilde{G}(0)P) \otimes I_n^{(k-i)}\right)N_k \quad (29)$$

where M_k and N_k are constant matrices whose elements are only 1 or 0 ,such that $x^{[k]} = M_k x^{(k)}$, $x^{(k)} = N_k x^{[k]}$.

step 5 Compute p_k as follows

$$p_k = -(Z_k + F_k + E_k + H_k)N_k U_k^{-1}, \quad k \geq 3 \quad (30)$$

where $Z_3 = 0$ and

$$Z_k = \frac{1}{4}\sum_{\substack{i+j=k+2\\ i,j \geq 3}} row(S_i\tilde{G}(0)[S_j]^T), \; k \geq 4$$

$$F_k = \frac{1}{4}\sum_{\substack{i+m+l=k+2\\ i,m \geq 2; l \geq 1}}\sum_{j=1}^{n} row([row(S_m\tilde{G}_l^j)]^T[S_i^j]^T)$$

$$E_k = \sum_{\substack{i+m=k+1\\ l,m \geq 2}} row(S_i A_m), \quad H_k = \frac{1}{k!}\frac{\partial^k \tilde{h}}{\partial x^k}(0)$$

where $S_2 = 2P$, S_i^j is the jth column vector of S_i and

$$A_m = \frac{1}{m!}\frac{\partial^m \tilde{f}}{\partial x^m}(0), \quad S_i = \sum_{w=1}^{i}(P_i^w)^T, \quad \tilde{G}_l^j = \frac{1}{l!}\frac{\partial^l \tilde{G}^j}{\partial x^l}(0)$$

where $\tilde{G}^j(x)$ is the jth column vector of $\tilde{G}(x)$ and $P_i^w \in R^{n\times n^{i-1}}$ is

$$P_i^w = \begin{bmatrix} P_{1\cdots 11} & P_{1\cdots 21} & \cdots & P_{n\cdots n1} \\ P_{1\cdots 12} & P_{1\cdots 22} & \cdots & P_{n\cdots n2} \\ \vdots & \vdots & \vdots & \vdots \\ P_{1\cdots 1n} & P_{1\cdots 2n} & \cdots & P_{n\cdots nn} \end{bmatrix} \quad (31)$$

where $P_{n_1 n_2 \cdots n_w}$ is $(n_1 \times n_2 \times \cdots \times n_w)$th row vector of the block matrix which is made by partition $p_i M_i$ into n^w block matrices;that is,

$$p_i M_i = [P_{1\cdots 11}\ P_{1\cdots 12}\ \cdots P_{1\cdots 1n}\ \cdots\cdots\ P_{n\cdots nn}] \quad (32)$$

step 6 Let $k = k + 1$ and go to the step 3.

5. A NUMERICAL EXAMPLE

Consider the second order system described by

$$\dot{x} = \begin{bmatrix} a \tan x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{b}{\cos x_2} \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d$$
$$y = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} x = Cx \quad (33)$$

where a and b are the uncertain parameters such that; $\hat{a} - \bar{\delta}_a \le a \le \hat{a} + \bar{\delta}_a$, $\hat{b} - \bar{\delta}_b \le b \le \hat{b} + \bar{\delta}_b$, where $\hat{a}$ and $\hat{b}$ are the nominal values of a and b, $\bar{\delta}_a$ and $\bar{\delta}_b$ are positive constants, c_1 and c_2 are penalties on x. The system (33) can be written in the form of the system (12) with $\Delta h(\cdot) = 0$, $J(\cdot) = 0$ and $\Delta J(\cdot) = 0$;that is,

$$\dot{x} = f(x) + \Delta f(x) + \{g(x) + \Delta g(x)\}u + kd, \quad y = Cx \quad (34)$$
$$f(x) = \begin{bmatrix} \hat{a} \tan x_2 \\ 0 \end{bmatrix}, \quad \Delta f(x) = \begin{bmatrix} \delta_a \tan x_2 \\ 0 \end{bmatrix}$$
$$g(x) = \begin{bmatrix} 0 \\ \frac{\hat{b}}{\cos x_2} \end{bmatrix}, \quad \Delta g(x) = \begin{bmatrix} 0 \\ \frac{\delta_b}{\cos x_2} \end{bmatrix}, \quad k = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $-\bar{\delta}_a \le \delta_a \le \bar{\delta}_a$ and $-\bar{\delta}_b \le \delta_b \le \bar{\delta}_b$. Let $\rho_f(x) = \bar{\delta}_a |\tan x_2|$ and $\rho_g(x) = \bar{\delta}_b |\frac{1}{\cos x_2}|$, then the corresponding Hamilton-Jacobi equation is as follows

$$\frac{\partial V}{\partial x}(x) f(x) + \frac{1}{4}\frac{\partial V}{\partial x}(x)\tilde{G}(x)\frac{\partial^T V}{\partial x}(x) + \tilde{h}(x) = 0 \quad (35)$$

where $\tilde{G}(x) = \frac{1}{\gamma^2}kk^T + \frac{1}{\delta}I - \tilde{m}^{-1}(x)g(x)g^T(x)$ and $\tilde{h} = x^T C^T C x + \delta(1+\theta)\bar{\delta}_a^2\ (\tan x_2)^2$, where $\tilde{m}(x) = \delta(1 + 1/\theta)\ \bar{\delta}_b^2\ (\frac{1}{\cos x_2})^2$. A H_∞ control law is $u = -\frac{1}{2}\ \tilde{m}^{-1}(x)g^T(x)\frac{\partial V}{\partial x}(x) = -\frac{1}{2}\tilde{m}^{-1}(x)g^T(x)(2Px + \sum_{k=3}^{6} p_k M_k \sum_{i=1}^{k} I^{(i-1)} \otimes I_n \otimes x^{(k-i)})$.

We will consider finding a six order approximate solution $V(x) = x^T P x + \sum_{k=3}^{6} p_k x^{[k]}$ of the Hamilton-Jacobi equation (35). According to the algorithm of the previous section, for $\hat{a} = 1, \hat{b} = 1, c_1 = 1, c_2 = 1, \bar{\delta}_a = 0.2, \bar{\delta}_b = 0.2, \gamma = 1.5, \delta = 8, \theta = 2$ we obtain that;$p_3 = 0, p_5 = 0$ and

$$P = \begin{bmatrix} 2.364 & 1.059 \\ 1.059 & 1.669 \end{bmatrix}$$
$$p_4 = [-0.1230\ \ -0.09067\ \ -0.1274\ \ 0.05413\ \ 0.1392]$$
$$p_6 = [0.01291\ \ 0.01705\ \ 0.01892\ \ 0.004155\ \ -0.005292\ \ 0.009019\ \ 0.03667]$$

Figure 1 shows the time responses of x_1 for a perturbed parameter $a = 1.2$ and $b = 0.8$ when $d(t) = 1$ if $0 \le t < 10$, $d = 0$ if otherwise, where the solid line is the response by the control law obtained above and the dashed line is the response by the control law designed for the nominal nonlinear system using the same algorithm.

6. CONCLUSION

Sufficient conditions with respect to L_2 gain and H_∞ control problem via the state feedback for nonlinear systems with norm bounded uncertainties are given. And an algorithm and data structure that generate a Taylor series approximate local solution of the HJEI are given.

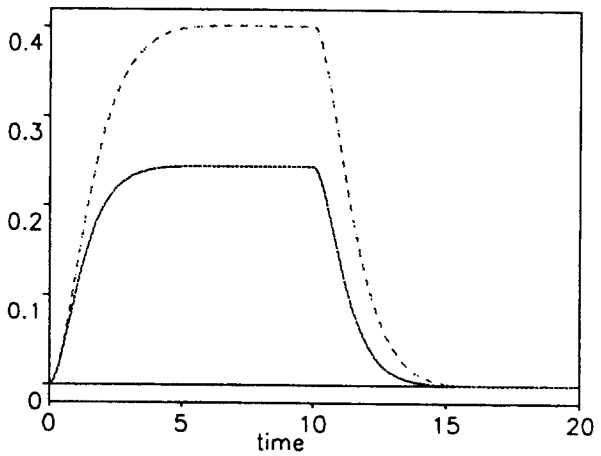

Figure 1: Time response of x_1 with a step disturbance($x_1(0) = 0, x_2(0) = 0$)

References

[1] J. A. Ball, J. W. Helton, and M. L. Walker, H_∞ control for nonlinear systems with output feedback, IEEE Trans. Automat. Contr., vol. 38, pp. 546 - 559, 1993

[2] Y. H. Chen, Design of robust controllers for uncertain dynamical systems, IEEE Trans. Automat. Contr., vol. 33, no. 5, pp. 487 - 491, 1988

[3] M. J. Corless and G. Leitmann, Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, IEEE Trans. Automat. Contr., vol. 26, no. 5, pp. 1139 - 1144, 1981

[4] J. Huang and C. F. Lin, An algorithm and data structure for approximately computing nonlinear H_∞ control laws, in proc. ACC, pp. 1133 - 1137, 1994

[5] J. Imura, T. Sugie and T. Yoshikawa, Bounded real lemma of nonlinear systems and its application to H_∞ state feedback control, in Proc. 32nd CDC, pp. 190 - 195, 1993

[6] A. Isidori, Dissipation inequalities in nonlinear H_∞-control, in Proc. CDC, pp. 3265 - 3270, 1992

[7] A. Isidori and A. Astolfi, Disturbance attenuation and H_∞ - control via measurement feedback in nonlinear systems, IEEE Trans. Automat. Contr., vol. 37, no.9, pp. 1283 - 1293, 1992

[8] Y. Ji and G. Gao, Nonlinear H_∞-control and estimation of optimal H_∞-gain, Syst. Contr. Lett., vol. 24, pp. 321 - 332, 1995

[9] Z. Qu, Global stabilization of nonlinear systems with a class of unmatched uncertainties, Syst. Contr. Lett., vol. 18, pp. 301 - 307, 1992

[10] A. Isidori and W. Kang, H_∞ control via measurement feedback for general nonlinear systems, IEEE Trans. Automat. Contr., vol. 40, No. 3, pp. 466 - 472, 1992

[11] W. Kang, P. K. De and A. Isidori, Flight control in a windshear via nonlinear H_∞ methods, in Proc. 31st CDC, pp. 1135 - 1142, 1992

[12] W. Lu and J. C. Doyle, H_∞ control of nonlinear systems: a convex characterization, in Proc. ACC, pp. 2098 - 2102, 1994

[13] A. J. van der Schaft, L_2- gain analysis of nonlinear systems and nonlinear state feedback H_∞ control, IEEE Trans. Automat. Contr., vol. 37, no.6, pp. 770 - 784, 1992

[14] L. Xie and C. E. Souza, Robust H_∞ control for linear systems with norm-bounded time-varying uncertainty, in Proc. 29th CDC, pp. 1034 - 1035, 1990