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Dynamics and Control of a Large Spacecraft with Flexible Appendages in Gravitational Field

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Abstract This paper describes dynamic analysis and attitude control of a large spacecraft with flexible appendages in gravitational field. The effect of attitude control and vibration control of flexible appendages in gravitational field has been clarified. We demonstrate some simulations in gravitational field for some cases, and suggest the effects of gravitational torque, parameters of flexible appendages, attitude control and vibration control of flexible appendages.

Key Words Attitude Control, Vibration Control, Flexible Appendages, Gravitational Field

1 INTRODUCTION

Recently, the demand for large spacecrafts is increasing. It is hard to keep high accuracy of the attitude because of their flexibility. But there have been done many studies about attitude control(1),(2) and vibration control of flexible structures(3),(4).

This paper describes the effects of the gravitational torque and the parameters of flexible appendages, and then investigates the effects of attitude control and vibration control of flexible appendages.

We suppose a simple model for a large spacecraft having a rigid body and flexible appendages attached to it. Although there are many external torques acting on the spacecraft, we consider the spacecraft in the gravitational field of only the earth (we ignore other torques)(5). In a system like this, it is expected that vibrations occur due to the small external forces and the flexibility of the appendages. These vibrations influence the attitude of the rigid body.

So, firstly, we investigate the effects of the gravitational torque and the parameters of the flexible appendages. After that, we apply attitude control of a rigid body and vibration control of flexible appendages to this system for suppressing both the vibrations of the body and the vibrations of the appendages.

2 ANALYTICAL MODEL

We consider a simple model of a large spacecraft, as shown in Fig. 1. This model consists of a rigid body and two flexible appendages. As for $O_b - X_b, Y_b, Z_b$ and $O - X, Y, Z$, we call each frame as the body-based system frame and the universal frame, respectively. The rigid body is free to undergo 3-dimensional attitude motion specified by the modified angles φ, θ, and ψ, called roll, pitch and yaw angles, respectively. Here O_b, O is the rigid body's center of mass. Each flexible appendage consist of a massless link whose one end is attached to the rigid body and the other has a pointmass. Each appendage is supported by a rotational spring at its root. In stationary state, one of the appendages is along $+Y_b$ direction, and the other along $-Y_b$ direction. The appendages can rotate in the Y_b-Z_b plane only. In the motion, angles between the appendages and axis Y_b are defined as α, β, respectively. In this paper, the parameters of the model are taken as follows. The moment of inertia about each axis of the rigid body is I_r. Both appendages have the same parameters, the moment of inertia about X_b and Z_b axis is I_a, the mass is m, the length of the link is ℓ, and the spring constant is k.

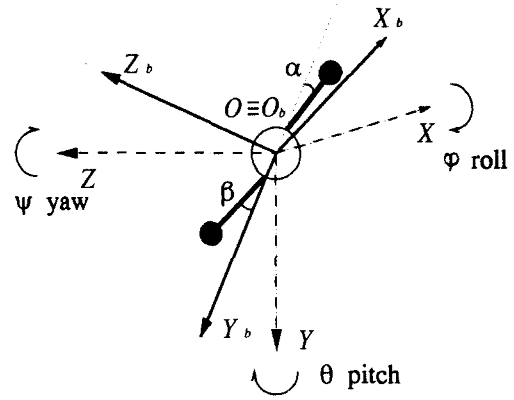

Fig. 1: A Large Spacecraft Model

In Figs. 2, 3 the spacecraft is shown in the gravitational field of the earth. We consider only the gravitational torque and ignore other external torques. The orbit is circular, and we consider only rotational motion, and ignore the translational one. We define the inertial reference frame $O_i - X_i, Y_i, Z_i$, where O_i is the earth's center of mass. Axis X is in the orbital plane. The angle between axis Z and the orbital plane is ϕ. R is the radius of the orbit.

In Fig. 2, the universal frame is in the rotational motion with respect to the inertial reference frame and the spacecraft attitude is constant with respect to the earth. Then, the orbital angular velocity $\boldsymbol{\Omega}$ in terms of the body based system frame, and the unit vector in the direction from the spacecraft to the earth $\boldsymbol{r}$, are given as follows:

$$\boldsymbol{\Omega} = (0, -\Omega\cos\phi, -\Omega\sin\phi),$$
$$\boldsymbol{r} = (0, \sin\phi, \cos\phi).$$

where Ω is orbital angular velocity in the inertial reference frame. We call this case *model – A*.

In Fig. 3, with respect to the inertial reference frame, the universal frame is in the translational motion and the spacecraft attitude is constant. Then, $\boldsymbol{\Omega}$ and $\boldsymbol{r}$ are given as follows:

$$\boldsymbol{\Omega} = (0, 0, 0),$$
$$\boldsymbol{r} = (\sin\Omega t, \sin\phi\cos\Omega t, \cos\phi\cos\Omega t).$$

We call this case *model – B*.

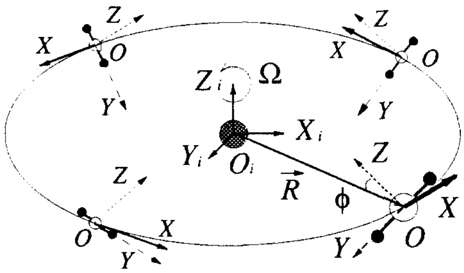

Fig. 2: *model – A*

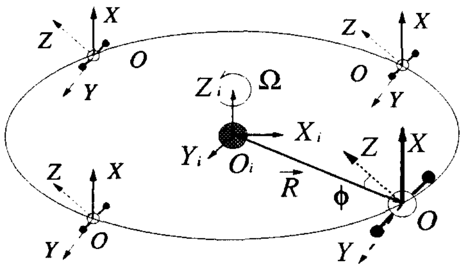

Fig. 3: *model – B*

3 MAIN EQUATIONS

The equations are obtained by using the conservation of energy[5]. Here, d/dt will be denoted by $(\dot{\ })$, d^2/dt^2 by $(\ddot{\ })$ and the vector product by $(\times)$.

We apply three-axis attitude control to the spacecraft which has a reaction wheel on each axis of roll, pitch and yaw. The equation for the rotational motion of the rigid body is written as follows.

$$\frac{d}{dt}\left(\frac{\partial \boldsymbol{T}}{\partial \dot{\boldsymbol{\omega}}}\right) + \boldsymbol{\omega} \times \left(\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{\omega}}\right) + \dot{\boldsymbol{H}} + \boldsymbol{\omega} \times \boldsymbol{H} = \boldsymbol{g}, \qquad (1)$$

where $\boldsymbol{T}$ is the kinetic energy of the complete system, $\boldsymbol{H}$ is the angular momentum of reaction wheels, $\boldsymbol{\omega}$ is the angular velocity of the spacecraft in terms of its body based frame. $\boldsymbol{g}$ is the gravity gradient torque given as:

$$\boldsymbol{g} = 3\left(\frac{\mu}{R^3}\right)\boldsymbol{r} \times \boldsymbol{I}\boldsymbol{r}, \qquad (2)$$

where $\mu = GM$, G is the universal gravitational constant and M is the mass of the earth. $\boldsymbol{I}$ is the inertia matrix of the complete system. We define $\dot{\boldsymbol{H}}$ as follows:

$$\dot{\boldsymbol{H}} = \begin{pmatrix} c_\varphi & \dot{\varphi} & + & h_\varphi & \varphi \\ c_\theta & \dot{\theta} & + & h_\theta & \theta \\ c_\psi & \dot{\psi} & + & h_\psi & \psi \end{pmatrix}.$$

$\boldsymbol{H}$ can be found as:

$$\boldsymbol{H} = \begin{pmatrix} c_\varphi & \varphi & + & h_\varphi & \int\varphi dt \\ c_\theta & \theta & + & h_\theta & \int\theta dt \\ c_\psi & \psi & + & h_\psi & \int\psi dt \end{pmatrix},$$

where $c_\varphi, c_\theta, c_\psi, h_\varphi, h_\theta$ and h_ψ are constants.

Next, we add a damping term to the appendages for control and obtain the equation of their rotational motion as follows:

$$\frac{d}{dt}\left(\frac{\partial \boldsymbol{L}}{\partial \dot{q}}\right) - \left(\frac{\partial \boldsymbol{L}}{\partial q}\right) + c\dot{q} = 0, \qquad (3)$$

where $\boldsymbol{L} = \boldsymbol{T} - \boldsymbol{V} - \boldsymbol{K}$, $\boldsymbol{V}$ is the potential energy due to gravity, $\boldsymbol{K}$ is the potential energy of the springs supporting the flexible appendages, q represents the attitude angle of flexible appendage (α or β). The control parameter c is constant.

Neglecting terms of order higher than $O(\epsilon^2)$ when the all angules and Ω are of order $O(\epsilon)$, and according to the representive time $T = \sqrt{m\ell^2/k}$, we obtain the nondimentional equation as follows:

$$\ddot{\varphi} + 2\kappa_\varphi\dot{\varphi} + \lambda_\varphi\varphi + \frac{I}{2(1+r^*)}(\ddot{\alpha} + \ddot{\beta}) = 0, \qquad (4)$$
$$\ddot{\theta} + 2\kappa_\theta\dot{\theta} + \lambda_\theta\theta = 0, \qquad (5)$$
$$\ddot{\psi} + 2\kappa_\psi\dot{\psi} + \lambda_\psi\psi = 0, \qquad (6)$$
$$\ddot{\alpha} + 2\gamma\dot{\alpha} + \alpha + (1+r^*)\ddot{\varphi} = 0, \qquad (7)$$
$$\ddot{\beta} + 2\gamma\dot{\beta} + \beta + (1+r^*)\ddot{\varphi} = 0, \qquad (8)$$

where r^* is r/ℓ. The nondimensional parameters are denoted as follows.

$$I = \frac{I_a}{I_s},\ 2\kappa_{\varphi,\psi} = \frac{c_{\varphi,\psi}}{I_s}T,\ 2\kappa_\theta = \frac{c_\theta}{I_r}T,$$

$$\lambda_{\varphi,\psi} = \frac{h_{\varphi,\psi}}{I_s},\ \lambda_\theta = \frac{h_\theta}{I_r},\ 2\gamma = \frac{c}{m\ell^2}T,$$

where $I_s = I_r + 2I_a$. According to the equations (7) and (8), the natural frequency of the appendages is 1 in any case.

4 SIMULATION

Figs. 4 through 10 show simulation results. According to the equations (4)~(8), we choose $\kappa_\theta = \kappa_\psi = 1$, $\lambda_\theta = \lambda_\psi = 1$ for a suitable control. Assume Ω, R and ϕ are chosen as follows:

$$\Omega = 0.001\,[\text{rad/s}],\ \phi = \pi/4\,[\text{rad}].$$

In Figs. 4 through 9, we choose $\kappa_\varphi = 1$, $\lambda_\varphi = 1$ and $\gamma = 0$. In Figs. 10, 11, we simulate each parameter $\kappa_\varphi, \lambda_\varphi$ and γ as given in each figure. The value of I is given in each figure. The vertical axis of each figure shows $\varphi, \theta, \psi, \alpha, \beta$ and each unit is [rad], and the horizontal one the nondimensional time t.

We shall discuss the effect of the gravitational torque, comparing Fig. 4 with Fig. 5 in case of *model – A*, and Fig. 7 with Fig. 9, in case of *model – B*. Considering the simulation results, we notice two points. The first one is

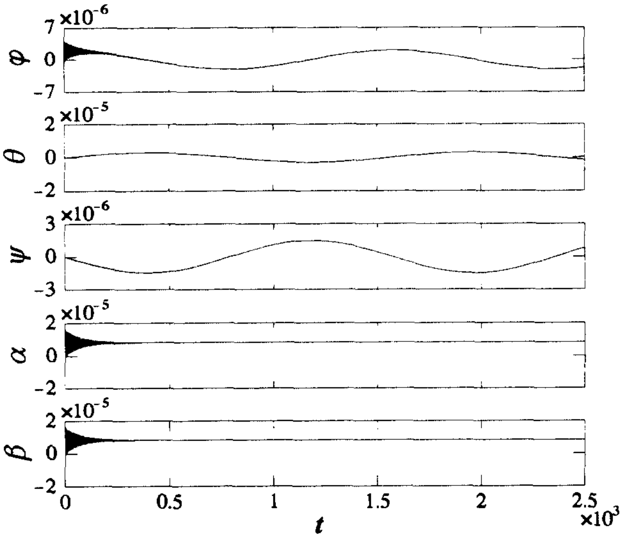

Fig. 4: $I = 0.5$ *model* − *A*

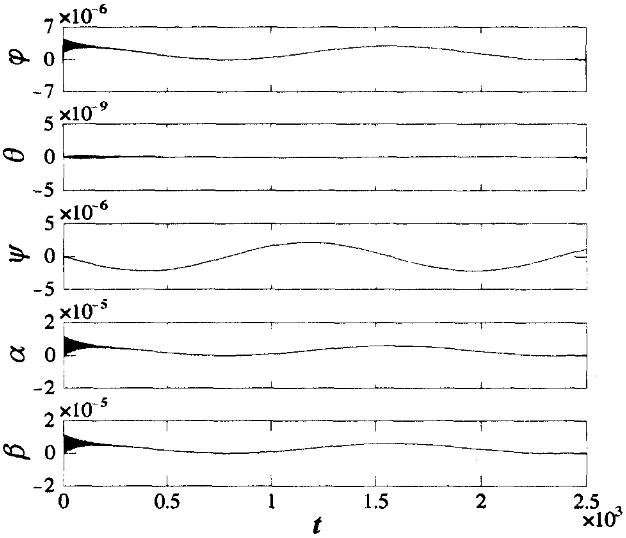

Fig. 7: $I = 0.5$ *model* − *B*

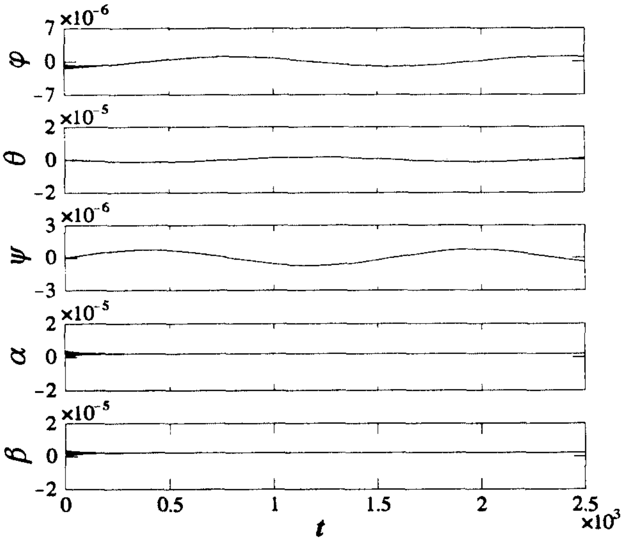

Fig. 5: $I = 0.5$ *model* − *A* (No Gravity)

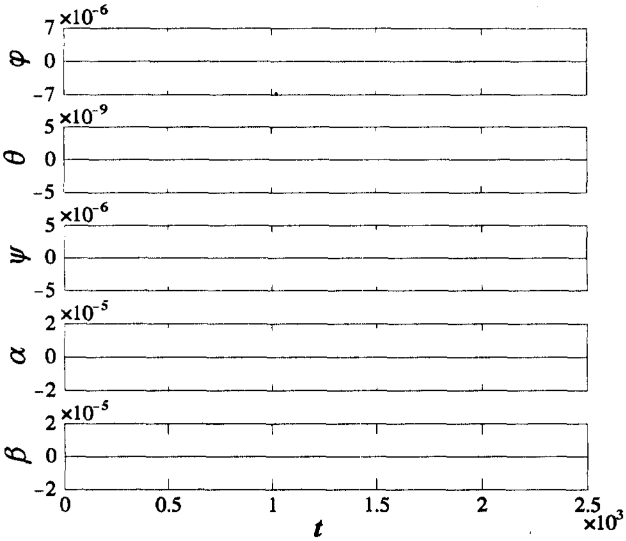

Fig. 8: $I = 0.5$ *model* − *B* (No Gravity)

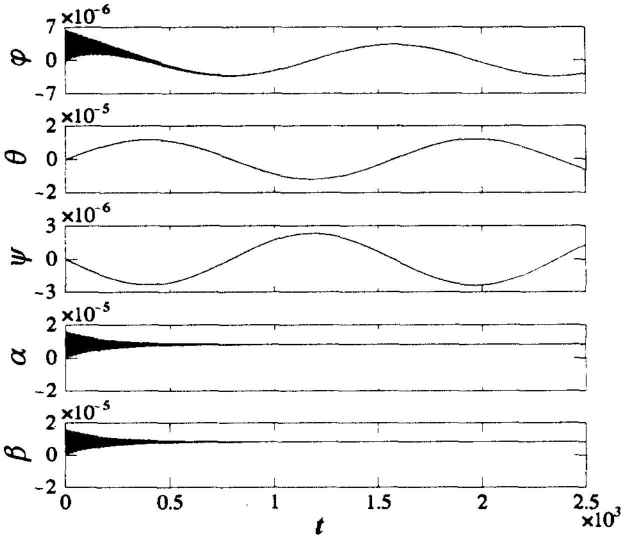

Fig. 6: $I = 0.8$ *model* − *A*

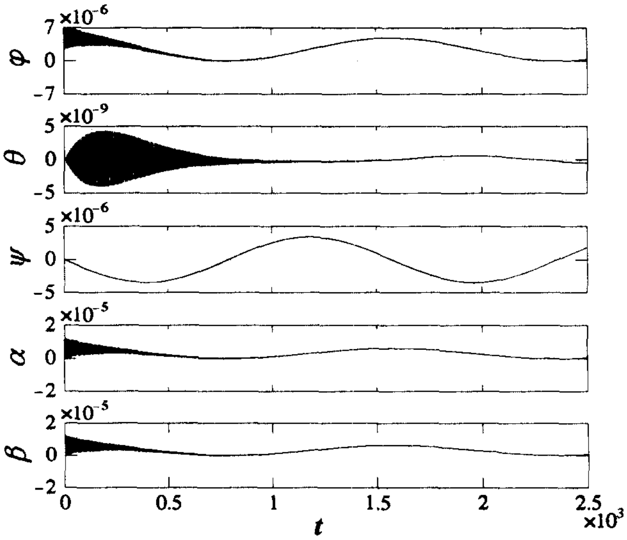

Fig. 9: $I = 0.8$ *model* − *B*

that orbital periodic vibrations occur in the rigid body, because of the influence of attitude control in case of *model – A*, and because of the influence of gravitational torque in case of *model–B*. In case of *model–B*, orbital periodic vibrations occur in the appendages, too. The second point is that appendage's vibrations influence the pitch θ, in case of *model – B*, but the amplitude of deviation in θ is small enough to be ignored, comparing it with φ, ψ.

Next we shall discuss the effect of the parameters of the flexible appendages, comparing Fig. 4 with Fig. 6, in case of *model – A*, and Fig. 7 with Fig. 9, in case of *model – B*. In all cases, T is the same value as $2\,[s]$. According to the simulation results, we notice that it takes a long time to damp the vibrations of the rigid body and appendages, in case when the appendage's moment of inertia is large. And the amplitude of deviation in φ for the rigid body is of higher order than that in the case of $I = 0.5$, though the amplitude of vibration of the appendages is of the same order as that in the case of $I = 0.5$. Especially the amplitude of θ in case of *model – B* is of higher order.

Finally we shall discuss the effect of attitude control of the rigid body and vibration control of the flexible appendages by investigating Fig. 10 (1), in case of *model–A*, and Fig.10(2), in case of *model–B*. In these cases, disturbances are given to $\dot{\alpha}$ and $\dot{\beta}$ on initial conditions. At first, we notice that it takes longer time to damp the vibrations of the appendages, though short time is required to damp angular vibrations of φ, in case when the damping term for φ is large. So, we can recognize that it is very effective to add the damping term for appendages.

5 CONCLUSIONS

From this paper we can conclude the following:

1. Orbital periodic vibrations occur in a rigid body because of the influence of attitude control in case of *model – A*, and because of the influence of gravitational torque in case of *model – B*.
2. It takes a long time to damp the vibrations of the rigid body and the appendages in the case when the appendage's moment of inertia is large.
3. The larger the amplitude of deviation in the rigid body, the larger is the moment of inertia of the appendages.
4. It takes a long time to damp the vibrations of the appendages though it takes shorter time to damp the vibrations of angle φ, in the case of when the damping term about φ is large.
5. It is very effective to add the damping term for the appendages.

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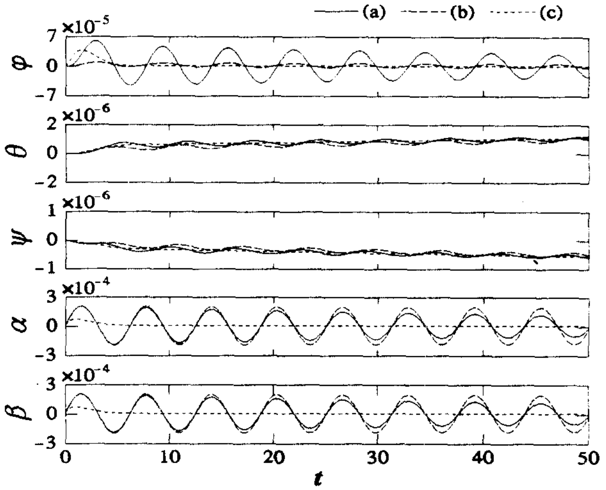
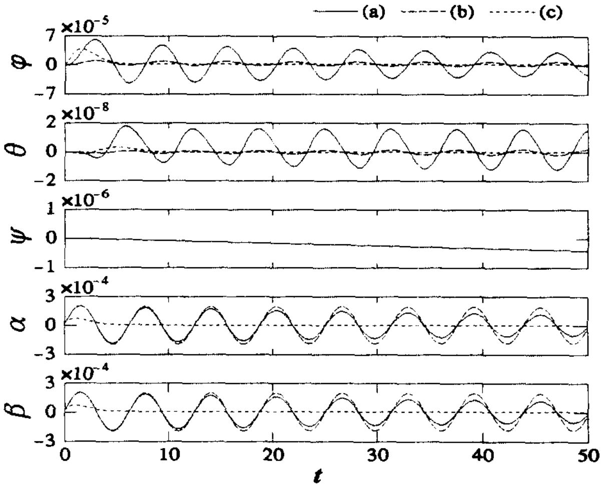


(1) *model – A*



(2) *model – B*

(a) $\kappa_\varphi = 1,\ \lambda_\varphi = 1,\ \gamma = 0$
(b) $\kappa_\varphi = 10,\ \lambda_\varphi = 1,\ \gamma = 0$
(c) $\kappa_\varphi = 1,\ \lambda_\varphi = 1,\ \gamma = 1$

Fig. 10: The Effect of Control $I = 0.5$