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Effect of Active Suspension Unit with H_∞ Robust Controller on the Vehicle Dynamics Performances

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Abstract This paper uses a new method to improve the performance criterion of an active suspension car. The used control strategy is based on robust $\mathcal{H}_\infty$ control theory taking into consideration the chassis flexibility. It will be shown that the modeling errors can be lumped into an unstructured uncertainty and the robust controller designed in the presence of these perturbations could maintain the stability and performance even for the controlled true system.

Keywords Mixed sensitivity, multiplicative uncertainty

1. INTRODUCTION

The customer's expectations of the performance of their vehicles is continually rising. However, in the last decay, the totally passive suspension systems have frozen the developments of the car dynamics, since they are very restricted in the quality of control it can provide, simultaneously, for both mass isolation and wheel hop. To avoid this problem, there can be investigated active suspension systems.

An active suspension can adapt to various levels of external forces and track irregularities so that it simultaneously appears "soft" to irregularities and "hard" to guidance forces. The electronic car suspension have been subject to the several publication.

However in the literature the system non-linearities and/or flexibility have been largely neglected, due to the difficulties met in calculations and the fact that at low frequencies the linear and non-linear systems don't differ so much. On the other hand it has been proven that the neglected non-linearities and modeling assumptions could lead to significant performance degradation and loss of stability, when the designed controller applied on the true system.

A new method of controller design was investigated last year by Kanbolat et al. to overcome this failure risk in the application of the controller to the real life. However, it was restricted with being applicable only to improve few performance output. Whereas, this work extends the theory to improve four performance outputs; acceleration transmission, wheel load fluctuation, suspension deflection and sprung mass displacement.

The framework of the design method is based on computation of the uncertainties by lumping the modeling errors into multiplicative error functions, then a robust controller computed by using $\mathcal{H}_\infty$-optimal control theory.

The paper is organized as follows. In Section 2, three car models are designed, a simple quarter car model, a rigid half car model and a flexible half car model. The last model is used to generate the data for the simulation of the true system and the others are employed as a nominal system in SISO feedback control systems. The final task of this part is the calculation of the robustness and performance weighting functions to use in the calculation of the mixed sensitivity approach. In Section 3, the nominal model, quarter car model is augmented with the weighting functions to handle both performance and robustness weighting functions. Then, $\mathcal{H}_\infty$-optimal control theory is used to calculate the robust controller for SISO feedback system. Finally, in Section 4 and Section 5, the behavior of the true controlled system is observed and discussed.

2. MODELING THE SUSPENSION SYSTEM

The aim of modeling is to produce nominal model and determine its quality for the robust control design. Quarter car and flexible half car models will be built and the frequency domain multiplicative error description between flexible and nominal model will be computed.

2.1 Quarter Car Model

The model of the car considered as the nominal model for the control design, is the so-called quarter car model, see Fig. 1.

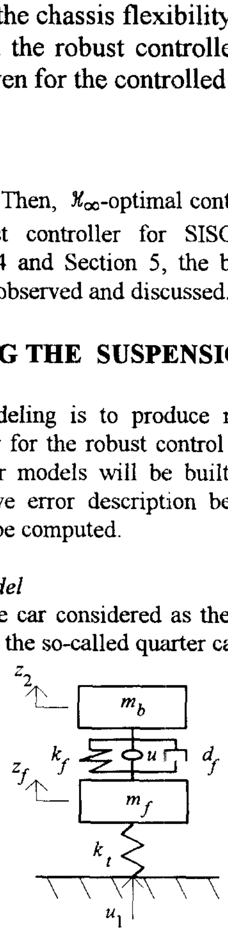

Fig. 1 Linear quarter car model

The equations of motion represented in a state-space form can be written as:

$$\dot{x}(t) = Ax(t) + B_1u_1(t) + B_2u_2(t)$$
$$x = [z_f \ z_b \ \dot{z}_f \ \dot{z}_b\ ] \tag{1}$$

where x denotes the state vector, u_1 the external disturbance and u_2 the control input. The matrices of the model are got from [2]. Assuming that an accelerometer is available to measure the sprung mass accelerations, the measured output is: $y = \ddot{z}_b$.

2.2 Flexible Half Car Model

High-order flexible model has been built up to be able to calculate the modeling error around the nominal model. A ten-degree model can be seen on Fig 2. The mass distribution of the main body satisfies the decoupling condition, so the front and rear wheels are decoupled.

Their reduced body masses are calculated [2]. The parameters of the front wheel station are identical with nominal model. This condition gives rise to the capability of comparison of the output of the nominal and flexible model.

The mathematical descriptions of the flexibility is computed using finite element methods, see [2] and is given by the following figure.

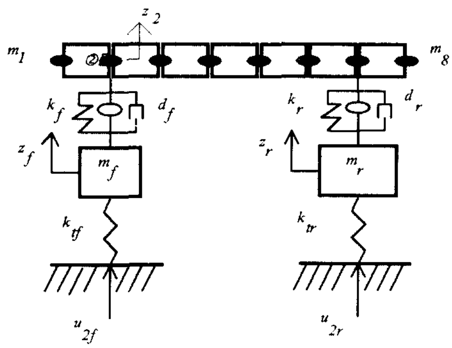

Fig. 2 Flexible Half Car Model

Fig. 3 shows the open loop responses of the quarter car and flexible half car models where the input is the distúrbance acting on the tyre (in the flexible model case it is the front wheel) and the output is the acceleration of the main body (in the flexible model case the sensor is located on point②, see Fig. 2).

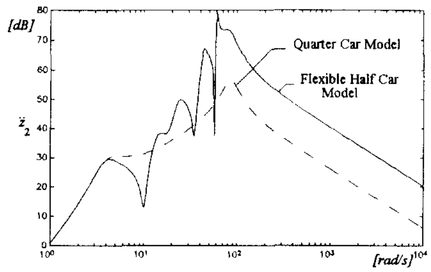

Fig. 3 Open Loop Response

From Fig. 3, it can be seen that the flexibility has not a negligible effect on the result.

2.3 Computation of Weighting Functions

The final step in the modeling part is the computation of weighting functions:

- Robustness weighting function, W_r is simply equal to an upper bound of the multiplicative error, Δ_M which is given by the following equation:

$$\Delta_M = \frac{G_{u2y2} - \hat{G}_{u2y2}}{\hat{G}_{u2y2}} \quad (2)$$

where $\hat{G}$ refers to the nominal model.

- The performance weight is selected considering the sensitivity region of the human body.

The multiplicative modeling error and the robustness and performance weighting functions are depicted in Fig. 5.

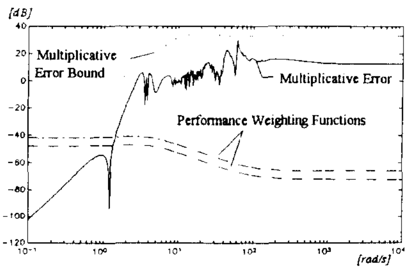

Fig. 4 Weighting functions

3. ROBUST CONTROL

In this section a robust controller will be computed to guaranty the stability of the true controlled plant and which can provide a close performance to the controlled nominal model even it is wrapped to the true system. Standard feedback configuration can be given as in Fig. 5.

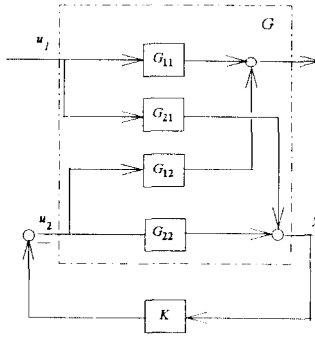

Fig. 5. General feedback configuration of ACS

It consists of the interconnected plant G given by following matrix and controller K. Both nominal mathematical models for both G and K are infinite dimensional time invariant (FDLTI) systems with transfer function matrices $G(s)$ and $K(s)$.

Considering to control loop, the sensitivity function $S(s)$ can be defined as from u_1 to y_p.

$$S(s) = G_{11} - G_{12}(I + G_{22}K)^{-1}G_{21} \quad (3)$$

Then it can be easily seen that the sensitivity operator, or inverse return difference operator is equal to $(I + G_{22}(s)K(s))^{-1}$. In SISO feedback case both G_{22} and $(I + G_{22}(s)K(s))^{-1}$ are scalar. While no nominal model $G(s)$ can emulate a physical plant perfectly, no model should be considered complete without assessment of its errors. On the other hand the representation of uncertainty vary primarily in terms of the amount of structure they contain. Since the rigidity assumption causes a large amount of modeling error, especially at high frequencies, in our calculations the multiplicative error function representation is used to models the perturbation acting on our controlled system, see Fig. 6.

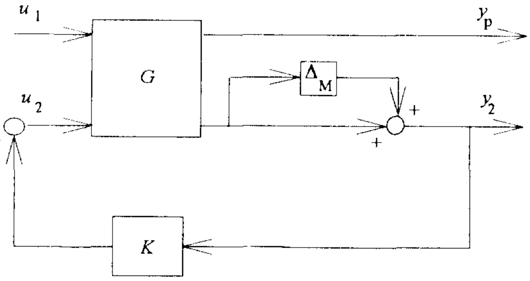

Fig. 6 Block diagram of controlled system

New function named as $M(s)$ and being the transfer function seen by the perturbation block Δ_M play an important role in robust control system design, see equation 4.

$$M(s) = W_r(I + G_{22}K)^{-1}G_{22}K \quad (4)$$

From point of view of disturbance attenuation, the task is to find a controller to lower the infinity norm of the sensitivity function as much as possible. In case of robust stability requirement of the closed loop system, Small-Gain theorem is used to check it. The control design requirements are concluded in the following equations:

$$\left\| W_r\left((I+G_{22}K)^{-1}G_{22}K\right)\right\|_\infty < 1$$
$$\left\| W_p\left(G_{11} - G_{12}K(I+G_{22}K)^{-1}G_{21}\right)\right\|_\infty \rightarrow \min_K \quad (5)$$

The mixed sensitivity of our controlled system is illustrated by the following function named as mixed sensitivity cost function, $T_{y_p u_1}$:

$$\left\| \begin{matrix} \delta\, W_p S \\ W_r M \end{matrix} \right\|_\infty < 1 \qquad \left\| T_{y_1 u_1} \right\| < 1 \quad (6)$$

where δ denotes the coefficients of response performance.

The open loop transfer function $G(s)$ is augmented with the weighting functions and with a new output referring to the robust stability by considering the mixed sensitivity formulation. The augmented transfer function is given by the following equation

$$P(s) = \left[\begin{array}{c|c} P_{11} & P_{12} \\ P_{21} & P_{22} \\ \hline P_{31} & P_{32} \end{array} \right] \quad (7)$$

The transfer function disturbance u_1 to the performance output y_p being $T_{u_1 2 y_p}$ and to the .robust output y_r being $T_{u_1 2 y_r}$ are given by the following equations respectively.

$$T_{u_1 2 y_p} = W_p(P_{11} - P_{12}K(I+P_{32}K)^{-1}G_{21})$$
$$T_{u_1 2 y_r} = W_r(P_{21} - P_{22}K(1+P_{32}K)^{-1}G_{31}) \quad (8)$$

The value of the elements concerning to the augmented transfer function are given on the following expressions.

$$P_{11} = G_{11} \qquad P_{12} = G_{12}$$
$$P_{21} = 0 \qquad P_{22} = \frac{G_{22}}{G_{21}} \quad (9)$$
$$P_{31} = G_{21} \qquad P_{32} = G_{22}$$

One can say that P_{11} is equal to zero since the disturbance has no effect on the robust output, additionally P_{12} is determined as G_{22}/G_{21}. The augmented transfer matrix in terms of nominal system is given on the following matrix.

$$P(s) = \left[\begin{array}{c|c} G_{11} & G_{12} \\ 0 & \frac{G_{22}}{G_{21}} \\ \hline G_{21} & G_{22} \end{array} \right] \quad (10)$$

By remembering that the designed feedback is SISO, it can be seen that the transfer functions given by equation (9) are identical with the equation set (5).

New column vector y is defined to be the mixed sensitivity output.

$$y = \begin{bmatrix} y_p \\ y_r \end{bmatrix} \quad (11)$$

The linear controller, $K(s)$, that is fed back from the measured signal to the control input, is designed so that the H_∞ norm of the transfer function, Gyu_1 from disturbance input u_1 to the mixed sensitivity output, y of the augmented model is minimized:

$$P_{yu_1} = P_{11} + P_{12}K[I - P_{22}K]^{-1}P_{21} \rightarrow \min_K \quad (11)$$

while satisfying the following condition:

$$\left\| P_{y_p u_1} \right\|_\infty \leq 1 \quad (12)$$

The augmented transfer function is illustrated on the Fig. 7

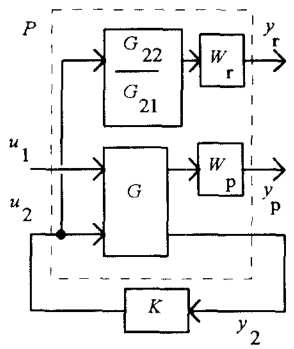

Fig. 7 The block diagram of the augmented diagram

The solution of the problem includes the combined solution of two Riccati equations with γ-iteration.

4. SIMULATION RESULTS

In this part, the characteristics and performance of the controllers are examined by carrying out computational simulations. The designed stabilizing controller considering to the nominal model (quarter car model) is applied on the flexible model. Then the nominal and robust stability of the closed loop system are checked. Our task is to modify the comfort of a family car, which has normally soft suspension, as much as possible while keeping on the driving stability.

Considering to the frequency domain simulations, see Fig. 8, it can be observed that the driving comfort and sprung mass motion can be lowered on width frequency range. However, a desire of an excellent performance for these outputs lead to an increase in the tyre load fluctuation and working space. Due to a transmission zeros located at 10.2Hz, the handling quality and suspension deflection couldn't be lowered under the peak value of the passive suspension around this frequency.

The coefficients of response performance are tested to obtain optimum comfort while keeping the driving stability as much as possible. The results are quite good for a family car. The passive suspension vibration isolation and sprung mass motion are improved in width frequency range. In spite of a big contradiction, the tyre load fluctuation almost remains constant in high frequency range and improved in the frequency range from 1.5 to 10Hz. On the other hand in the working space of active suspension a little augmentation is observed. In conventional car suspensions, a performance level reached by our electronic suspension requires a big amount of increase in working space and tyre load fluctuation. In general, a little amount of increase in suspension travel is not quite important in the case of family cars, but this is an important consideration when available suspension working space is severe design constraint..

The hypothesis of no suspension minimize (optimize) all four of the performance parameters simultaneously [1], [4], [6] is confirmed by this paper. The state space matrices of the nominal and flexible model, and their design parameters are taken from [2].

5. CONCLUSIONS

Our paper followed and developed the active vehicle suspension design method based on the $\mathcal{H}_\infty$ robust control theory that was investigated in [2]. Due to the difficulties, coming from the complexities of the nominal model, in the control design calculations the designers have been forced to make several assumptions in the modeling part. However this fact produces several uncertain system elements leading sometimes the compensated system to behave even worse than the original plant [5]. This paper used the robust control design to evaluate a controller providing stability of the true controlled system and a little degradation of performance comparing to the designed one. The perturbations have been lumped into an unstructured uncertainty. This gives the possibility of consideration of the effects concerning to all uncertain parameters in one matrix of function.

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7. APPENDIX

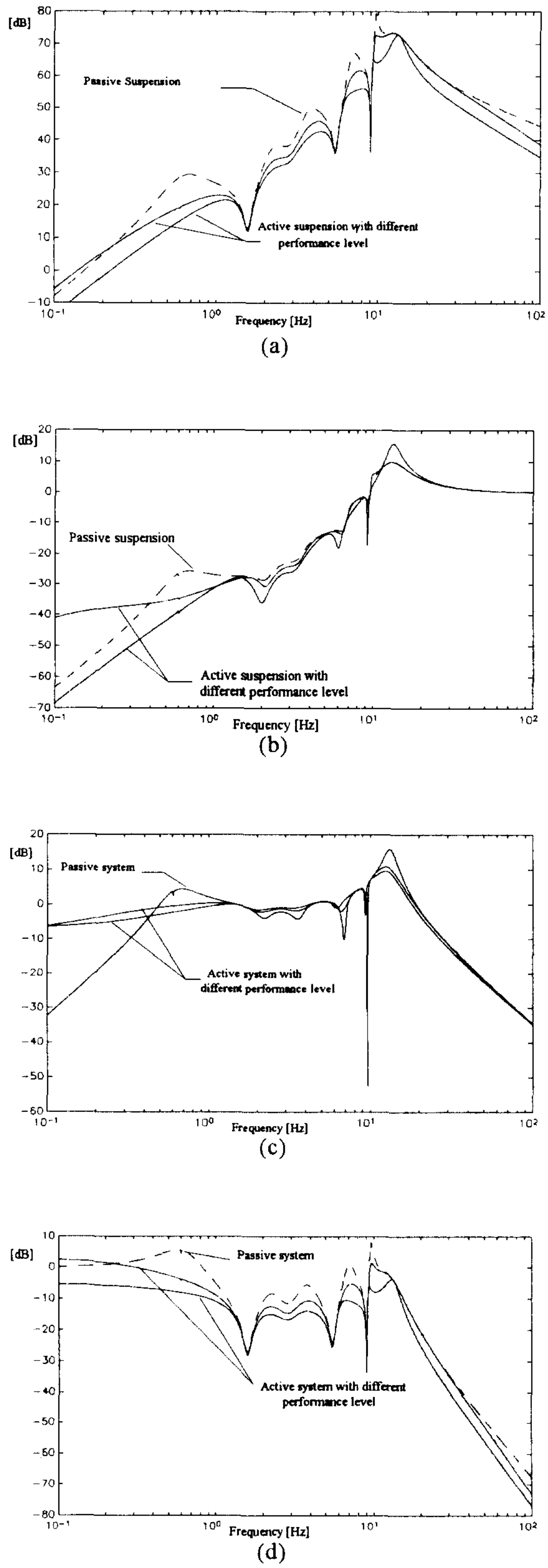

Fig.8 Disturbance to vertical acceleration of body a), to tyre load fluctuation b), to working space c) and to sprung mass displacement d) transfer functions.