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Coprime factor reduction of plant in H^∞ mixed sensitivity problem

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Abstract In this paper, we get a reduced order controller in H^∞ mixed sensitivity problem with weighting functions. For this purpose, we define frequency weighted coprime factor of plant in H^∞ mixed sensitivity problem and reduce the coprime factor using the frequency weighted balanced truncation technique. Then we design the controller for plant with reduced order coprime factor using J-lossless coprime factorization technique. Using this approach, we can derive the robust stability condition and achieve good performance preservation in the closed loop system with reduced order controller. And it behaves well in both stable plant and unstable plant.

Keywords Model reduction, Coprime factor, Mixed sensitivity, Robust stability, Weighting function

1. Introduction

Mathematical models of physical systems often have very high order system transfer functions. Because of computational and other practical limitations, the high order model should be reduced for synthesis, analysis, and implementation. The approximation problem of a high order model by a lower order one has attracted much attention in the past two decades and many techniques have been proposed[1].

Indeed, using model reduction is a technique for designing a reduced order controller. In addition to guarantee the closed loop stability, the reduced order model must characterize the physical system as closely as possible such that performance objectives for the controlled physical system can be met with reduced order model.

A method using the concept of balanced realization has been proposed by Moore[5] for the reduction of linear continuous system. Based on different truncation criteria, a number of model reduction methods[6] using balanced realizations are available. Another popular methods are Hankel approximation[3] and g-COVER method[8]. Hankel approximation method has a closed form error criterion for the optimal reduced order model. And model reduction method using coprime factorization technique has also been developed. It has a good property that it can directly reduce the unstable model. On the other hand, the balanced realization method was extended to include the frequency dependent weighting functions. The weighting functions are used to meet the design specifications[2].

Though the H^∞ mixed sensitivity minimization has various structures, we consider a frequency weighted H^∞ mixed sensitivity problem which was treated by Tsai et al[9]. The problem can be thought as disturbance rejection and stabilization in the face of unstructured perturbations. It is known that H^∞ mixed sensitivity problem can have pole-zero cancellations and be used in partial pole placement in the closed loop system.

In here, we obtain the reduced order controller in H^∞ mixed sensitivity problem using coprime factor reduction of plant and J-lossless coprime factorization design method[10]. The robust stabilization condition of the closed loop system with reduced order controller is found. This approach can be directly used in the unstable plant as well as stable plant and achieve partial pole placement in the closed loop system using weighting functions. And it also preserves good performances of closed loop system with full order plant and controller.

2. H^∞ mixed sensitivity problem

Consider the linear, time-invariant feedback system in Fig.1. For notational convenience, the Laplace variable s will be dropped. G is the nominal plant and K is the controller; W_1, W_2, and W_d are weighting functions. And v_d, e_1, and e_2 are the external disturbance, output from disturbance considering sensitivity function, and output from disturbance through controller in the feedback system,respectively

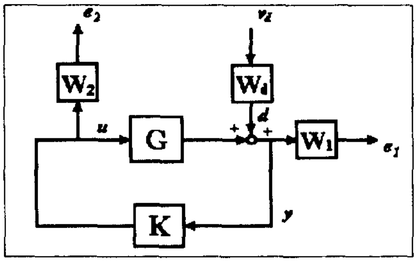

Fig. 1. Block diagram for H^∞ mixed sensitivity problem.

The transfer function from v_d to e_1 and e_2 is given by

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} W_1(I-GK)^{-1}W_d \\ W_2K(I-GK)^{-1}W_d \end{bmatrix} v_d = Tv_d. \tag{1}$$

Then the H^∞ mixed sensitivity problem is defined to be given by

$$\min_{\text{stabilizing } K} \| T \|_\infty \tag{2}$$

where W_1, W_2, and W_d are chosen to meet the given design specifications[7].

The above H^∞ mixed sensitivity problem is formulated to achieve good disturbance rejection and to maintain stability in the face of unstructured additive perturbations. These purposes cannot be done simultaneously over all frequencies, but the weighting functions W_1 and W_2 can be used to emphasize one or other objective over significant frequency ranges.

Typically W_1 is chosen as a high gain, low pass filter in order to reject output disturbances in the low frequency range. Otherwise W_2 is chosen as a high pass filter to maintain stability in the face of high frequency additive perturbations. The weighting function W_d can be regarded as a generator which characterizes all relevant disturbances. W_d is also used as designer freedom for partial pole placement and stopband shaping.

Generally, H^∞ mixed sensitivity problem can have pole-zero cancellation in the closed-loop transfer functions. And it is noted that H^∞ mixed sensitivity problem can be transformed to robust stabilization problem for coprime factor uncertainties. We will consider the coprime factor uncertainties which are used as errors of model reduction.

Throughout this paper, weighting functions are assumed to have the following conditions

i) $W_1, W_2 \in RH^\infty, \quad W_1^{-1}, W_2^{-1} \in RH^\infty$

ii) $W_d^{-1} \in RH^\infty$.

3. Coprime factor reduction

3.1 Frequency weighted model reduction

There are two approaches for designing reduced order controllers; one is to simplify the high order plant and design the controller for it, the other is to get the full order controller for the high order plant and then reduce the controller. Both have some advantages and disadvantages, respectively. In designing reduced order controller, closed loop stability should be guaranteed by any methods and original performance must be preserved as closely as possible. Here we will get the reduced order controller using the plant order reduction method.

With the plant order reduction, the error due to the reduced order model is associated with the perturbation of the stability robustness theorem[4] like shown in Fig. 2.

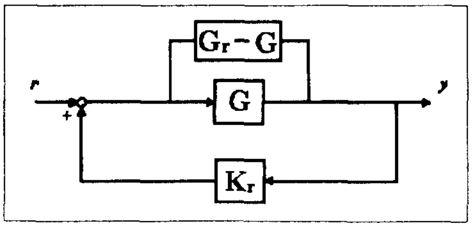

Fig. 2. Block diagram for model reduction of nominal plant G.

The closed loop stability of the nominal plant G with reduced order controller K_r is guaranteed by

$$E = \| [G - G_r] K_r (I - G_r K_r)^{-1} \|_\infty < 1. \tag{3}$$

Unfortunately, the weighting functions for reducing the order of nominal plant G are not known a priori. The weighting functions depend on both the reduced order model and the reduced order controller which are not known before the model reduction.

Enn[2] proposed the frequency weighted balanced truncation model reduction method and recently many researchers are trying to find simple error bound.

3.2 Frequency weighted coprime factor reduction

H^∞ mixed sensitivity problem in Fig. 1 can be changed to the robust control problem with coprime factor uncertainties in Fig. 3. We assume that the poles of W_d are those of plant, then there is no addition in controller order by W_d. Using special W_d, we can get some advantages, that is, it is able to reduce unstable plant, to consider the controller design method using normalized coprime factorization and to replace the given fixed poles in mixed sensitivity problem with desired poles.

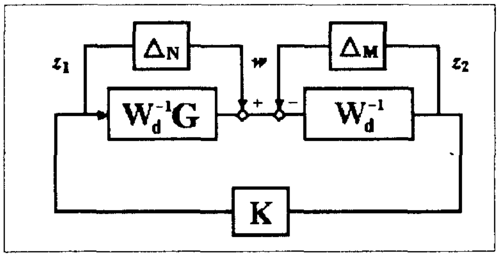

Fig. 3. Closed loop system with coprime factor uncertainties.

Let the condition for coprime factor uncertainties be

$$\left\| [\Delta_N \ \Delta_M] \begin{bmatrix} W_2^{-1} & 0 \\ 0 & W_1^{-1} \end{bmatrix} \right\|_\infty < \varepsilon_{\max}. \tag{4}$$

Then the necessary and sufficient condition for robust stability in the face of $[\Delta_N \ \Delta_M]$ is given by

i) (G, K, W_1, W_2, W_d) is internally stable

ii) $\left\| \begin{matrix} W_1(I - GK)^{-1} W_d \\ W_2 K (I - GK)^{-1} W_d \end{matrix} \right\|_\infty \le \varepsilon_{\max}^{-1}$

where $\varepsilon_{\max}$ is a maximum stability margin.

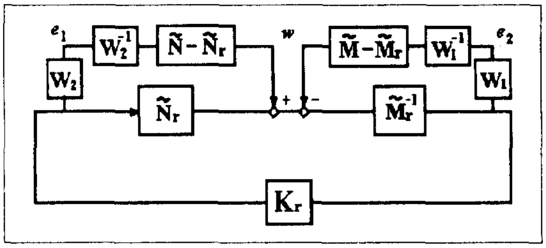

Fig. 4. Block diagram for coprime factor reduction.

In this structure, H^∞ mixed sensitivity problem is equivalent to robust stabilization problem with coprime factor uncertainties. And robustness problem of Fig. 3 can be transformed to coprime factor reduction problem of Fig. 4. Where the coprime factor $[\tilde{N}\ \tilde{M}]$ is defined by $[W_d^{-1}G\ \ W_d^{-1}]$.

Let the frequency weighted coprime factor model be

$$[\tilde{N}W_2^{-1}\ \ \tilde{M}W_1^{-1}], \tag{5}$$

then the reduced coprime factor $[\tilde{N}_r\ \tilde{M}_r]$ of $[\tilde{N}\ \tilde{M}]$ is obtained by frequency weighted balanced truncation of eqn. (5).

Theorem 1

The closed loop stability of $[\tilde{N}\ \tilde{M}]$ with K_r will be guaranteed if

$$\left\| [(\tilde{N}-\tilde{N}_r)W_2^{-1}\ \ (\tilde{M}-\tilde{M}_r)W_1^{-1}] \times \begin{bmatrix} W_2K_r(I-G_rK_r)^{-1}\tilde{M}_r^{-1} \\ W_1(I-G_rK_r)^{-1}\tilde{M}_r^{-1} \end{bmatrix} \right\|_\infty < 1 \tag{6}$$

is satisfied.

(proof) It is directly obtained by small gain theorem. ■

Like the eqn. (3), the weighting functions for reducing the order of $[\tilde{N}\ \tilde{M}]$ are not known a priori. Assume that

$$\varepsilon_1 = \| [(\tilde{N}-\tilde{N}_r)W_2^{-1}\ \ (\tilde{M}-\tilde{M}_r)W_1^{-1}] \|_\infty \tag{7}$$

$$\left\| \begin{bmatrix} W_2K_r(I-G_rK_r)^{-1}\tilde{M}_r^{-1} \\ W_1(I-G_rK_r)^{-1}\tilde{M}_r^{-1} \end{bmatrix} \right\|_\infty < \varepsilon_{r\max} \tag{8}$$

are given, then $\varepsilon_{r\max} > \varepsilon_1$ is a sufficient condition for robust stability. If this condition is satisfied, the closed loop stability of $[\tilde{N}\ \tilde{M}]$ with K_r is maintained. Minimum H^∞-norm of the closed loop system with G and K_r is defined by

$$\varepsilon_r^{-1} = \min_k \left\| \begin{bmatrix} W_1(I-GK_r)^{-1}\tilde{M}^{-1} \\ W_2K_r(I-GK_r)^{-1}\tilde{M}^{-1} \end{bmatrix} \right\|_\infty \tag{9}$$

where k is a reducing order.

To guarantee the robust stability, lower bound of ε_r should be $\varepsilon_r \geq \varepsilon_{\max} - \varepsilon_1 > 0$, but it is not a priori bound because we have to find $\varepsilon_{r\max}$ from the reduced order plant. If we assume to be $\varepsilon_{\max} \leq \varepsilon_{r\max}$, a priori bound $\varepsilon_r \geq \varepsilon_{\max} - \varepsilon_1$ is satisfied. Therefore, $\varepsilon_{\max} > \varepsilon_1$ is a robust stability condition and $(\varepsilon_{\max} - \varepsilon_1)^{-1}$ is a performance bound with reduced order controller. We cannot always replace $\varepsilon_{r\max}$ with $\varepsilon_{\max}$, because $\varepsilon_{\max} \leq \varepsilon_{r\max}$ is not proved. But we can see that it is satisfied in many cases.

For controller design, though any H^∞ design methods can be used, we use the J-lossless coprime factorization approach in here. We construct the CSD(Chain Scattering Description) standard plant of H^∞ mixed sensitivity problem, find J-lossless left and right coprime factors by two Riccati equations and get the H^∞ controller from the J-lossless right coprime factor. In H^∞ mixed sensitivity problem, if we use the weighting function W_d, the standard plant in CSD is always stable and has a good property that H^∞ controller can be designed by solving just one Riccati equation.

4. Numerical example

To illustrate the appropriateness of the proposed approach, we use a depth model of underwater vehicle. The plant is given by

$$G = \frac{2.8649(s+0.6159)(s+4.7767)(s-14.06358)}{s(s+0.063584)(s+0.4734)(s+0.6299)(s+6.6856)}.$$

Though this plant has a pole on jw-axis, coprime factors of plant with W_d is always stable. Therefore the proposed method can be directly used to get the reduced order controller without stable and antistable partial fraction of plant. We compare the robust stability conditions in both cases; one is the reduction of plant in eqn. (3), the other is frequency weighted coprime factor reduction in eqn. (6), and the left side norm of each equations is shown in table 1.

Table 1. Robust conditions

order(K_r)	6	5	4	3
eqn. (3)	0.0004	0.0227	1.0332	>1
eqn. (6)	0.0126	0.0266	0.9755	>1

The result of eqn. (6) represents that the robust stability conditions are satisfied. But the result of eqn. (3) shows that the robust stability conditions are not satisfied in using 4th order controller. Even if the condition is not satisfied, the implemented closed loop system with 4th order controller is stable. Therefore the proposed robust stability condition is more reliable.

To observe the preservation of performance, Fig 5 shows the time response of closed loop systems with full order and reduced order controller.

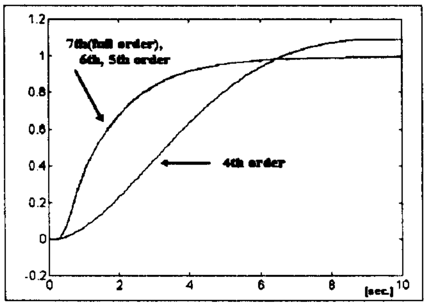

Fig. 5. Time responses of closed loop systems.

In this plant, time response of the reduction of plant itself is similar to one of frequency weighted coprime factor reduction of plant

5. Conclusions

We derived a robust stability condition for the frequency weighted coprime factor reduction of plant in H^∞ mixed sensitivity problem. The nominal plant is factorized to coprime fctors by W_d and we can get the reduced order controller without stable and antistable partial fraction of plant. The weighting functions are impotant factors to improve the performance of closed loop with the reduced order controller, but it is very hard to select. In the furture reserach, It is needed to derive a tight priori bound for the robust stability and preservation of performance. And we have to find a good method to reduce the controller by frequency weighted coprime factor reduction of compensator.

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