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Effects of Time-to-go Freezing on PN Guidance Loop Stability

Dong-Young Rew*, Min-Jea Tahk

Dept. of Aerospace Eng., KAIST, Yusung Ku, Kusung Dong, Taejon 305-701, KOREA
Tel: +82-42-869-3758; Fax: +82-42-869-3710; E-mail: dyrew@fdcl.kaist.ac.kr

Abstracts Due to finite bandwidth of missile dynamics, guidance commands in PN guidance tend to diverge as the missile approaches to the target. In this paper, a new method based on the short-time stability theorem is introduced to extend the stability region.

Keywords guidance, stability, time-to-go

1 Introduction

It is known that PN guidance loops suffer from command divergence under non-ideal missile dynamics. Command divergence characteristics of a homing guidance loop were analyzed by stability analyses. Finite-time Lyapunov approach[1], hyperstability theory [2] were used for finding stability conditions of the guidance loop. These approaches give sufficient conditions for stability which are conservative for real application. Recently, a new approach based on short-time stability theory was introduced[3].

In the intercept engagement, guidance loop instability results performance degradation due to control limitations. And methods preventing command divergence should be devised in the guidance loop design. One of the methods to overcome this stability problem inherent in PN is time-to-go freezing technique. To apply time-to-go freezing technique, it should be determined when to freeze t_{go} in the intercept engagement. However, there is no known method to determine t_{go} freezing time. Moreover, how t_{go} freezing affects the guidance loop performance has not been reported.

In this paper, effects of time-to-go freezing is studied based on the guidance loop stability aspects. Short-time stability theory is applied in analyzing the guidance loop performance, because the change of guidance gain with a preprogrammed schedule can be dealt with ease in the short-time stability approach.

2 Guidance model

Consider a two-dimensional missile/target engagement as shown in Figure 1, where R is the relative range between the target and missile, σ is the LOS angle, and a_m and a_t represent the missile and target acceleration normal to the LOS line, respectively.

Kinematic relation between the target and missile with constant closing velocity V_c is described by the linear equation

$$V_c t_{go}\ddot{\sigma} - 2V_c\dot{\sigma} = -a_m + a_t. \quad (1)$$

Define a new variable δ as $\delta \equiv V_c\dot{\sigma}$, then Eq.(1) can be

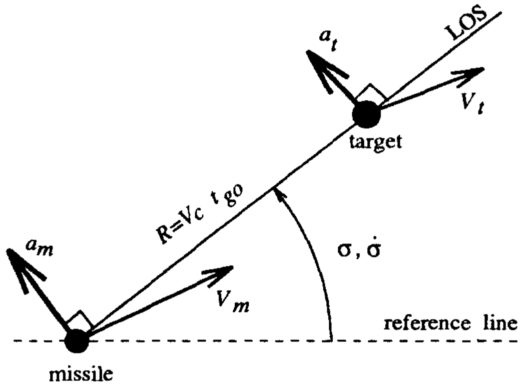

Figure 1: missile and target engagement geometry

rewritten as

$$\dot{\delta} = \frac{2}{t_{go}}\delta + \frac{1}{t_{go}}(-a_m + a_t). \quad (2)$$

In TPN [4], the guidance command, a_{mc}, is calculated from

$$a_{mc} = \Lambda V_c\dot{\sigma} = \Lambda\delta \quad (3)$$

where Λ is the effective guidance gain for which a value between 3 and 5 is frequently chosen in practice.

Missile/autopilot dynamics may be represented as

$$\dot{x}_m = F_m x_m + G_m a_{mc} \quad (4)$$
$$a_m = H_m x_m. \quad (5)$$

By combining Eqs.(2) thru (5), and assuming a non-maneuvering target, PN guidance loop dynamics can be expressed by the following homogeneous linear time-varying differential equation:

$$\begin{bmatrix} \dot{\delta} \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} \frac{2}{t_{go}} & -\frac{1}{t_{go}}H_m \\ \Lambda G_m & F_m \end{bmatrix} \begin{bmatrix} \delta \\ x_m \end{bmatrix}. \quad (6)$$

Zero effort miss is defined as the distance by which the missile will miss the target if the target flies along its present course without maneuver and the missile makes no further corrective maneuvers. For a constant closing velocity, $R(t)$ is given as $R(t) = V_c t_{go}$, and it can be proved that the zero effort miss becomes

$$M(t) \approx V_c\dot{\sigma}(t)t_{go}^2. \quad (7)$$

By substituting $\delta = V_c\dot{\sigma}$ into Eq.(7), we obtain

$$\delta(t) \approx \frac{\text{ZEM}}{t_{go}^2}. \tag{8}$$

Eq.(8) shows that $\delta(t)$ remains finite if the zero effort miss decreases faster than t_{go}^2 as the time to go approaches to zero.

3 Short-time stability of PN guidance loop

The concept of short-time stability[3] is of practical use for qualitative studies of PN since the guidance loop operates during a finite interval of time. PN guidance loop dynamics are reformulated here to apply the short-time stability theorem. Sufficient conditions for short-time stability of PN guidance loop with 1st-order missile/autopilot dynamics are then derived.

Define a new state vector ξ as

$$\xi \equiv \dot{x}_m, \tag{9}$$

then the PN guidance loop dynamical equation is rewritten in terms of ξ and δ as

$$\begin{bmatrix} \dot{\delta}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} F_{11}^1 & F_{12}^1 \\ F_{21}^1 & F_{22}^1 \end{bmatrix} \begin{bmatrix} \delta(t) \\ \xi(t) \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} F_{11}^1 &= \frac{2-\Lambda(t)}{t_{go}} \\ F_{12}^1 &= -\frac{1}{t_{go}} H_m F_m^{-1} \\ F_{21}^1 &= \left(\frac{\Lambda(t)\,(2-\Lambda(t))}{t_{go}} + \dot{\Lambda}(t)\right) G_m \\ F_{22}^1 &= F_m - \frac{\Lambda(t)}{t_{go}} G_m H_m F_m^{-1}. \end{aligned}$$

Consider 1st-order missile/autopilot dynamics and apply the state transformation

$$\delta(t) = \delta(t) \tag{11}$$

$$\xi(t) = \epsilon(t)\zeta(t) \tag{12}$$

to Eq.(10) to obtain

$$\begin{bmatrix} \dot{\delta}(t) \\ \dot{\zeta}(t) \end{bmatrix} = \begin{bmatrix} F_{11}^2 & F_{12}^2 \\ F_{21}^2 & F_{22}^2 \end{bmatrix} \begin{bmatrix} \delta(t) \\ \zeta(t) \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} F_{11}^2 &= \frac{2-\Lambda(t)}{t_{go}} \\ F_{12}^2 &= \frac{\tau}{t_{go}}\epsilon(t) \\ F_{21}^2 &= \left(\frac{\Lambda(t)\,(2-\Lambda(t))}{\tau t_{go}} + \frac{\dot{\Lambda}(t)}{\tau}\right)\frac{1}{\epsilon(t)} \\ F_{22}^2 &= -\frac{1}{\tau} + \frac{\Lambda(t)}{t_{go}} + \frac{\dot{\epsilon}(t)}{\epsilon(t)}. \end{aligned}$$

Note that the physical meaning of short-time stability depends upon the parameters such as $P(t)$, $C(t)$, and C_0. In this paper, $P(t)$, $C(t)$, and C_0 are determined based on the following properties for guidance loop stability.

Guidance loop stability

Property 1. Missile/autopilot states are bounded.

Property 2. Acceleration command is bounded.

Property 3. Zero effort miss decreases with the order of t_{go}^2 as time to go approaches to zero.

For PN guidance loops, Property 2 implies Property 3 because the zero effort miss should decrease at least with the order of t_{go}^2 for the acceleration command to remain finite. These properties can be formulated as a short-time P-stability problem by selecting a suitable weight $P(t)$ and a suitable bound for $x^T P x$.

For example, we can select $P(t)$ as an identity matrix and $C(t)$ as a constant, then we obtain $U(t)$ as

$$U(t) = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \tag{14}$$

where

$$\begin{aligned} U_{11} &= \frac{2\,(2-\Lambda(t))}{t_{go}} \\ U_{12} &= \frac{\Lambda(t)\,(2-\Lambda(t)) + \tau^2\epsilon(t)^2 + t_{go}\dot{\Lambda}(t)}{\epsilon(t)\tau t_{go}} \\ U_{21} &= \frac{\Lambda(t)\,(2-\Lambda(t)) + \tau^2\epsilon(t)^2 + t_{go}\dot{\Lambda}(t)}{\epsilon(t)\tau t_{go}} \\ U_{22} &= \frac{2\,(\tau\Lambda(t) - t_{go})}{\tau t_{go}} - \frac{2\dot{\epsilon}(t)}{\epsilon(t)}. \end{aligned}$$

4 Time-to-go freezing

One of the strategies to schedule guidance gain is freezing t_{go} at a certain time. Time-to-go freezing, in this paper, is assumed to have the following form:

$$a_{mc} = \frac{\Lambda}{t'_{go}} M(t_{go}) \tag{15}$$

where t'_{go} is the time to go at the t_{go} freezing time and $M(t_{go})$ is the zero effort miss at t_{go}. Effects of time-to-go freezing in Eq.(15) can be interpreted as if the guidance gain decreases as shown in figure 2.

The guidance gain is expressed by usig the unit step function.

$$\begin{aligned} \Lambda(t_{go}) &= \bar{\Lambda}\left(\frac{t_{go}}{t'_{go}}\right)^2 u(t_{go}) \\ &\quad + \bar{\Lambda}\left[1 - \left(\frac{t_{go}}{t'_{go}}\right)^2\right] u(t_{go} - t'_{go}) \end{aligned} \tag{16}$$

Time derivative of $\Lambda(t_{go})$ is obtained as

$$\frac{d\Lambda}{dt}(t_{go}) = \begin{cases} -\frac{2\Lambda(t_{go})}{t_{go}} & \text{for } t_{go} \le t'_{go} \\ 0 & \text{for } t_{go} > t'_{go} \end{cases}. \tag{17}$$

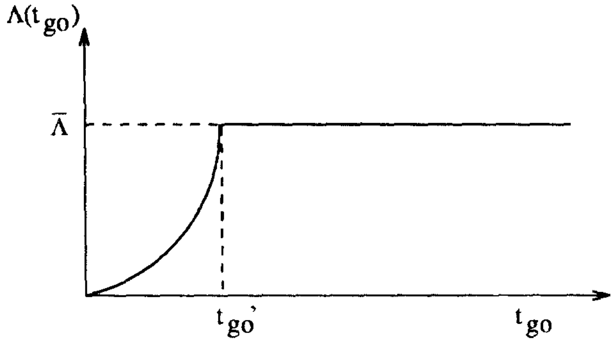

Figure 2: guidance gain schedule for freezing t_{go}

5 Stability conditions

Selection of $\epsilon(t)$ is important for reducing conservatism of the stability condition. It can be shown that the selection of $\epsilon(t)$ so as to make the matrix U be a diagonal form gives least conservative stability conditions. But there are some restrictions in selecting $\epsilon(t)$. Several choices of $\epsilon(t)$ can be considered. In this paper, stort-time stability conditions for

$$\epsilon(t) = \frac{\sqrt{\bar{\Lambda}(\bar{\Lambda}-2)}}{\tau}, \quad \bar{\Lambda} > 2 \tag{18}$$

is derived.

Substituting ϵ defined in Eq.(18) into Eq.(14), we obtain $U(t)$ as follows: For $t_{go} > t'_{go}$,

$$U(t) = \begin{bmatrix} \frac{2(2-\bar{\Lambda})}{t_{go}} & 0 \\ 0 & \frac{2(\tau\bar{\Lambda} - t_{go})}{\tau t_{go}} \end{bmatrix}. \tag{19}$$

For $0 \le t_{go} < t'_{go}$,

$$U(t) = \begin{bmatrix} \frac{2(2-\Lambda(t))}{t_{go}} & \frac{-\Lambda(t)^2+\kappa}{\tau t_{go}\sqrt{\kappa}} \\ \frac{-\Lambda(t)^2+\kappa}{\tau t_{go}\sqrt{\kappa}} & \frac{2(\tau\Lambda(t)-t_{go})}{\tau t_{go}} \end{bmatrix}, \tag{20}$$

where $\kappa \equiv \bar{\Lambda}(\bar{\Lambda}-2)$.

For convenience, we introduce some non-dimensional parameters. t_{go} at the initial time, or total engagement time is nomalized as $\eta_0 \equiv \frac{t_{go}(t_0)}{\tau}$. t_{go} at the t_{go} freezing time is represents by the normalized parameter $\eta' \equiv \frac{t'_{go}}{\tau}$. Stability limit is defined as $\eta^* \equiv \frac{t^*_{go}}{\tau}$ where t^*_{go} represents the least value of t_{go} that guarantees guidance loop stability. Stability region, denoted by SR, is defined by using these non-dimensional parameters as

$$\mathrm{SR} = [\eta^*, \eta_0].$$

Stability condition I

In real guidance engagements, t_{go} will be frozen before guidance command divergence occurs. Therefore, it is sufficient to check the stability condition for $0 \le t_{go} < t'_{go}$.

Stability condition I can be evaluated based on Theorem I which states non-negative definiteness of $-U(t)$ for stability. For $-U(t)$ being non-negative definite, the following two inequality conditions should be satisfied simultaneously:

$$\frac{2(\Lambda(t)-2)}{t_{go}} \ge 0 \tag{21}$$

$$\frac{4(2-\Lambda(t))(\tau\Lambda(t)-t_{go})}{\tau t_{go}} - \frac{(\Lambda(t)^2-\kappa)^2}{\tau^2 t_{go}^2 \kappa} \ge 0. \tag{22}$$

The condition given by Eq.(21) is simplified as

$$\Lambda(t) \ge 2. \tag{23}$$

But the condition given by Eq.(22) can be evaluated by numerical method.

Eq.(22) is rewritten as

$$4\tau\kappa(\Lambda(t)-2)(\tau\Lambda(t)-t_{go}) \le -(\Lambda(t)^2-\kappa)^2. \tag{24}$$

Noting that the right hand side of Eq.(24) is positive and that $\Lambda(t) \ge 2$ from Eq.(23), we obtain $\eta \ge \Lambda(t)$. As $\Lambda(t)$ is given by $\Lambda(t) = \bar{\Lambda}\eta/\eta'$, Eq.(23) becomes

$$\eta \ge \sqrt{2\bar{\Lambda}\eta'}. \tag{25}$$

If we assume that t_{go} is frozen before instability occurs, η' should satisfiy $\eta' > \bar{\Lambda}$ which is the stability condition I based on a constant guidance gain. Therefore, Eq.(25) becomes

$$\eta > \sqrt{2\bar{\Lambda}\bar{\Lambda}}. \tag{26}$$

Note that the condition given by Eq.(26) is not a sufficient condition but a necessary condition for short-time stability.

Stability condition II

Stability condition II for a PN guidance loop with t_{go} freezing can be derived using Theorem II. Stability conditions are calculated by direct integration of maximum eigenvalue of $U(t)$ given by Eq.(19) and (20).

Stability condition II varies with η'. Figures 3 and 4 show the change of the stability limit with η' for $\bar{\Lambda} = 3$, $\bar{\Lambda} = 4$, respectively. Dotted line in each figure represents the guidance gain at the stability limit.

Figures 5 and 6 show the change of the stability limit as a function of η' for $\bar{\Lambda} = 3$, $\bar{\Lambda} = 4$, and $\bar{\Lambda} = 5$, respectively.

Figures of the stability limit show that t_{go} freezing method can extend the stability region and that selection of t_{go} freezing time is important. Too fast t_{go} freezing reduces the stability region and too late t_{go} freezing also reduces the stability region. When t_{go} is frozen so late, the stability limit becomes the same as that of conventional PN. Straight line before $\eta' = 0$, in each figure, corresponds to the stability limit of conventional PN.

Effect of t_{go} freezing is larger when total engagement time is small and constant guidance gain is large.

6 Conclusion

In this paper, effects of t_{go} freezing is analyzed based on the short-time stability theory. It may be the first attempt to obtain the effects of t_{go} freezing without computer simulation study.

Stability conditions obtained in this paper show that the method of t_{go} freezing help to extend stability region and that t_{go} freezing time is most important. It is also shown that effects of t_{go} freezing increase when total engagement time is short and constant guidance gain is large.

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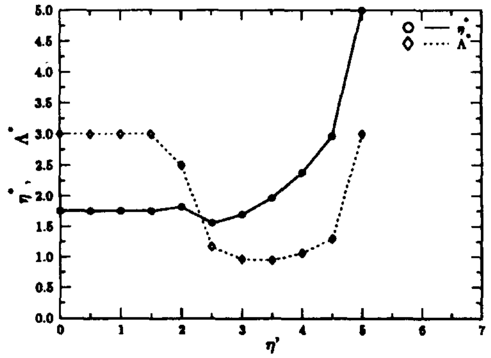

Figure 3: relationship between η' and stability limit(η^*) for $\eta_0 = 5$ and $\bar{\Lambda} = 3$

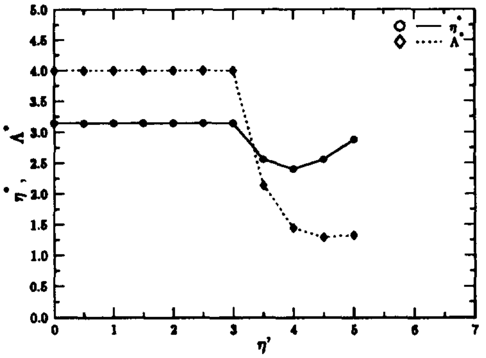

Figure 4: relationship between η' and stability limit(η^*) for $\eta_0 = 5$ and $\bar{\Lambda} = 4$

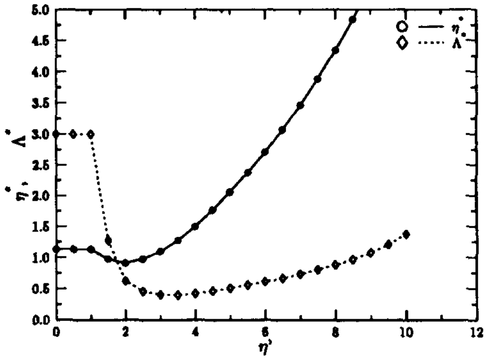

Figure 5: relationship between η' and stability limit(η^*) for $\eta_0 = 10$ and $\bar{\Lambda} = 3$

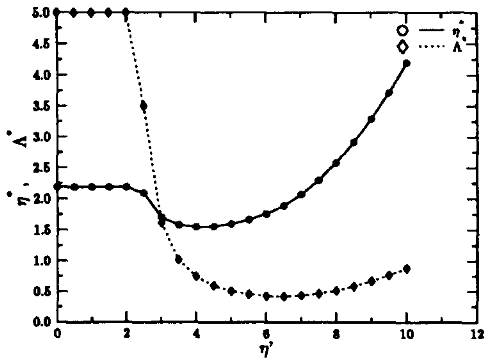

Figure 6: relationship between η' and stability limit(η^*) for $\eta_0 = 10$ and $\bar{\Lambda} = 5$