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Terminal Sliding Mode Control of Robot Manipulators for PTP Task

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Abstract In this paper, a variable structure control scheme with a terminal sliding mode is proposed for robot manipulators. The proposed control scheme guarantees that the output tracking error converges to zero in finite time, and the overall system shows robust property against parametric uncertainties and external disturbances all the time.

Keywords Variable Structure Control(VSC), Sliding Mode Control(SMC), Robust Control.

1. INTRODUCTION

Recently, Variable Structure Control (VSC) has received increasing interest [1]-[3]. In the field of the VSS application, the robot manipulators have been studied a lot for a long time [4]-[6]. However, since these VSS controllers have been designed with simple linear sliding surfaces, the system has a reaching phase problem. In addition, in order to get a fast transient response and fast output error convergence, the poles of the sliding mode dynamics have to be located far from the origin on the open left half s-plane, and thus it increases control gains. Furthermore, in some cases, it is preferable that the relaxation time can be adjustable [7]-[8].

To overcome these shortcomings, some terminal sliding mode controllers have been proposed based on the Zak's study [9]-[10]. However, the controllers produce unbounded control signals around $x = 0$ axis and they have to be able to evaluate a nonlinear function $x^{\frac{p}{q}}$; moreover, a nonlinear function $x^{\frac{p}{q}}$ has to satisfy the additional conditions which were not noted on their works [11].

Thus, in this paper, a variable structure control scheme with the terminal sliding mode is proposed for robot manipulators using new nonlinear sliding surfaces. The proposed controller guarantees that the output tracking error converges to zero in finite time; besides, it assures the occurrence of the sliding mode all the time. Therefore, the overall system always shows robust property against parametric uncertainties and external disturbances, that is, the tracking error curve can be predetermined on the entire time interval regardless of the existence of the modeling uncertainties and disturbances.

2. ROBOT DYNAMICS

The dynamic equation of an n degree-of-freedom robot manipulator can be derived using the Lagrangian formulation as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + d, \qquad (1)$$

where $M(q)$ is an $n \times n$ inertia matrix, $C(q,\dot{q})$ is an $n \times n$ matrix corresponding to Coriolis and centrifugal factors, $G(q)$ is an $n \times 1$ vector of gravitational torques, d is an $n \times 1$ bounded disturbance vector, q is an $n \times 1$ vector of joint angular positions, and u is an $n \times 1$ input torque vector.

Let us define each matrices as

$$\begin{aligned} M &= M^{0} + \Delta M, \\ C &= C^{0} + \Delta C, \\ G &= G^{0} + \Delta G, \end{aligned}$$

where "^{0}" denotes the mean value and "Δ" denotes the estimation error. Assume that the $\Delta M_{ij}, \Delta C_{ij}$, and ΔG_i are bounded by M_{ij}^m, C_{ij}^m, and G_i^m, respectively as

$$\begin{aligned} |\Delta M_{ij}| &\le M_{ij}^m, \\ |\Delta C_{ij}| &\le C_{ij}^m, \\ |\Delta G_i| &\le G_i^m, \end{aligned}$$

where "m" denotes the maximal absolute estimation error of each element, and $i, j = 1, 2, \cdots, n$. At the same time, we assume that $|d_i| \le d_i^m$, where $i = 1, 2, \cdots, n$.

3. CONTROL SYSTEM DESIGN

Let us define the trajectory tracking error as

$$e(t) = q(t) - q_d(t),$$

where $q_d(t)$ represents the desired trajectory, and propose a following terminal sliding surface:

$$s_i = \dot{e}_i + h_i(t), \tag{2}$$

where the function h_i is assumed to satisfy the following assumption.

Assumption 1 $h_i : \Re_\dagger \rightarrow \Re$, $h_i \in C^1[0,\infty)$, $\dot{h}_i \in L^\infty$ and $h_i \in L^p \cap L^\infty$ for some $p \in [1,\infty)$. Where, $C^1[0,\infty)$ represents the set of all first differentiable continuous functions defined on $[0,\infty)$.

The function h_i can be chosen arbitrarily such that the tracking error goes to zero in finite time, that is, h_i represents the desired error velocity curve. And the initial value of h_i is set as $h_i(0) = \dot{e}(0)$.

Let us define the following positive-definite function as a Lyapunov function candidate:

$$V = \frac{1}{2} s^T M s. \tag{3}$$

Differentiating (3) with respect to time and adopting the skew-symmetricity of $\dot{M}(q) - 2C(q,\dot{q})$, we have

$$\begin{aligned}
\dot{V} &= s^T M \dot{s} + s^T C s \\
&= s^T (M\dot{s} + Cs) \\
&= s^T \left(M\ddot{q} - M\ddot{q}_d - M\dot{h} + Cs\right) \\
&= s^T \left(u + d - C\dot{q} - G - M(\ddot{q}_d + \dot{h}) + Cs\right) \\
&= s^T \left(u + d - G - M(\ddot{q}_d + \dot{h}) + C(s - \dot{q})\right), \qquad (4)
\end{aligned}$$

From (4), the equivalent control law is defined as

$$u_{eq} = M^0 (\ddot{q}_d + \dot{h}) - C^0 (s - \dot{q}) + G^0. \tag{5}$$

Now, we introduce the control scheme such as

$$u = u_{eq} - K \bullet sgn(s), \tag{6}$$

where "$\bullet$" means the element-by-element multiplication of two vectors, and

$$\begin{aligned}
K &= M^m \left|\ddot{q}_d + \dot{h}\right| + C^m |s - \dot{q}| + G^m + d^m + \eta, \\
\eta &= [\,\eta_1,\ \eta_2,\ \cdots,\ \eta_n\,]^T, \qquad \eta_i > 0, \\
sgn(s) &= [\,sgn(s_1),\ sgn(s_2),\ \cdots,\ sgn(s_n)\,]^T, \\
sgn(s_i) &= \begin{cases} 1 & \text{if } s_i > 0 \\ 0 & \text{if } s_i = 0 \\ -1 & \text{if } s_i < 0 \end{cases} \qquad i = 1,\ 2,\ \cdots,\ n,
\end{aligned}$$

and the absolute of a vector denotes the vector whose element has its absolute value, i.e. $|x| = [\,|x_1|,\ |x_2|, \cdots, |x_n|\,]^T$.

Using the above control law, we can derive a following theorem for the occurrence of the sliding mode.

Theorem 1 For the robot manipulator (1) with the control law (6), the system is in the sliding mode all the time.

Proof: By inserting (6) in (4), we can obtain the following inequality:

$$\begin{aligned}
\dot{V} &= s^T \Big\{ M^0 \left(\dot{h} + \ddot{q}_d\right) - C^0 (s - \dot{q}) + G^0 \\
&\quad - \left(M^m \left|\dot{h} + \ddot{q}_d\right| + C^m |s - \dot{q}| + G^m + d^m + \eta\right) \\
&\quad \bullet sgn(s) + d - G - M\left(\dot{h} + \ddot{q}_d\right) + C(s - \dot{q}) \Big\} \\
&= s^T \Big\{ \left(M^0 - M\right)\left(\dot{h} + \ddot{q}_d\right) + \left(C - C^0\right)(s - \dot{q}) \\
&\quad + \left(G^0 - G\right) - M^m \left|\dot{h} + \ddot{q}_d\right| \bullet sgn(s) \\
&\quad - C^m |s - \dot{q}| \bullet sgn(s) - G^m \bullet sgn(s) \\
&\quad + d - d^m \bullet sgn(s) - \eta \bullet sgn(s) \Big\} \\
&\le -\sum_{i=1}^{n} \eta_i |s_i| .
\end{aligned}$$

From the above inequality, it is clear that $\dot{V} = 0$ if and only if $s = 0$, and so $\dot{V} \le 0$. Therefore, $V(t) = 0\ \forall t \ge 0$ because $s(0) = 0$ from (2). This also implies that $s(t) \equiv 0$. Thus, the system is forced to stay in the sliding mode all the time.

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Therefore, the following theorem can be derived for the stability of the overall system and the error convergence time.

Theorem 2 If the control law (6) is applied to the robot manipulator (1), then the overall system is globally exponentially stable and the tracking error converges to zero in finite time.

Proof: From the Theorem 1, we know that $s_i(t) \equiv 0\ \forall i$. Furthermore, it is clear that the sliding mode ($s_i = 0$) for the proposed terminal siding surface is stable and has a terminal attractor $e_i = 0\ \forall i$. Therefore, the overall system is globally exponentially stable and the tracking error e converges to the terminal attractor $e = 0$ in finite time. It completes the proof.

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Remark 1: Since $h_i(t)$ can be designed arbitrarily by the control system designer, one can design a control system such that the tracking error converges to zero within a given desired relaxation time for *any* given initial condition. For example, in case of the motor control, the desired velocity curve is usually given as a trapezoidal form. Then $h(t)$ is given as in Fig. 1.

Remark 2: In the proposed sliding surface, a saturation function can be used instead of the signum function, $sgn(\cdot)$. When a saturation function is used, the tracking error cannot converge to *zero* in finite time.

4. SIMULATION

The simulation has been carried out for a two-link robot manipulator model used by Yeung and Chen [12]. The dynamic equation is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + d,$$

where $q = [q_1\ q_2]^T$,

$$\begin{aligned}
M_{11} &= (m_1+m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos q_2 + J_1 \\
M_{12} &= M_{21} = m_2 r_2^2 + m_2 r_1 r_2 \cos q_2 \\
M_{22} &= m_2 r_2^2 + J_2 \\
C_{11} &= -2m_2 r_1 r_2 \dot{q}_2 \sin q_2 \\
C_{12} &= -m_2 r_1 r_2 \dot{q}_2 \sin q_2 \\
C_{21} &= m_2 r_1 r_2 \dot{q}_1 \sin q_2 \\
C_{22} &= 0 \\
G_1 &= \{(m_1+m_2)r_1 \cos q_1 + m_2 r_2 \cos(q_1+q_2)\}\, g \\
G_2 &= m_2 r_2 g \cos(q_1+q_2).
\end{aligned}$$

The parameter values are also the same as those of Yeung and Chen.

$$\begin{gathered}
r_1 = 1m, \quad r_2 = 0.8m, \\
J_1 = 5kg\cdot m, \quad J_2 = 5kg\cdot m, \\
m_1 = 0.5kg, \\
0.5kg < m_2 < 6.25kg, \\
|d_1|,\ |d_2| < 20N\cdot m.
\end{gathered}$$

In the simulation, we chose two $h_i(t)$'s as following:

$$h_1(t) = \begin{cases} 2t & \text{if } 0 \le t \le 0.5 \\ 1 & \text{if } 0.5 \le t \le 1 \\ 1-2(t-1) & \text{if } 1 \le t \le 1.5 \\ 0 & \text{otherwise} \end{cases}$$

$$h_2(t) = \begin{cases} -6\left(\dfrac{t}{1.5}\right) + 6\left(\dfrac{t}{1.5}\right)^2 & \text{if } 0 \le t \le 1.5 \\ 0 & \text{otherwise} \end{cases}$$

That is to say, for the first link of the robot manipulator h is chosen such that the constant acceleration, constant velocity, constant deceleration which means that the h is chosen for the motor to operate in the manner of time sub-optimal. And for the second link h is chosen such that the motor command is generated with smooth form.

The results are shown in Figs. 2–4. In these figures, the solid lines represent the response of the first link and the dashed curves represent those of second link.

The tracking errors of the proposed controller are presented in the Fig. 2. It can be seen that the tracking error converges to zero in finite time (1.5 *sec*).

Fig. 3 shows the velocity curves. As can be seen in this figure, $\dot{e}_1$ shows trapezoidal curve, that is, acceleration part, constant velocity part, and deceleration part. And $\dot{e}_2$ represents more smooth velocity curve than that of $\dot{e}_1$.

In the Fig. 4, phase portrait is given. In the figure, it is clear that the sliding mode always occurrs. Thus, the overall system shows robust property against external disturbances and parameter uncertainties.

5. CONCLUSIONS

In this paper, a variable structure control scheme with the terminal sliding mode is proposed for robot manipulators. The proposed sliding surface guarantees that the output tracking error converges to zero in finite time under parametric uncertainties and external disturbances. The simulation results showed that the tracking error successfully converges to zero in finite time. Furthermore, since the occurrence of the sliding mode can be guaranteed all the time, the overall system always shows robust property against parametric uncertainties and external disturbances.

6. REFERENCES

[1] V. I. Utkin, "Variable Structure Systems with Sliding Mode," *IEEE Trans. Automat. Contr.*, vol. AC-22, no. 2, pp. 212–222, 1977.

[2] R. A. DeCarlo, S. H. Zak, and G. P. Matthews, "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial," *Proc. IEEE*, vol. 76, no. 3, pp. 212–232, 1988.

[3] J. Y. Hung, W. Gao, and J. C. Hung, "Variable Structure Control: A Survey," *IEEE Trans. Ind. Electron.*, vol. IE-40, no. 1, pp. 2–22, 1993.

[4] F. Harashima, H. Hashimoto, and S. Kondo, "MOSFET converter-fed position servo system with sliding mode control," *IEEE Trans. Ind. Electron.*, vol. IE-32, no. 3, pp. 238–244, 1985.

[5] Y. Chen, T. Mita, and S. Wakui, "A New and Simple Algorithm for Sliding Mode Trajectory Control of the Robot Arm," *IEEE Trans. Automat. Contr.*, vol. AC-35, no. 7, pp. 828–830, 1990.

[6] W. Zhu, H. Chen, and Z. Zhang, "A Variable Structure Robot Control Algorithm with an Observer," *IEEE Trans. Robotics and Automation*, vol. RA-8, no. 4, pp. 486–492, 1992.

[7] M. Zak, "Terminal Attractors for Addressable Memory in Neural Networks," *Physics Letters A*, vol. 133, no. 1,2, pp. 18–22, 1988.

[8] S. Jayasuriya and A. R. Diaz, "Performance Enhancement of Distributed Parameter Systems by a Class of Nonlinear Controls," *Proc. IEEE Conf. Decision and Control*, pp. 2125–2126, 1987.

[9] S. T. Venkataraman and G. Gulati, "Control of Nonlinear Systems using Terminal Sliding Modes," in *Proc. Amer. Contr. Conf.*, pp. 891–893, 1992.

[10] Man Zhihong, A. P. Paplinski, and H. R. Wu, "A Robust MIMO Terminal Sliding Mode Control Scheme for Rigid Robotic Manipulators," *IEEE Trans. Automat. Contr.*, vol. AC-39, no. 12, pp. 2464–2469, 1994.

[11] K. B. Park and J. J. Lee, "Comments on "A Robust MIMO Terminal Sliding Mode Control Scheme for Rigid Robotic Manipulators"," submitted to *IEEE Trans. Automat. Contr.*, 1995.

[12] K. S. Yeung and Y. P. Chen, "A new controller design for manipulators using the theory of vari-

able structure systems," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 2, pp. 200–206, 1988.

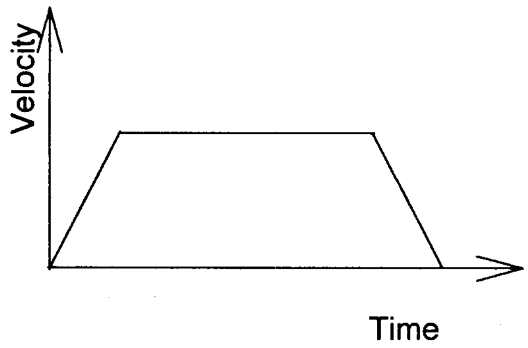

Fig. 1. Trapezoidal Velocity Command

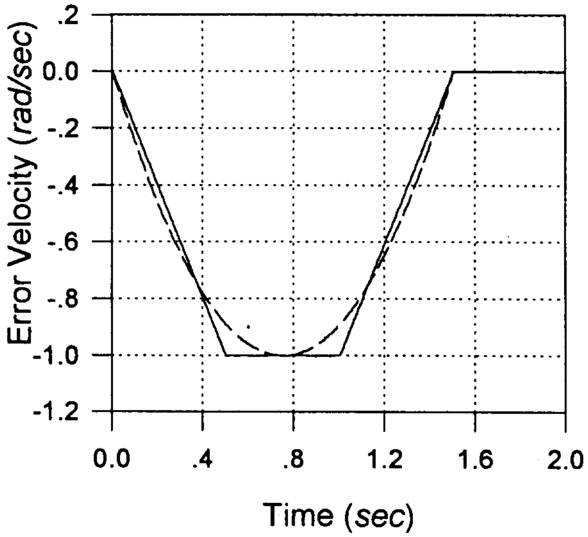

Fig. 3. Error Velocity

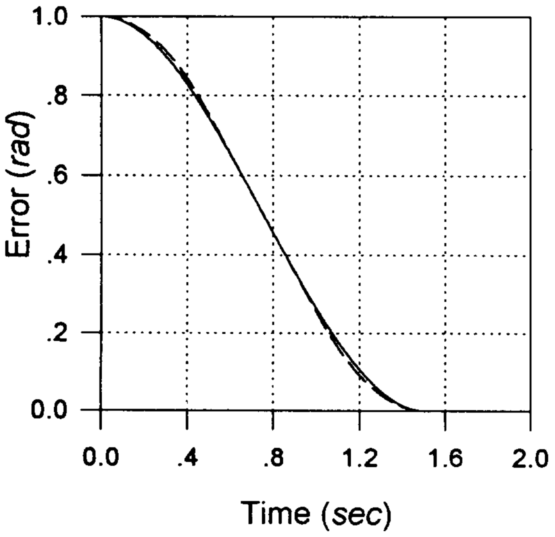

Fig. 2. Tracking Error

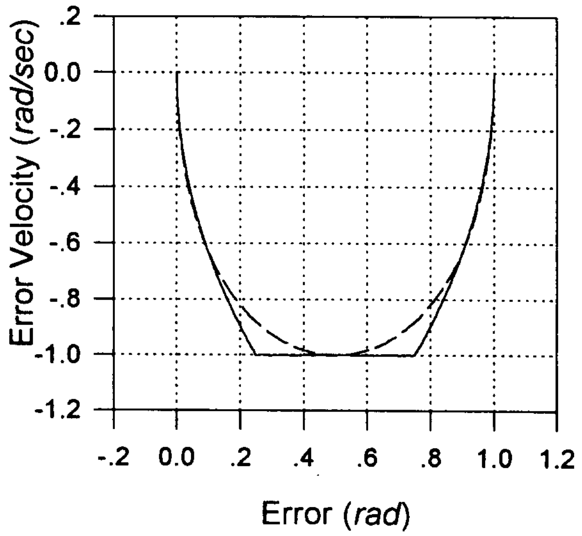

Fig. 4. Phase Portrait