

OPTIMAL LOCATION OF ACOUSTIC SENSORS AND OPTIMAL OBSERVATION POLICY FOR DETECTING ANOMALOUS PLANE OBJECT IN SHIELD CONSTRUCTION METHOD

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Abstract In excavating tunnels, shield tunneling machines having many cutters on their cutter planes are used. Not many observation data being available in the detection system, optimal observation policy is very important. From this viewpoint, we previously considered the optimal location of acoustic sensors on the cutter plane and also the optimal observation policy for the case where three receiving transducers were used, and showed that the optimal sensor location was given as arbitrary equally-spaced points on the cutter plane circle, and that the optimal rotating angles were also found to be arbitrary. In application, however, it is often difficult to locate sensors at arbitrary positions or to use three sensors from the viewpoints of machine structure and cost. This paper considers the optimal observation policy for detecting anomalous plane objects for the case where two receiving transducers are used and the case where three receiving transducers are located only on a diameter of the cutter plane.

Keywords Tunnel excavating, Anomalous plane objects, Acoustic sensors, Optimal sensor location, Optimal observation policy

1. INTRODUCTION

In constructing subways and laying power cables under the ground, tunnels are excavated by means of shield tunneling machines. If the excavation is done without being aware of existence of rocks that are too big to be excavated, underground structures, foundation pillars of the buildings on the ground, concrete walls and so on, the shield tunneling machines will be damaged. Furthermore, the term of the construction will be greatly delayed to repair the shield tunneling machines. It is, therefore, a very important to detect in advance where the anomalous objects are located in front of the shield tunneling machines. From this viewpoint, the author previously developed an effective detection system⁴⁾ to detect anomalous objects ahead. The system made use of acoustic sensors located on the cutter plane of the machine to get information on the anomalous objects. Not many observation data being available in the detection, it is important to optimize the location of receiving transducers on the cutter plane and to determine optimal rotating angles of cutter plane to get information on anomalous objects. The authors also considered, with this point as background, the optimization problem for the case where three receiving transducers were available^{5),6)}. As the result, the optimal location of receiving transducers was shown to be given as arbitrary equally-spaced points on the circle of cutter plane, and the optimal rotating angles were found to be arbitrary.

In applications, however, it is often difficult to locate sensors at arbitrary positions of the cutter plane and to use three sensors from the viewpoints of machine structure and cost. Accordingly, it is significant to clarify the optimal location of acoustic sensors and the optimal observation policy for detecting anomalous objects under such a practical situation. For want of a space, the paper deals with the detection of anomalous plane objects and discusses the optimization

problem for the two cases ; 1)two receiving transducers are available on the cutter plane, 2)three receiving transducers are available only on a diameter of the cutter plane.

2. ESTIMATION ERROR OF THE ANOMALOUS PLANE OBJECT

The detection system collects information on the sound propagation distance from the transmitting transducer(on the cutter plane) to the receiving one via the reflection point on the anomalous plane object. The distance is obtained as the product of the propagation time measured and the velocity of the sound under ground. The number of distance data on the anomalous plane object is given by the product of the number of sensors used and the number of times the tunneling machine stops. For simplicity of discussion, acoustic sensors are assumed to have double functions of transmitting and receiving.

An orthogonal coordinate system is defined as Fig.1. Here x axis is in the direction of the machine's forward movement. Fig.1 shows the relation between cutter plane of the machine and the anomalous plane object \mathcal{P} . In the figure, the vector $\vec{OA} = \mathbf{a}$ denotes the normal vector of the anomalous plane object. Thus the anomalous plane \mathcal{P} is expressed by the equation

$$\mathbf{a}^T \mathbf{x} = \|\mathbf{a}\|^2, \quad \mathbf{x} \triangleq (x, y, z)^T \quad (1)$$

where $\|\cdot\|$ and T denote the Euclidean norm and the transpose respectively. Therefore characterizing the anomalous plane \mathcal{P} is equivalent to determining the normal vector \mathbf{a} .

By the way, in the detection system for the anomalous plane object, the sound propagation distance s_j up to the reflection point is measured. On the other hand, if the normal vector \mathbf{a} is given, the theoretical reflection point is deter-

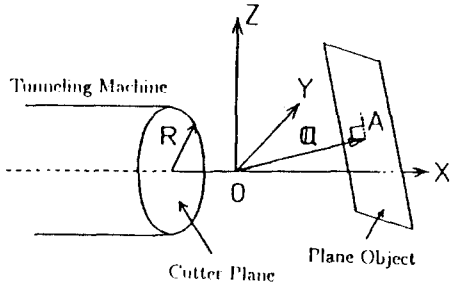


Fig.1. Relation between cutter plane and the anomalous plane object

mined by the Snell's law³⁾ corresponding to the sensor position $\mathbf{x}_j (1 \leq j \leq M)$, where M is the number of observation points. Therefore, by applying the least squares method to the measured and theoretical values, the normal vector \mathbf{a} is estimated⁶⁾. If measured values of $\{s_j\} (1 \leq j \leq M)$ have no errors, true normal vector \mathbf{a}_o is obtained by the least squares method. In practice, however, the sound propagation times and the underground sound velocity having errors, the normal vector \mathbf{a} obtained by the least squares method has the error $\Delta \mathbf{a}$ from the true vector \mathbf{a}_o . The covariance matrix of the error $\Delta \mathbf{a}$ defined by $\Sigma(\mathbf{a}_o) \triangleq E[\Delta \mathbf{a} \Delta \mathbf{a}^T]$ plays an important role in determining the optimal sensor location and the optimal observation policy. The covariance matrix is given by⁶⁾

$$\Sigma(\mathbf{a}_o) = \sigma^2 Q^{-1}(\mathbf{a}_o) \quad (2)$$

where σ^2 is the error variance of the measured values of $\{s_j\}$ and $Q(\mathbf{a}_o)$ is given by

$$Q(\mathbf{a}_o) \triangleq \sum_{j=1}^M \mathbf{b}_j(\mathbf{a}_o) \mathbf{b}_j^T(\mathbf{a}_o) \quad (3)$$

Here the vector $\mathbf{b}_j(\mathbf{a}_o)$ is defined by

$$\mathbf{b}_j(\mathbf{a}_o) \triangleq \frac{2}{a} \left\{ \mathbf{a}_o + \left(\frac{1}{a^2} \mathbf{a}_o \mathbf{a}_o^T - I_3 \right) \mathbf{x}_j \right\}; 1 \leq j \leq M \quad (4)$$

where $a = \|\mathbf{a}_o\|$. From the equations, it is found that the covariance matrix $\Sigma(\mathbf{a}_o)$ is varied with the true normal vector \mathbf{a}_o and the sensor positions $\{\mathbf{x}_j\}$.

3. OPTIMAL SENSOR LOCATION AND OPTIMAL OBSERVATION POLICY

Taking the cost and the constraint on the machine structure in mind, we here discuss the optimal sensor location and the optimal observation policy for the case where two sensors are available on the cutter plane and the case where three sensors are available only on a diameter of the cutter plane. As the optimization criterion we adopt the minimization of $\det[\Sigma(\mathbf{a}_o)]$ as well as in the previous paper⁶⁾. From (2), it is equivalent to the maximization of $\det[Q(\mathbf{a}_o)]$. For convenience of the subsequent analysis we represent the normal vector \mathbf{a}_o in polar coordinate as follows.

$$\mathbf{a}_o = a (\cos \theta, \sin \theta \cos \varphi, \sin \theta \sin \varphi)^T \quad (5)$$

where a is the shortest distance from the origin to \mathcal{P} .

3.1 The Case Two Sensors Available

We derive the optimal sensor location on the cutter plane and the optimal rotating angles of the cutter plane in the sense of minimax criterion. To get the optimal solution, the cutter plane is rotated at the same position and plural times of observations are carried out for getting optimal sensor location and optimal observation policy detecting for an anomalous plane object at a long distance from the machine.

Let the center of the cutter plane be the origin and the distances between the center O and the sensor S_1, S_2 be R_1, R_2 , respectively, with α the angle between OS_1 and OS_2 .

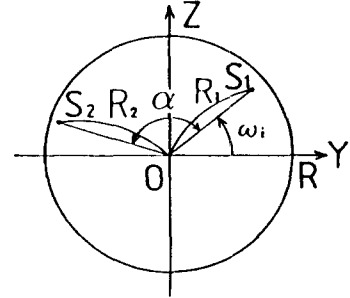


Fig.2. Location of sensors

ω_i denotes the angle between OS_1 and y axis, in the i -th rotating position, where ω_i is expressed in the range $0 \leq \omega_i < 2\pi$ without loss of generality and p denotes the number of rotation of the cutter plane. The position vectors $\mathbf{x}_1^i, \mathbf{x}_2^i$ of the sensor S_1, S_2 at the i -th rotation are given by

$$\mathbf{x}_1^i = \begin{bmatrix} 0 \\ R_1 \cos \omega_i \\ R_1 \sin \omega_i \end{bmatrix}, \quad \mathbf{x}_2^i = \begin{bmatrix} 0 \\ R_2 \cos(\omega_i + \alpha) \\ R_2 \sin(\omega_i + \alpha) \end{bmatrix} \quad (6)$$

Substituting (5),(6) into (4), we get the following vectors $\mathbf{b}_1^i(\mathbf{a}_o)$ and $\mathbf{b}_2^i(\mathbf{a}_o)$

$$\mathbf{b}_i^k(\mathbf{a}_o) = (2/a) \times \begin{bmatrix} a \cos \theta & + R_k \cos \theta \sin \theta \cos(\varphi - \xi_i^k) \\ a \sin \theta \cos \varphi + R_k \sin^2 \theta \cos \varphi \cos(\varphi - \xi_i^k) - R_k \cos \xi_i^k \\ a \sin \theta \sin \varphi + R_k \sin^2 \theta \sin \varphi \cos(\varphi - \xi_i^k) - R_k \sin \xi_i^k \end{bmatrix} \quad (k=1, 2; 1 \leq i \leq p) \quad (7)$$

corresponding to $\mathbf{x}_1^i, \mathbf{x}_2^i$, where $\xi_i^1 \triangleq \omega_i, \xi_i^2 \triangleq \omega_i + \alpha (1 \leq i \leq p)$. Therefore, defining the matrix $B(\mathbf{a}_o) \triangleq [\mathbf{b}_1^1(\mathbf{a}_o), \mathbf{b}_1^2(\mathbf{a}_o), \dots, \mathbf{b}_p^1(\mathbf{a}_o), \mathbf{b}_p^2(\mathbf{a}_o)]$, the matrix $Q(\mathbf{a}_o)$ defined by (3) is represented by

$$Q(\mathbf{a}_o) = B(\mathbf{a}_o) B^T(\mathbf{a}_o) \quad (8)$$

Applying a formula¹⁾ on determinant of positive definite matrix to $Q(\mathbf{a}_o)$ and also applying fundamental row operation on the matrix $B(\mathbf{a}_o)$, we get

$$\begin{aligned} \det[Q(\mathbf{a}_o)] &\leq (4/a^2)^3 \times (2pa^2 \cos^2 \theta) \\ &\times \left\{ \sum_{k=1}^2 \left(R_k^2 \sum_{i=1}^p \cos^2 \xi_i^k \right) \right\} \left\{ \sum_{k=1}^2 \left(R_k^2 \sum_{i=1}^p \sin^2 \xi_i^k \right) \right\} \\ &\leq (4/a^2)^3 (2pa^2 \cos^2 \theta) \{ (pR_1^2 + pR_2^2)/2 \}^2 \\ &\leq (4/a^2)^3 2p^3 a^2 R^4 \cos^2 \theta \end{aligned} \quad (9)$$

where the three equalities hold when the following equations are satisfied.

$$R_1 = R_2 = R \quad (10)$$

$$\sum_{i=1}^p \cos \omega_i + \sum_{i=1}^p \cos(\omega_i + \alpha) = 0 \quad (11)$$

$$\sum_{i=1}^p \sin \omega_i + \sum_{i=1}^p \sin(\omega_i + \alpha) = 0 \quad (12)$$

$$\sum_{i=1}^p \sin 2\omega_i + \sum_{i=1}^p \sin 2(\omega_i + \alpha) = 0 \quad (13)$$

$$\sum_{i=1}^p \cos 2\omega_i + \sum_{i=1}^p \cos 2(\omega_i + \alpha) = 0 \quad (14)$$

The solutions of the equations (10)–(14) are given as follows.

a) $R_1 = R_2 = R$, $\alpha = \pi$. Rotating angles $\{\omega_i\}$ such that

$$\sum_{i=1}^p \cos 2\omega_i = 0, \quad \sum_{i=1}^p \sin 2\omega_i = 0 \quad (15)$$

b) $R_1 = R_2 = R$, $\alpha = \pi/2$. Rotating angles $\{\omega_i\}$ such that

$$\sum_{i=1}^p \cos \omega_i = 0, \quad \sum_{i=1}^p \sin \omega_i = 0 \quad (16)$$

c) $R_1 = R_2 = R$, α : any other than $\pi/2$ and π . Rotating angles $\{\omega_i\}$ such that

$$\begin{aligned} \sum_{i=1}^p \cos 2\omega_i = 0, \quad \sum_{i=1}^p \sin 2\omega_i = 0 \\ \sum_{i=1}^p \cos \omega_i = 0, \quad \sum_{i=1}^p \sin \omega_i = 0 \end{aligned} \quad (17)$$

Since it is difficult to derive generally optimal rotating angles $\{\omega_i\}$ for any p , we give below the optimal sensor location on the cutter plane and optimal rotating angles concretely for the case of $p = 2$ and $p = 3$ from the practical standpoint. Here, we assume $\omega_1 = 0$ without loss of generality.

(1) The Case of $p = 2$

The optimal α and ω_2 are given by

$$\omega_2 = \pi/2 \text{ or } 3\pi/2 \text{ for } \alpha = \pi \quad (18)$$

$$\omega_2 = \pi \text{ for } \alpha = \pi/2 \quad (19)$$

corresponding to the solution a) and b) respectively.

(2) The Case of $p = 3$

The optimal α , ω_2 , and ω_3 are given by

$$(\omega_2, \omega_3) = (\pi/3, 2\pi/3) \text{ or } (2\pi/3, 4\pi/3) \text{ for } \alpha = \pi \quad (20)$$

$$(\omega_2, \omega_3) = (2\pi/3, 4\pi/3) \text{ for } \alpha = \pi/2 \quad (21)$$

$$(\omega_2, \omega_3) = (2\pi/3, 4\pi/3) \text{ for any } \alpha \text{ other than } \pi/2 \text{ and } \pi \quad (22)$$

corresponding to the solution a), b) and c) respectively. Of course in these equations ω_2, ω_3 are commutable in order.

As a result of above discussion, two sensors should be located at both ends of a diameter of the cutter plane i.e. $\alpha = \pi$ so that the optimal observation policy can be realized in both cases of $p = 2$ and $p = 3$, since it is not determined beforehand to adopt $p = 2$ or $p = 3$ in practical applications. Under the location, the optimal rotating angle is $\omega_2 = \pi/2$ for $p = 2$, and optimal angles ω_2, ω_3 are $\pi/3, 2\pi/3$ or $2\pi/3, 4\pi/3$ in commutable order for $p = 3$. The location

given by $\alpha = \pi$ and the corresponding observation policy are confirmed to be optimal for the detection of an anomalous plane object at a short distance (numerically shown later).

3.2 The Case of Three Sensors on a Diameter

We derive the optimal location of three sensors on a diameter of the cutter plane and the optimal rotating angles of the cutter plane in the sense of minimax criterion. To get the solution, the cutter plane is assumed to be rotated at the same position and plural times of observations are made for an anomalous plane object at a long distance.

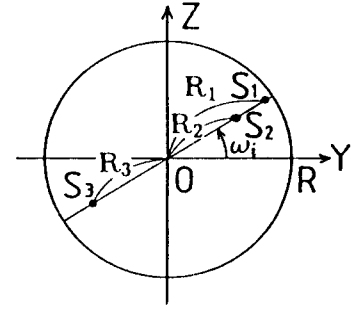


Fig.3. Location of sensors

Let the angle between y axis and the diameter at the i -th rotating position be ω_i ($0 \leq \omega_i < 2\pi$; $1 \leq i \leq p$) (Fig.3). Therefore, the positions of the three sensors S_1, S_2 and S_3 are represented by

$$\mathbf{x}_i^k = \begin{bmatrix} 0 \\ R_k \cos \omega_i \\ R_k \sin \omega_i \end{bmatrix} \quad (k = 1, 2, 3; 1 \leq i \leq p) \quad (23)$$

where R_1, R_2 and R_3 take the values from $-R$ to $+R$ as shown in Fig.3. The corresponding vector denoted by (4) is given by

$$\begin{aligned} \mathbf{b}_i^k(\mathbf{a}_o) = (2/a) \\ \times \begin{bmatrix} a \cos \theta + R_k \cos \theta \sin \theta \cos(\varphi - \omega_i) \\ a \sin \theta \cos \varphi + R_k \sin^2 \theta \cos \varphi \cos(\varphi - \omega_i) - R_k \cos \omega_i \\ a \sin \theta \sin \varphi + R_k \sin^2 \theta \sin \varphi \cos(\varphi - \omega_i) - R_k \sin \omega_i \end{bmatrix} \\ (k = 1, 2, 3; 1 \leq i \leq p) \end{aligned} \quad (24)$$

Therefore when we define the matrix $B(\mathbf{a}_o) \triangleq [b_1^1(\mathbf{a}_o), b_1^2(\mathbf{a}_o), b_1^3(\mathbf{a}_o), \dots, b_p^1(\mathbf{a}_o), b_p^2(\mathbf{a}_o), b_p^3(\mathbf{a}_o)]$ and apply the similar operation used in the preceding section to this matrix, the matrix $Q(\mathbf{a}_o)$ expressed by (8) can be reduced to the equation including $R_1, R_2, R_3, \{\omega_i\} (1 \leq i \leq p)$, p, a , and θ . Since it is difficult to derive a general optimal solution which maximizes the minimum value of $\det[Q(\mathbf{a}_o)]$ on the direction angles θ, φ of an anomalous plane, we consider to obtain the optimal solution for $p = 2$, because it is important in practical applications. Assuming $\omega_1 = 0$ as in the preceding section, the following equation is obtained.

$$\begin{aligned} \det[Q(\mathbf{a}_o)] \\ = 4(4/a^2)^3 (a \cos \theta)^2 (\sin^2 \omega_2) (R_1^2 + R_2^2 + R_3^2) \\ \times (R_1^2 + R_2^2 + R_3^2 - R_1 R_2 - R_2 R_3 - R_3 R_1) \\ \leq 4(4/a^2)^3 (a \cos \theta)^2 (R_1^2 + R_2^2 + R_3^2) \\ \times (R_1^2 + R_2^2 + R_3^2 - R_1 R_2 - R_2 R_3 - R_3 R_1) \end{aligned} \quad (25)$$

Here the equality holds for $\omega_2 = \pi/2$ or $3\pi/2$. On the other hand, the quadratic form $R_1^2 + R_2^2 + R_3^2 - R_1 R_2 - R_2 R_3 - R_3 R_1$

in (25) is found to have the maximum (under the constraint $-R \leq R_1, R_2, R_3 \leq R$) with R_1, R_2 and R_3 given by²⁾

$$(R_1, R_2, R_3) = (R, -R, -R), (-R, R, -R), (-R, -R, R), (-R, R, R), (R, -R, R), (R, R, -R) \quad (26)$$

Since $R_1^2 + R_2^2 + R_3^2$ has the maximum value simultaneously with the R_i 's, the upper bound of $\det[Q(\alpha_o)]$ is maximized with the R_i 's of (26). Therefore, the optimal sensor location is given with two sensors on one end of the diameter and the remaining one on another end of the diameter. Furthermore, the optimal rotating angle is given by $\omega_2 = \pi/2$ (or $3\pi/2$).

This solution for an anomalous plane object at a long distance is numerically shown to be optimal even for a plane object at a short distance.

4. NUMERICAL ANALYSIS AND DISCUSSION

By evaluating the minimum value of $\det[Q(\alpha_o)]$, we show numerically that the solution for an anomalous plane object at a long distance obtained in the preceding chapter is also optimal for a plane object at a short distance. With any anomalous plane object at a short distance in mind, we choose $R=2.5\text{m}$ and $a=4\text{m}$, and the ranges $0 \leq \theta \leq \pi/4$ and $0 \leq \varphi < 2\pi$ are considered. Furthermore, the observations are assumed to be done each 1 meter movement of the tunneling machine.

4.1 The Case of Two Sensors

Details being omitted for want of a space, the optimal sensor location for the plane object at a short distance was also shown numerically to be given by $\alpha = \pi$ on the cutter plane circle and the optimal rotating angle was shown to be given by $\omega_2 = \pi/2$ (or $3\pi/2$) for $p=2$ and $(\omega_2, \omega_3) = (2\pi/3, 4\pi/3)$ (or $(\pi/3, 2\pi/3)$) in commutable order for $p=3$. These agree actually with the solutions (18) and (20) respectively for the plane object at a long distance.

4.2 The Case of Three Sensors(on a Diameter)

The observations are done at the two positions of 4m and 3m's distances from the anomalous plane object. Details being omitted, the maximum value of the criterion function (the minimum of $\det[Q(\alpha_o)]$) is obtained with two sensors located at the one end of the diameter and the remaining one at another end, and furthermore with the rotating angle $\omega_2 = \pi/2$ (or $3\pi/2$). Fig.4 shows, for reference, the contour lines of the criterion function when the sensors S_1 and S_3 are located at both ends of the diameter and the remaining

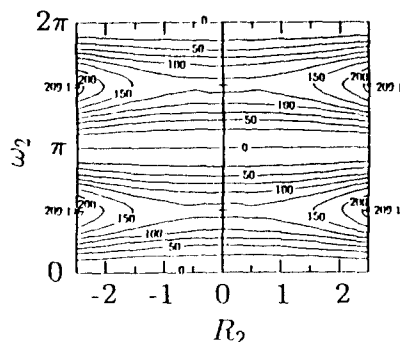


Fig.4. Contour lines of the criterion function

sensor S_2 is moved on the diameter. R_2 and ω_2 denote, respectively, the distance of S_2 from the origin and the relative rotating angle. From Fig.4, it is actually confirmed that the maximum value is obtained with the optimal observation policy described above. That is, the solution agrees with the one of the anomalous plane object at a long distance obtained in section 3.2.

From the results of 4.1 and 4.2 we found numerically that, in the detection system of either two sensors or three sensors on a diameter, the solutions obtained analytically for the plane object at a long distance were also optimal even for the machine's movement in a short distance from the anomalous plane object (Note that not all the solutions are optimal).

5. CONCLUSION

This paper dealt with the optimal sensor location on the cutter plane and the optimal rotating angles of the cutter plane in the shield construction method from the viewpoint of minimax criterion. The discussion was done, from the standpoint of practical applications, for the two cases where two sensors are available on the cutter plane and three sensors are available only on a diameter of the cutter plane under the assumption that the sensors had double functions of transmitting and receiving. The optimality was analytically discussed for an anomalous plane object at a long distance and numerically for one at a short distance from the tunneling machine.

As a result, for the system with two sensors, the optimal location turned out to be given with both ends of a diameter of the cutter plane and the optimal relative rotating angles were $\pi/2$ (or $3\pi/2$) for $p=2$ and $2\pi/3$'s (or $\pi/3$'s) for $p=3$ regardless of the distance up to the anomalous plane object.

On the other hand, for the system with three sensors on the diameter, the optimal location was given with two sensors located at the one end of the diameter and remaining one at the other end, and furthermore the optimal rotating angle was $\pi/2$ (or $3\pi/2$) for $p=2$ (this time too, regardless of the distance).

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