# STABILITY IMPROVEMENT OF INDUCTION MOTOR VECTOR CONTROL SYSTEM WITHOUT SPEED SENSOR

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<u>Abstracts</u> In this paper, two representative schemes for vector control of induction motor without speed sensor are studied. First, the two sensorless systems which are implemented by voltage and current sources are presented with new ideas and interpretations. Then a linear model around an operating point is proposed. Finally, the stability improvement of these systems are studied and evaluated by computing the trajectories of poles and zeros.

**Keywords** Induction motor, Sensorless Vector Control, Stability Analysis, MRAS

#### 1. INTRODUCTION

The vector control of an induction motor without a speed sensor is drawing attention because of its features of simplified vector control and high-performance general-purpose inverter using a constant V/f control scheme[1]. The sensorless control is one of the most up-to-date researching subjects to the engineers in the field of power electronics and control theory. In this paper, two basic methods which are controlled by voltage source or current one are newly interpreted and studied from the viewpoint of stability improvement.

### 2. VECTOR CONTROL

The d-q axis model of an induction motor with the reference axes rotating at arbitrary speed  $\omega$  is Voltage model:

$$\begin{bmatrix} e_{sd} \\ e_{sq} \end{bmatrix} = \begin{bmatrix} r_s + \sigma L_s p - \omega \sigma L_s \\ \omega \sigma L_s & r_s + \sigma L_s p \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} M p / L_r - \omega M / L_r \\ \omega M / L_r & M p / L_r \end{bmatrix} \begin{bmatrix} \psi_{rd} \\ \psi_{rq} \end{bmatrix} \tag{1}$$

Current Model:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = -\sigma_r M \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} \sigma_r + p & \omega_r - \omega \\ \omega - \omega_r & \sigma_r + p \end{bmatrix} \begin{bmatrix} \psi_{rd} \\ \psi_{rq} \end{bmatrix}$$
(2)

where,  $\sigma = 1 - M^2/(L_r L_r)$ ,  $\sigma_r = r_r/L_r$ , p = d/dt

Electromagnetic torque:

$$T_{e} = (P/2) (M/L_{r}) (i_{sq} \psi_{rd} - i_{sd} \psi_{rq})$$
(3)

The concept of vector control is to control the stator

currents according to the information of the rotor flux direction. In order to lock the d-axis of the arbitrary reference frame to the rotor flux vector, the q-component of this flux vector is defined equal to zero:

$$\psi_{ra} = 0 \tag{4}$$

Substitution of Eq.(4) into Eq. (2) gives

$$p \psi_{rd} = -\sigma_r \psi_{rd} + \sigma_r M i_{sd}$$
 (5)

$$\omega = \omega_r + \sigma_r M i_{sa} / \psi_{rd}$$
 (6)

Conversely, if the Eqs. (5) and (6) are satisfied in control, it is expected to get Eq. (4). In this condition, the electromagnetic torque is proportional to the q-axis stator current:

$$T_{e} = (P/2) (M/L_{r}) i_{sa} \psi_{rd}$$
 (7)

## 3. CONTROLLED CURRENT SOURCE TYPE

Figure 1 shows the controlled-current source-fed vector control system without using a speed sensor[2] [3]. If the magnetizing current  $i_{sd}^{*}$  is constant, Eq. (5) becomes

$$\psi_{nd}^* = M i_{sd}^* \tag{8}$$

where, the superscript \* denotes the values in the controller.

Many schemes of the speed estimation are based on the comparison between the outputs of the voltage and current models. This is an application of Model Reference Adaptive System (MRAS). In this case, the current model is considered as the adjustable model and the voltage model and actual induction motor are considered as the reference model.

To calculate the rotor fluxes from the voltage model, it needs integrating process. However, it is difficult to preset the initial value and to prevent the drift of a pure integrator. Therefore, the voltage model is modified by the error between the estimated rotor fluxes obtained by the two models from the viewpoint of observer theory as follows:

$$p \ \hat{\psi}_{rd} = (L_r/M) \{ e_{sd} - (r_s^* + \sigma L_s p) i_{sd} + \omega^* \sigma L_s i_{sa} \} + \omega^* \hat{\psi}_{ra} + (\psi_{rd}^* - \hat{\psi}_{rd}) / T_c$$
 (9)

$$p \ \hat{\psi}_{rq} = (L_r/M) \{ e_{sq} - (r_s^* + \sigma L_s p) i_{sq} - \omega^* \sigma L_s i_{sd} \} - \omega^* \hat{\psi}_{rd} + (\psi_{rd}^* - \hat{\psi}_{rq}) / T_c$$
 (10)

where, the voltages and currents are transformed by using  $\theta^*$ . The fluxes  $\psi_{rd}^*$  and  $\psi_{rq}^*$  represent those based on the current model.  $\psi_{rq}^* = 0$  and  $\psi_{rd}^*$  is given by Eq.(8). Taking the coordinate transformation, the actual torque current in Fig.1 is expressed as

$$\hat{l}_{sq} = \frac{i_{sq} \hat{\psi}_{rd} - i_{sd} \hat{\psi}_{rq}}{\sqrt{\hat{\psi}_{rd}^2 + \hat{\psi}_{rq}^2}}$$
(11)

The rotor speed is estimated as follows:

$$\hat{\omega}_r = (K_i + \frac{K_i}{T_i s}) (i_{sq}^* - i_{sq}^*)$$
(12)

If we use the MRAS theory directly, the rotor speed is estimated as follows:

$$\hat{\omega}_r = (K_i + \frac{K_i}{T_i s}) \hat{\psi}_{rq}$$
(13)

# 4. CONTROLLED VOLTAGE SOURCE TYPE

We consider a full order observer in Eqs.(1) and (2), by setting Eq.(4). In general, the currents and the rotor fluxes are estimated from the observer. On the contrary, in this paper the currents are given as the commanded values  $i_{sd}^*$  and  $i_{sq}^*$  and the voltages  $e_{sd}^*$  and  $e_{sq}^*$  are impressed to the motor. In this case, Eq.(6) is used for the coordinate transformation. As the result, the currents  $i_{sd}^*$  and  $i_{sq}^*$  become the rotor flux oriented values. Also, if the current  $i_{sd}^*$  is constant, Eq.(8) is valid. By neglecting the derivative of the currents, the following equations are obtained from Eq.(1).

$$e_{sd}^* = r_s^* i_{sd}^* - K_{w1} \omega^* \sigma L_s i_{sq}^* + K_{v1} (i_{sd}^* - i_{sd}) + K_{v2} (i_{sq}^* - i_{sq})$$
(14)

$$e_{sq}^{*} = r_{s}^{*} i_{sq}^{*} + \omega^{*} L_{s} i_{sd}^{*} + K_{v3} (i_{sd}^{*} - i_{sd}) + K_{v4} (i_{sq}^{*} - i_{sq})$$
(15)

Figure 2 shows the controlled-voltage source-fed vector control system without using a speed sensor. In conventional system[3],  $K_{w1} = 1$  and  $K_{v2} = K_{v3} = 0$ . In order to improve system stability, we propose the current feedback and a stator flux control ( $K_{w1} = 0$ ). The controller is a full order observer and the unknown speed is estimated from the viewpoint of MRAS. By considering the controller is the adjustable model

and the actual induction motor is the reference model, the speed is estimated as follows:

$$\hat{\omega}_{r} = (K_{c} + \frac{K_{i}}{S}) \{ (i_{sq}^{*} - i_{sq}) \psi_{rd}^{*} - (i_{sd}^{*} - i_{sd}) \psi_{rq}^{*} \}$$

$$= (K_{c} + \frac{K_{i}}{S}) (i_{sq}^{*} - i_{sq})$$
(16)

### 5. STABILITY ANALYSIS

In this paper, the analysis of the system shown in Fig. 2 is presented. To simplify the analysis of the system in Fig.2, the following assumptions are made:

(1) The voltages are controlled ideally, or, more precisely,

$$e_{ss}^* = e_{ss}, e_{sb}^* = e_{sb}, e_{sc}^* = e_{sc}$$
 (17)

- (2) Among motor constants, only the primary resistance  $r_s$  and secondary resistance  $r_s$  vary.
- (3) The reference magnetizing current  $i_{st}^*$  is constant.

To derive the equation for the induction motor in the d-q axis revolving synchronously with  $\theta^*$ , the assumption (1) is used to obtain

$$e_{sd}^* = e_{sd}, e_{sa}^* = e_{sa}$$
 (18)

The induction motor can be described by

$$p i_{sd} = \frac{e_{sd}^*}{\sigma L_s} - \frac{1}{\sigma L_s} (r_s + \frac{M^2}{L_r} \sigma_r) i_{sd} + \omega^* i_{sq} + \frac{\sigma_r M}{\sigma L_s L_r} \psi_{rd} + \frac{\sigma_r \omega_r}{\sigma L_s L_r} \psi_{rq}$$
(19)

$$p i_{sq} = \frac{e_{sq}^{\star}}{\sigma L_{s}} - \frac{1}{\sigma L_{s}} (r_{s} + \frac{M^{2}}{L_{r}} \sigma_{r}) i_{sq} - \omega^{\star} i_{sd}$$
$$- \frac{\sigma_{r} \omega_{r}}{\sigma L_{s} L_{r}} \psi_{rd} + \frac{\sigma_{r} \sigma_{r}}{\sigma L_{s} L_{r}} \psi_{rq}$$
(20)

$$p \psi_{rd} = -\sigma_r \psi_{rd} + \sigma_r M i_{sd} + (\omega^* - \omega_r) \psi_{rd}$$
 (21)

$$p \psi_{ra} = -\sigma_r \psi_{ra} + \sigma_r M i_{sa} - (\omega^* - \omega_r) \psi_{rd}$$
 (22)

$$p \, \omega_r = \frac{P^2 M}{4 J L_r} \left( i_{sq} \, \psi_{rd} - i_{sd} \, \psi_{rq} \right) - \frac{R_w}{J} \, \omega_r - \frac{P}{2 J} \, T_L \tag{23}$$

where, J is the moment of inertia; and  $T_L$  is the load torque. Frequency controller:

$$p \theta^* = \omega^* = \hat{\omega}_r + \frac{r_r^* i_{sq}^*}{L_r i_{sd}^*}$$
(24)

PI speed controller:

$$p e_s = (K_s / T_s) (\omega_r^* - \hat{\omega}_r)$$
 (25)

$$i_{sq}^* = e_s + K_s \left( \omega_r^* - \hat{\omega}_r \right) \tag{26}$$

PI speed estimator:

$$p e_c = K_i (i_{sa}^* - i_{sa}) \tag{27}$$

$$\hat{\omega}_r = e_c + K_c \left( i_{sa}^* - i_{sa} \right) \tag{28}$$

The steady-state solution can be obtained by setting p=0. Stability of the system under consideration is analyzed using a model which is obtained by linearizing the original nonlinear equation around the operating point. Equations (19) to (23) give

$$p \Delta x_{s} = A_{s} \Delta x_{s} + B_{s} \Delta u_{s} + B_{L} \Delta T_{L}$$

$$where, \Delta x_{s} = \begin{bmatrix} \Delta i_{sd}, \Delta i_{sq}, \Delta \psi_{rd}, \Delta \psi_{rq}, \Delta \omega_{r} \end{bmatrix}^{T}$$

$$\Delta u_{s} = \begin{bmatrix} \Delta e_{sd}, \Delta e_{sq}, \Delta \omega^{*} \end{bmatrix}^{T}$$

$$A_{s} = \begin{bmatrix} a_{3} & \omega^{*} & a_{1} \sigma_{r} M \\ -\omega^{*} & a_{3} & -a_{1} M \omega_{r} \\ \sigma_{r} M & 0 & -\sigma_{r} & * \\ 0 & \sigma_{r} M & -(\omega^{*} - \omega_{r}) \\ -a_{2} \psi_{rq} & a_{2} \psi_{rd} & a_{2} i_{sq} \end{bmatrix}$$

$$a_{1} M \omega_{r} & a_{1} M \psi_{rq} \\ a_{1} M \sigma_{r} & -a_{1} M \psi_{rd} \\ * \omega^{*} - \omega_{r} & -\psi_{rq} \\ -\sigma_{r} & \psi_{rd} \\ -a_{2} i_{sd} & -R_{w} / J \end{bmatrix}$$

$$a_{1} = \frac{1}{\sigma L_{s} L_{r}}, a_{2} = \frac{P^{2} M}{4 J L_{r}}, a_{3} = -a_{1} (r_{s} L_{r} + \sigma_{r} M^{2})$$

$$B_{s} = \begin{bmatrix} 1/(\sigma L_{s}) & 0 & i_{sq} & 0 \\ 0 & 1/(\sigma L_{s}) & -i_{sd} & 0 \\ 0 & 0 & -\psi_{rq} & 0 \\ 0 & 0 & -\psi_{rq} & 0 \\ 0 & 0 & -\psi_{rd} & 0 \\ 0 & 0 & 0 & -P/(2J) \end{bmatrix}$$

The output equation is given by

$$\Delta y = C_s \Delta x_s \tag{3Q}$$

where, 
$$\Delta y = \begin{bmatrix} i_{sd}, i_{sq} \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The linearized state equation of the controller is given by

$$p \Delta z = A_z \Delta z + A_y \Delta y + B_r \Delta r \tag{31}$$

where, 
$$\Delta z = \left[ \Delta e_s, \Delta e_c \right]^T$$
,  $\Delta r = \left[ \Delta \omega_r^* \right]$ 

The input vector  $\Delta u_s$  is expressed in terms of  $\Delta y$ ,  $\Delta z$  and  $\Delta r$  as follows:

$$\Delta u_s = F_v \Delta y + F_r \Delta z + F_r \Delta r \tag{32}$$

Equations (29) to (32) give the linearized model of the whole system such that

$$p \Delta x = A \Delta x + B \Delta r + B_T \Delta T_L \tag{33}$$

where, 
$$\Delta \mathbf{x} = \begin{bmatrix} \Delta \mathbf{x}_{s}^{T}, \Delta z^{T} \end{bmatrix}^{T}$$

$$A = \begin{bmatrix} A_{s} + B_{s} F_{s} C_{s} & B_{s} F_{s} \\ A_{s} C_{s} & A_{s} \end{bmatrix}$$

$$B = \begin{bmatrix} B_{s} F_{r} \\ B_{r} \end{bmatrix}, B_{T} = \begin{bmatrix} B_{L} \\ 0 \end{bmatrix}$$

# 6. RESULTS OF THE ANALYSIS

The tested induction motor has the following ratings and constants:

2.2 kW; 
$$p = 4$$
;  $r_s = 0.662 \Omega$ ;  $r_r = 0.645 \Omega$ ;  $L_s = L_s = 0.086 H$ ;  $M = 0.082 H$ ;  $J = 0.0617 kg - m^2$ 

Trajectories of poles and zeros of speed transfer function are computed by using the linear model.

In the case of the current source type in which  $T_c = \infty$ , it is learned that pole  $p_1$  and zero are canceled completely on imaginary axis if the reference resistance  $r_s^*$  coincides with actual resistance  $r_s$  as shown in Fig.3. However, if  $r_s$  is smaller than  $r_s^*$ , the system becomes unstable. Effects of time constant  $T_c$  on the trajectories of poles and zeros are shown in Fig.4. Pole  $p_1$  and zero on the imaginary axis tend to be stabilized more effectively as time constant  $T_c$  decreases.

Figure 5 shows the root trajectories for the conventional voltage source type in which  $K_{w1} = 1$ . It is learned from this figure that the system becomes unstable as the rotor speed increases. On the other hand, the proposed method stabilizes the system as shown in Fig.6.

By using these models, the influences for the change of the stator and rotor resistances can be computed. Stability is improved by choosing smaller reference rotor resistance.

#### 7. CONCLUSION

Two representative approaches for sensorless vector control of the induction motor and the stability improvement of these systems have been described and discussed. Conventional sensorless methods are interpreted from the viewpoints of the observer and MRAS theories.

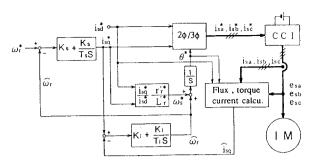


Fig.1 Controlled current source (CCS) sensorless system.

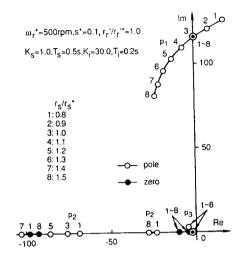


Fig.3 Root loci of CCS system  $(T_c = \infty)$ 

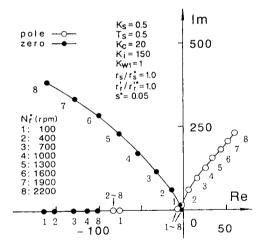


Fig. 5 Root loci of CVS system (Conventional method).

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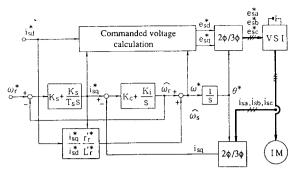


Fig.2 Controlled voltage source (CVS) sensorless system.

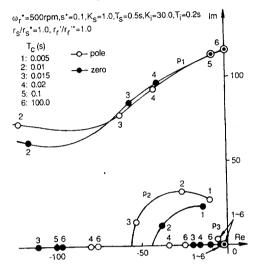


Fig.4 Root loci of CCS system (T<sub>c</sub>: variable)

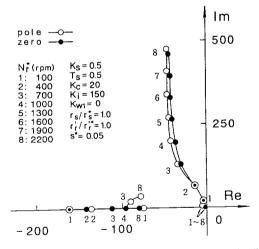


Fig.6 Root loci of CVS system (Proposed method).

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