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The Problem of Stability and Uniform Sampling in the Application of Neural Network to Discrete-Time Dynamic Systems

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Abstract Neural network has found wide applications in the system identification, modeling, and realization based on its function approximation capability. The system governed by nonlinear dynamics is hard to be identified by the neural network because there exist following difficulties. First, the training samples obtained by the state trajectory are apt to be nonuniform over the region of interest. Second, the system may become unstable while attempting to obtain the samples. This paper deals with these problems in discrete-time system, and suggest effective solutions which provide stability and uniform sampling by the virtue of robust control theory and heuristic algorithms.

Keywords Neural Network, Discrete-Time System, Stability

1. Introduction

Since the artificial neural network was modeled form the biological neurons, it has been widely used due to its properties of approximation, generalization, *etc.* There have been various researches [1][3] to analyze the function approximation capability of neural network. But it is still remained unsolved to compute the network structure as a function of the complexity of the problem.

Besides the classical problem of neural network mentioned above, additional problems arise in the applications for dynamic systems. In the early days of neural network, the applications were primarily in the area of pattern recognition and hence pertains to static systems. Since dynamics constitutes an essential part of all practical systems, it was tried by many authors [4] to use neural networks as components in dynamical systems. The various designs of control architectures are studied and extensive simulations are carried out to show the models proposed are particularly effective for the identification and control of nonlinear systems. However, much of the work is of a heuristic nature. Narendra *et al.* [5] presented the first attempt to relate the experimental studies to theoretical developments and tried to propose a general methodology by which control methods based on neural networks can be made more rigorous.

In most of the papers which seek for the general methodology in using neural networks for dynamic systems, the efforts are mainly contributed to the stability analysis. These analyses are based on the assumptions endowed to systems, in the other words, if neural network is applied to a design methodology, the function in the system should be examined whether it satisfies the prescribed assumptions or not. But, it does not seem always possible to judge the satisfaction of such assumptions proposed by Narendra. Hence, it is needed to relax the stringent assumptions to the simplified ones such as the boundedness of the function, *etc.*

Narendra [6] also suggested the successive identification and control strategy to guarantee the stability and the uniform sampling in the process of identifying a nonlinear function in the dynamic equation. However, they induced the strong results only for the first order system and employed ambiguous assumptions to be trapped into mass of problems. To solve these problems in the continuous-time system, we adopted the supervisory control algorithm of Wang [7] and used this algorithm to guarantee the stability during the training time of neural network [10]. We also modified the supervisory algorithm and applied it to robot dynamics to provide the uniform sampling as well as the stability [12]. In this paper, we present a reasonable solution for the problem of uniform sampling and stability which occurs in certain class of discrete-time systems.

2. Establishment of the base region

The class of system in which we are interested is described by

$$\mathbf{x}(k+1) = f(\mathbf{x}(k)) + \mathbf{u} \tag{1}$$

where $\mathbf{x}, \mathbf{u} \in \Re^n$ and f is an unknown mapping. The merit of the equation (1) is that f can be directly canceled if we know the information of f in some area.

The theorem introduced by Corless and Manela [8] provide a sufficient condition for g.u.a.s., so enable bounded random input to be used for identification. Iterative perturbation algorithm suggested by Eom [11] can be applied to identify the base region in this case. However, without the knowledge of satisfying the condition a heuristic approach named linearized model approach with pseudo-inverse is preferred. The target system must show the stable fashion before our control action, *i.e.*, the system stays in the basin of a stable equilibrium point, a limit cycle, or a chaotic attractor. Especially in this paper, the following is assumed.

Assumption 1 The system described by (1) initially stays in one of its stable equilibrium points with zero control input.

For the first order system, the following useful theorem is proved to judge the satisfaction of Assumption 1.

Theorem 1 (First Order Case) If the function f in (1) has as many stable equilibrium points as critical points on the interval I, and satisfies the Schwartzian derivative

$$S_f(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2 < 0 \tag{2}$$

then there are no limit cycles.

Proof The proof is given in [9].

□

For higher order systems, there exists no such criterion like Schwartzian derivative. So even the known systems are hard to be confirmed to satisfy Assumption 1.

If a system described by (1) satisfies Assumption 1, it can be linearized at the stable equilibrium point:

$$\bar{\mathbf{x}}(k+1) = A\bar{\mathbf{x}}(k) + B\mathbf{u} \tag{3}$$

where A has eigenvalues inside the unit circle, and B is the matrix whose elements are all 1.

Assumption 1 is needed to start identification process at the basin of the stable equilibrium point. Furthermore to get the information of any state around that point by the perturbation of u, equation (3) should be locally controllable. For local controllability, the following is assumed.

Assumption 2 Linearized system (3) is controllable.

And the following theorem assures the local controllability of the original nonlinear equation (1).

Theorem 2 If equation (3) satisfies Assumption 2, the original nonlinear equation (1) is locally controllable.

Proof The proof is given by Ňarendra.

□

By theorems and assumptions it is clear that there exists stable and controllable region around a stable equilibrium point. But, finding the range where stability and controllability hold is very difficult. So we propose a heuristic algorithm to approximate the matrix A with the samples around the equilibrium point.

First, gather the finite number of $(\bar{\mathbf{x}}_i(k), \bar{\mathbf{x}}_i(k+1))$ pairs by adjusting incremental input after the state converges sufficiently close to the stable equilibrium point. Then, an estimation of A_{crt} is calculated based on the late N sequence of $(\bar{\mathbf{x}}_i(k), \bar{\mathbf{x}}_i(k+1))$ pairs:

$$X(k) \equiv [\bar{\mathbf{x}}_N(k), \cdots, \bar{\mathbf{x}}_1(k)] \tag{4}$$

$$X(k+1) \equiv [\bar{\mathbf{x}}_N(k+1), \cdots, \bar{\mathbf{x}}_1(k+1)] \tag{5}$$

$$X(k+1) = A_{crt}X(k) \tag{6}$$

$$A_{crt} = X(k+1)X(k)^+ \tag{7}$$

$$= X(k+1)X(k)^T(X(k)X(k)^T)^{-1} \tag{8}$$

Now, A_{crt} is used to check the degree of stability. The exploration continues toward all possible directions around the equilibrium point as long as

$$\bar{\sigma}(A_{crt}) < \sigma_{max} < 1 \tag{9}$$

where $\bar{\sigma}(A_{crt})$ is the maximum eigenvalue of A_{crt} and σ_{max} is determined by the designer's choice. If the condition (9) is successively conformed, the whole explored region satisfies $\|\bar{\mathbf{x}}(k+1)\| < \|\bar{\mathbf{x}}(k)\|$, in the other words, the state converges to zero.

Simulation :(*1st order case*) We will show the validity of linearized model approach with pseudo-inverse for the 1st order system described by

$$x(k+1) = \cos[x(k)]\sqrt[3]{4x(k)} \tag{10}$$

The system initially stays at $x = -0.92$, one of its stable equilibrium points. We choose $N = 1$ and $\sigma_{max} = 0.9$. $N = 1$ means we use the usual inverse instead of the pseudo-inverse. Fig.1 shows the exploration around the stable equilibrium point. The identification is performed first to the positive direction and next to the negative direction. If state trajectory comes back to the stable equilibrium point, the same procedure is repeated with a larger magnitude of input. Doing this, A_{crt} is calculated to judge the relative stability. Fig.2 shows the growth of A_{crt} as the system becomes unstable. Identification is terminated when it exceeds σ_{max}. The sign of eigenvalues are matched to the direction of identification.

3. Successive Identification and Control Algorithm

Robust control theory and several heuristic algorithms is verified helpful for the establishment of the base region of successive identification and control algorithm. However, they do not guarantee the uniform sampling so additional processing is needed to get uniformly distributed samples form randomly distributed data. The wide range of the base region sacrifices more time. In successive identification and control algorithm, the region available for the next step of identification depends on the range of the pre-identified region. Too narrow base region may be harmful for successive identification and control algorithm. It should be properly determined in comparison with the desired region in the whole state space.

Successive identification and control algorithm can be applied to any kind of function f in (1) provided that the knowledge of f is sufficiently accurate over some region. The establishment of such an region in a stable fashion is explained in the previous sections.

The each step of successive identification and control algorithm is backed up by the following theorems and assumptions.

Assumption 3 There exist a function N_f which approximates f such that for $\forall \mathbf{x}$ which satisfies $\|\mathbf{x} - \mathbf{x}^*\| < D$,

$$\|f(\mathbf{x}) - N_f(\mathbf{x})\| < \delta. \tag{11}$$

The simple feedforward neural network(FNN) is used as N_f because it has the following property.

Theorem 3 FNN described by

$$\sum_{j=1}^{N} c_{j,k}\sigma(\mathbf{y}_j \cdot \mathbf{x} + \theta_j) \tag{12}$$

where $N, c_{j,k}, \theta_j \in \Re^n$ and $\mathbf{y}_j, \mathbf{x} \in \Re^n$, is an universal approximator in $C[D]$ space.

Proof It is proved by many authors, see for example [2]

□

It is demanded to make the local stable equilibrium point at the center of the known region in our successive identification and control algorithm. Therefore the following theorem is developed.

Theorem 4 If a system described by (1) satisfies Assumption 3, it is possible to make local stable equilibrium point at any state $\mathbf{x}$ in $\{\mathbf{x} | \|\mathbf{x} - \mathbf{x}^*\| < D\}$.

Proof Adjusting control input $\mathbf{u} = -N_f + A\mathbf{x} + \mathbf{x}^*$ where the eigenvalues of A are inside the unit circle, $\mathbf{x}^*$ becomes the local stable equilibrium point.

□

To guarantee the extension of the identified region in the next iteration, the following theorem is needed.

Theorem 5 There exists a scalar $L > 0$ such that for $\forall \mathbf{x}$ which satisfies $\|\mathbf{x} - \mathbf{x}^*\| < L$

$$\|f(\mathbf{x}) - N_f(\mathbf{x})\| < D. \tag{13}$$

Proof It is induced from the basic property of the continuous function. Because $\|f(\mathbf{x}) - N_f(\mathbf{x})\| < \delta$ in the region, $\|\mathbf{x} - \mathbf{x}^*\| < D$, it is straightforward that there exists some region which satisfies $\delta < \|f(\mathbf{x}) - N_f(\mathbf{x})\| < D$ outside of it.

□

Note that $\|f(\mathbf{x}) - N_f(\mathbf{x})\| < D$ is equivalent to $\|\mathbf{x}(k+1) - \mathbf{x}^*(k+1)\| < D$ when input, $\mathbf{u}$ is zero.

The procedure of our successive identification and control algorithm is as follows.

- **STEP 1** Make a local stable equilibrium point, $\mathbf{x}^*$, at the center of the known region, $\|\mathbf{x} - \mathbf{x}^*\| < D$, by adjusting $\mathbf{u} = -N_f + \mathbf{x}^* + \mathbf{v}$.
(*guaranteed by* Theorem 4)
- **STEP 2** Adjust an input $\mathbf{v} = D + k \cdot \delta$ where δ is the vector which has the random direction and the fixed magnitude, and the initial value of k is 1. Next adjust a zero input $\mathbf{v} = 0$. If $\mathbf{x}(k+1)$ is inside the region, $\|\mathbf{x} - \mathbf{x}^*\| < D$, add $f(\mathbf{x}(k))$ to a training set of neural network, increase k by one, and return to STEP 1. If not, proceed to STEP 3 with $D = D + k \cdot \|\delta\|$.
(*guaranteed by* Theorem 3)
- **STEP 3** Using the training set obtained in STEP 2, train a neural network until it approximates f in the region, $D_{old} < \|\mathbf{x} - \mathbf{x}^*\| < D_{new}$, above the accuracy of $\|f(\mathbf{x}) - N_f(\mathbf{x})\| < \delta$ and then go back STEP 1.
(*guaranteed by* Theorem 5)

As we can see clearly in STEP 2, the information of $f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}^* + k \cdot \delta$ can be directly obtained, so unfirmly distributed sample, equally space by δ, are available for training. Cause $\mathbf{x}$ comes back to the local stable equilibrium point at least in 2-step when the zero input is adjusted, The identification speed is very fast.

Simulation :(*1st order case*) For the simulation of successive identification and control algorithm we choose the same dynamic equation as used for the linearized model approach with pseudo-inverse in the previous section.

$$x(k+1) = \cos[x(k)]\sqrt[3]{4x(k)} \quad (14)$$

It is assumed that the region $[-0.98 - 0.86]$ is successfully identified by a neural network $A_c rt$ as a measure of relative stability. Then, successive identification and control algorithm enables us to identify the region of interest, $[-0.98, 5]$, with three neural networks. Fig.3 shows the result of the identification.

4. Conclusion and Further Study

This paper is devoted to guarantee the stability in the application of neural network to dynamics system identification. For the certain class of discrete-time systems, successive identification and control algorithm is developed for uniform sampling and stability after base region is identified. The base region is trained by neural network with iterative perturbation algorithm when the system equation satisfies a certain condition borrowed from the robust control theory. Otherwise the base region is carefully explored using estimated jacobian, A_{crt}, as a criterion for relative stability.

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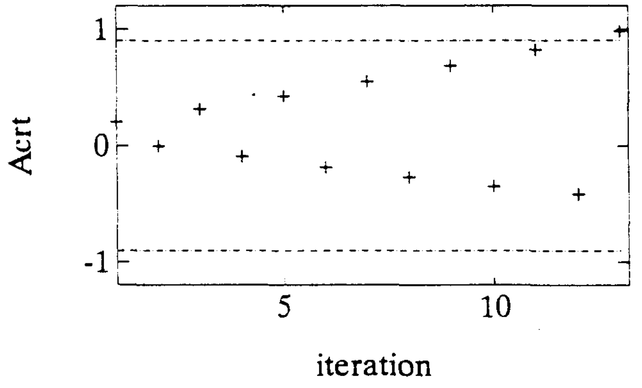

Fig.1 Exploration around the stable equilibrium point while calculating A_{crt}.

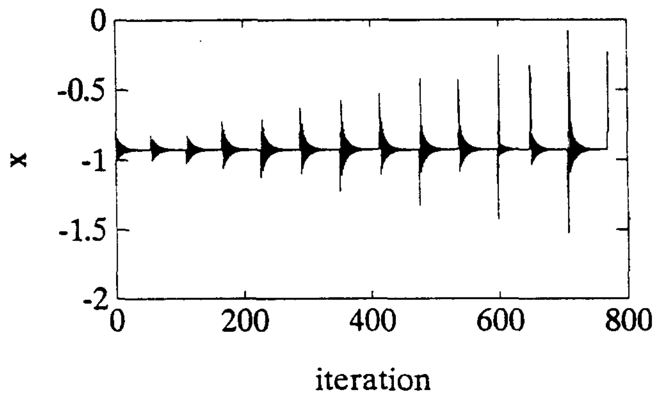

Fig.2 The growth of eigenvalues of A_{crt} as system becomes unstable.(The si gn is chosen to display the direction of identification, - -: δ_{max})

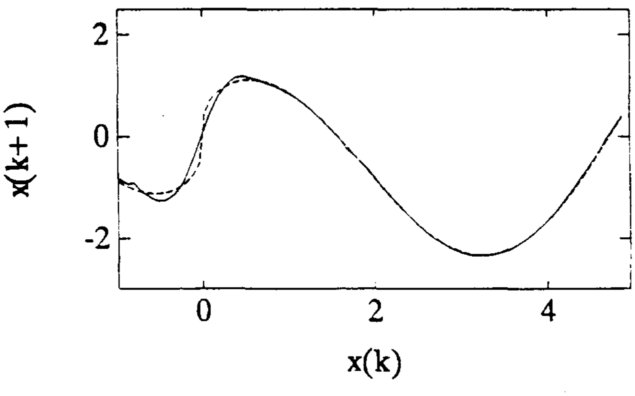

Fig.3 Approximation of equation (14) by three neural netw orks using SCA.