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Design of Unknown–Input PI Observer and Exact LTR

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Abstract In this paper, an unknown–input proportional integral (PI) observer is presented and its applicability to the design of exact loop transfer recovery (Exact LTR) is shown.

First, a sufficient condition for the PI observer to estimate the states of systems without knowledge of unknown input is derived. A simple existence condition of the observer is given. Under the conditions, the Exact LTR with specified observer's poles is achieved by the unknown–input PI observer.

Keywords Unknown–input observer, PI observer, Exact LTR, Invariant zeros

1. Introduction

The problem of estimating the state of system requires the assumption that all system inputs are known or measurable. In practice, however, there exist many situations in which some system inputs are completely unknown. For the system with unmeasurable inputs, many papers by conventional observer have been recently published [1]–[3].

On the other hand, a proportional integral (PI) observer was proposed by Wojciechowski [4] with the aim of desensitizing the observer by asymptotic regulation of observer error in the face of small parameter variations and step disturbances, and it was shown by Kawaji and Sawada [5] that the PI observer has the equivalent relation to disturbance observer. Recently, the PI observer is applied to the design of loop transfer recovery (LTR) [6]–[8] and the *time-recovery* is proposed by Niemann et al. [7]. However, there is no mention on PI observer with unknown inputs.

In this paper, we discuss the design method of unknown–input PI observer and show its applicability to the exact loop transfer recovery (Exact LTR) problem. The sufficient condition for the unknown–input PI observer is given. And a simple existence condition of the observer is presented. Under the conditions, the Exact LTR is perfectly achieved by the unknown–input PI observer without any necessary condition except for left invertible and minimum phase system.

Notation

I_n	n–square matrix with 1's on the diagonal and 0's elsewhere
$I_{n\times m}$	$n\times m$ dimension matrix with 1's on the diagonal of $\min(n,m)$ and 0's elsewhere
0_n	n–square matrix with 0's
$0_{n\times m}$	$n\times m$ dimension matrix with 0's
A^g	Generalized inverse matrix of A

2. Design of Unknown–Input PI Observer

Consider a linear time–invariant system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the unknown input, and $y(t) \in \Re^p$ is the output. A, B and C are known constant matrices of appropriate dimensions, and rank $B = m$ and rank $C = p$. It is assumed that $p \geq m$ and the pair (C, A) is observable. Further, assume without loss of generality the matrix C has the form

$$C = \begin{bmatrix} C_m & 0_{p\times(n-p)} \end{bmatrix}$$

where C_m is nonsingular.

Consider a related system represented by

$$\begin{cases} \dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}\omega(t) \\ \hat{x}(t) = \hat{C}z(t) + \hat{D}y(t) \\ \dot{\omega}(t) = y(t) - C\hat{x}(t) \end{cases} \tag{2}$$

where $\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{H}$, and $\hat{J}$ are unknown matrices of appropriate dimensions.

Definition 1 : The system (2) is said to be a proportional integral observer (PI observer) for the system (1) if and only if

$$\lim_{t\to\infty} e(t) = 0, \quad \forall\, x(0_-),\ z(0_-),\ u(\cdot) \tag{3}$$

$$\lim_{t\to\infty} \omega(t) = 0, \quad \forall\, \omega(0_-) \tag{4}$$

where $e(t) = \hat{x}(t) - x(t)$ represents the observer error.

We can have the following relations between the system and the observer.

Theorem 1 : *The system (2) is an unknown–input PI observer for the system (1) if*

$$\mathrm{Re}\ \lambda_i \begin{bmatrix} \hat{A} & \hat{H} \\ -C\hat{C} & 0 \end{bmatrix} < 0, \quad i = 1, \ldots, n+p \tag{5}$$

and if there exists a matrix $U \in \Re^{n\times n}$ such that

$$\hat{A}U + \hat{B}C = UA \tag{6}$$

$$\hat{J} = UB = 0 \tag{7}$$

$$\hat{C}U + \hat{D}C = I_n \tag{8}$$

where Re $\lambda_i[\cdot]$ *denotes the real part of the i–th eigenvalue.*

Proof : Define the estimation error by

$$\xi(t) = z(t) - Ux(t)$$

From (1) and (2), the dynamics of this error obeys

$$\dot{\xi}(t) = \hat{A}\xi(t) + (\hat{A}U + \hat{B}C - UA)x(t) + (\hat{J} - UB)u(t) + \hat{H}\omega(t) \tag{9}$$

And eq. (2) leads to

$$\hat{x}(t) = \hat{C}\xi(t) + (\hat{C}U + \hat{D}C)x(t) \tag{10}$$

$$\dot{\omega}(t) = C(x(t) - \hat{x}(t)) \tag{11}$$

By substituting (6) - (8) into (9) - (11), we have

$$\dot{\xi}(t) = \hat{A}\xi(t) + \hat{H}\omega(t)$$
$$\hat{x}(t) = \hat{C}\xi(t) + x(t)$$
$$\dot{\omega}(t) = -C\hat{C}\xi(t)$$

or

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{H} \\ -C\hat{C} & 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \omega(t) \end{bmatrix}$$
$$\hat{x}(t) = \hat{C}\xi(t) + x(t)$$

Thus, under the condition (5), $\omega(t) \to 0$ and $e(t) \to 0$ ($t \to \infty$). ■

Remark 1 : For designing an unknown-input PI observer, the matrix U is selected such that (7) is satisfied. This is directly related with designing the Exact LTR and shown in Section 3.

In the following, we let $\hat{C} = I_n$ for simplicity. Then, from (8)

$$U = I_n - \hat{D}C \tag{12}$$

By substitution of (12) into (6), we have

$$\hat{A} = UA - KC \tag{13}$$
$$\hat{B} = \hat{A}\hat{D} + K \tag{14}$$

where

$$K = \hat{B} - \hat{A}\hat{D}$$

Also the condition (5) is rewritten as

$$\mathrm{Re}\,\lambda_i \begin{bmatrix} UA - KC & \hat{H} \\ -C & 0 \end{bmatrix} < 0, \quad i = 1, \ldots, n+p \tag{15}$$

The remained problem is how to find the matrices K and $\hat{H}$ with the designed matrix U. This is the standard problem for designing the PI observer, and was solved systematically by Kawaji and Kim [9]. The procedure of solving the problem is summarized in Appendix.

Lemma 1 : For the unknown-input PI observer (2), there exist the matrices K and $\hat{H}$ if

(i) pair (C, UA) is observable (16)

(ii) $\mathrm{rank} \begin{bmatrix} UA & I_{n\times p} \\ C & 0_p \end{bmatrix} = n+p$ (17)

□

Next, we will consider the existence condition of unknown-input PI observer. Substituting (12) into (7), we have

$$\hat{D}CB = B \tag{18}$$

In order for the matrix $\hat{D}$ satisfying (18) to exist

$$\mathrm{rank}\, CB = \mathrm{rank}\, B = m \tag{19}$$

must hold. The condition (19) requires that $p \geq m$.

The general solution of (18) can be written as

$$\hat{D} = B(CB)^g + G\{I_p - CB(CB)^g\} \tag{20}$$

where G is an arbitrary matrix.

By substituting (20) into (12), we can get

$$U = (I_n - GC)\{I_n - B(CB)^g C\}$$

From the above equation, there exists a matrix G which makes $(I_n - GC)$ nonsingular, and then the rank $U = n-m$.

Since rank $B = m$, there exists the left-inverse of matrix B, *i.e.*,

$$B^g B = I_m$$

Under the condition of rank $U = n-m$, we have Ker $U \cap$ Ker $B^g = \{0\}$, *i.e.*,

$$\mathrm{rank} \begin{bmatrix} U \\ B^g \end{bmatrix} = n$$

Then, we have the relation

$$\begin{bmatrix} U & 0 \\ B^g & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I_n & 0 \\ -B^g(sI_n - A) & I_m \end{bmatrix}$$
$$= \begin{bmatrix} U(sI_n - A) & 0 \\ 0 & I_m \\ C & 0 \end{bmatrix}$$
$$= \begin{bmatrix} I_n & 0 & -s\hat{D} \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} \begin{bmatrix} sI_n - UA & 0 \\ 0 & I_m \\ C & 0 \end{bmatrix}$$

It follows that

$$\mathrm{rank} \begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} = m + \mathrm{rank} \begin{bmatrix} sI_n - UA \\ C \end{bmatrix}$$

Consequently, for $\forall s \in \mathbf{C}$

$$\mathrm{rank} \begin{bmatrix} sI - A & B \\ C & 0 \end{bmatrix} = n+m \tag{21}$$

which means that the invariant zeros of the system (1) must be stable.

Let the matrices be defined as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad UA = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix}$$
$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \text{and} \quad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

Then, from (17)

$$\mathrm{rank} \begin{bmatrix} UA & I_{n\times p} \\ C & 0_p \end{bmatrix} = \mathrm{rank} \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} & I_p \\ \widetilde{A}_{21} & \widetilde{A}_{22} & 0 \\ C_m & 0 & 0 \end{bmatrix} = n+p$$

The matrix $\widetilde{A}_{22}$ is rewritten as

$$\widetilde{A}_{22} = A_{22} - LA_{12}$$

where

$$L = \{B_2(C_m B_2)^g + G_2 + C_m B_2 (C_m B_1)^g\} C_m$$

From the assumption that the pair (C, A) is observable, the pair (A_{12}, A_{22}) is observable. So, the matrix $\widetilde{A}_{22}$ is of full rank by proper choice of K_2.

From the above statements, we summary the following theorem.

Theorem 2 : *For the system (1), the unknown-input PI observer (2) exists if*

i) $\mathrm{rank}\, CB = \mathrm{rank}\, B = m$

ii) $\mathrm{rank}\begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} = n + m, \quad \forall s \in \mathbf{C}$

hold. □

Remark 2 : In Theorem 2, the condition ii) is equivalent to following condition

i) $\mathrm{rank}\begin{bmatrix} sI_n - UA \\ C \end{bmatrix} = n, \ \forall s \in \mathbf{C}$

ii) pair (C, UA) is observable

If $I_p - C\hat{D} = 0$, then the pair (C, UA) is unobservable. The case is obtained for example $p = m$, and CB is nonsingular [3].

3. Exact LTR by Unknown–Input PI Observer

In this section, we will show that the Exact LTR can be achieved by the unknown-input PI observer. Let the system (1) be controlled by an observer based controller.

$$u(t) = -F\hat{x}(t)$$

where F is the state feedback gain and $\hat{x}(t)$ the estimated state vector. The PI observer based control system is illustrated by Fig. 1.

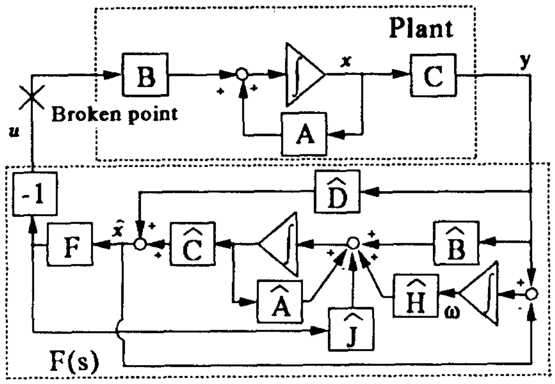

Fig. 1. PI observer based feedback control system

Assuming that the broken point is located in plant input, the loop transfer function is given by

$$L_{obs}(s) = F(s)G(s) \tag{22}$$

where

$$G(s) = C\Phi(s)B$$
$$: \quad \Phi(s) = (sI - A)^{-1}$$

is the transfer function of plant, and

$$F(s) = -F\{\phi(s) + \hat{J}K + s^{-1}\hat{H}C\}^{-1}(\phi(s)\hat{D} + \hat{B} + s^{-1}\hat{H})$$
$$: \quad \phi(s) = (sI - \hat{A})$$

is the transfer function of the PI observer based controller. The corresponding loop transfer function with state feedback law is given by

$$L_{sf}(s) = -F\Phi(s)B \tag{23}$$

The difference between the two loop transfer functions $L_{obs}(s)$ and $L_{sf}(s)$ is defined as the loop recovery error at the input loop broken point

$$\begin{aligned} E_I(s) &= L_{obs}(s) - L_{sf}(s) \\ &= M_I(s)\{I + M_I(s)\}^{-1}\{I + F\Phi(s)B\} \end{aligned} \tag{24}$$

where

$$M_I(s) = F(sI - \hat{A} + s^{-1}\hat{H}C)^{-1}\hat{J} \tag{25}$$

So, it is obvious that the Exact LTR is achieved if $M_I(s) = 0$. Eq. (25), called the recovery matrix for the Exact LTR, is rewritten as

$$M_I(s) = F(sI - UA + KC + s^{-1}\hat{H}C)^{-1}UB \tag{26}$$

It follows that if $UB = 0$, the recovery matrix $M_I(s)$ equal zero exactly.

Theorem 3 : *The Exact LTR for the system (1) is achieved, if unknown-input observer is constructed.* □

Remark 3 : The condition of Exact LTR in Theorem 3 is equivalent to

$$\langle \hat{A}, Im\hat{J} \rangle \subset \mathrm{Ker}\, \hat{C}$$

where, $\langle \hat{A}, Im\hat{J} \rangle$ denotes the controllable subspace for the system $(\hat{A}, \hat{J})$. This result was shown in Niemann et al. [7]. But, method for designing the observer was not shown.

4. Numerical example

We consider the following system which was used as an example in Kawaji and Kim [9]

$$\left.\begin{aligned} A &= \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ -1.6 & -2.3 & -1.2 \end{bmatrix}, \quad B = \begin{bmatrix} 2.7 \\ 0.5 \\ -1.2 \end{bmatrix} \\ C &= \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \end{aligned}\right\}$$

The state feedback gain matrix F is designed by the conventional LQR method as

$$F = \begin{bmatrix} 0.9678 & 0.2692 & 0.0409 \end{bmatrix}$$

In this system, the conditions of Theorem 2 are satisfied, so that there exists the matrix G such that PI observer is stable. Thus, the unknown-input PI observer can be designed.

The matrix $\hat{D}$ is calculated from (20)

$$\hat{D} = \begin{bmatrix} 0.821 & 0.967 \\ 0.603 & -2.259 \\ -0.218 & -1.225 \end{bmatrix}$$

with

$$G = \begin{bmatrix} 1.000 & 1.000 \\ 2.000 & -2.000 \\ 1.000 & -1.000 \end{bmatrix}$$

Then, the matrix U is obtained from (12) as

$$U = \begin{bmatrix} 0.179 & -0.967 & 0.000 \\ -0.603 & 3.259 & 0.000 \\ 0.218 & 1.225 & 1.000 \end{bmatrix}$$

Let the eigenvalues of PI observer be $\{-1, -2, -3, -4, -5\}$. Then, the PI observer gains K and $\hat{H}$ are obtained from Appendix A as

$$K = \begin{bmatrix} 6.011 & -0.544 \\ -0.090 & 8.411 \\ -1.768 & 186.685 \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} 7.872 & 178.529 \\ 0.296 & -591.894 \\ 0.000 & 0.000 \end{bmatrix}$$

The other unknown parameters are calculated from (13) and (14).

$$\hat{A} = \begin{bmatrix} -6.011 & 0.723 & -0.967 \\ 0.090 & -9.014 & 3.259 \\ 0.168 & -188.767 & 0.025 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} 1.723 & -6.805 \\ -6.165 & 24.865 \\ -115.547 & 613.169 \end{bmatrix}$$

The result shown in Fig. 2 is the loop transfer recovery by unknown-input PI observer, which is equal to that of state feedback control system perfectly.

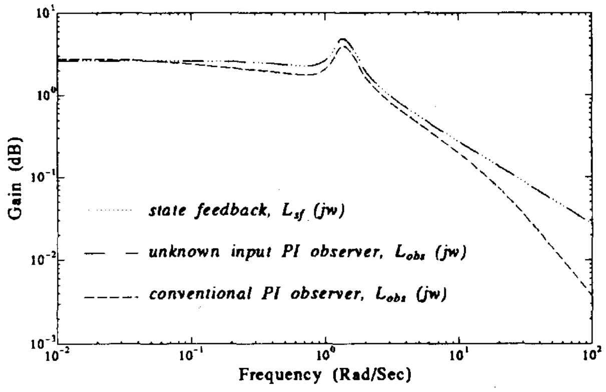

Fig. 2. Gain plots for loop transfer functions

5. Conclusions

In this paper, we have presented a simple design method of unknown-input PI observer. The sufficient condition for the unknown-input PI observer is derived. And simple existence condition of the observer is given and can be checked by rank conditions. Also, it was shown that the Exact LTR by unknown-input PI observer is achieved without any condition except for left invertible and minimum phase system.

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Appendix A

< Design algorithm of PI observer >

Step 1 : Construct augmented matrices as

$$A_e = \begin{bmatrix} UA & I_{n\times p} \\ I_{p\times n} & 0_p \end{bmatrix}, \quad C_e = \begin{bmatrix} C & 0_p \end{bmatrix}$$

Step 2 : Design a matrix L_e by conventional pole assignment, LQG, or etc.

$$\mathrm{Re}\,\lambda_i \left[A_e - L_e C_e \right] < 0, \quad i = 1, \ldots, n+p$$

where

$$L_e = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

Step 3 : Calculate the matrices K and $\hat{H}$

$$K = L_1$$
$$\hat{H} = I_{n\times p}(L_2 - I_{p\times n} C^g)$$