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Likelihood Search Method with Variable Division Search

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Abstracts Various methods and techniques have been proposed for solving optimization problems; the methods have been applied to various practical problems. However the methods have demerits. The demerits which should be covered are, for example, falling into local minima, or, a slow convergence speed to optimal points. In this paper, Likelihood Search Method (L.S.M.) is proposed for searching for a global optimum systematically and effectively in a single framework, which is not a combination of different methods. The L.S.M. is a sort of a random search method (R.S.M.) and thus can get out of local minima. However exploitation of gradient information makes the L.S.M. superior in convergence speed to the commonly used R.S.M..

Keywords Optimization, Random search method, Gradient information, Search range, Search direction

1. INTRODUCTION

In this paper, Likelihood Search Method (L.S.M.)[1] is proposed for searching for a global optimum.

Simulations are carried out to search for the variables which minimize a complicated multi variable nonlinear function with local minima. Simulation results show that the L.S.M. is superior to both the commonly used R.S.M. and Gradient Method.

2. LIKELIHOOD SEARCH METHOD (L.S.M.)

The L.S.M., a sort of random search method, utilizes gradient information; the range and the direction of the search are determined by the gradient information so that the searching efficiency becomes high; i.e. the better solutions can be found quickly by fine searching.

A likelihood of finding good solutions is estimated by the gradient information; a sharp gradient means that it is very likely that a better solution exists in the vicinity of the current solution, while given a gentle gradient we do not expect that a better solution is near the current solution. Therefore, in case of high likelihood, the search is done within a short range in the opposite direction of the gradient vector. On the other hand, in case of low likelihood, the search does not much depend on the gradient; the range is wide, and the direction includes even those which make the objective function worse.

The gradient information D is defined as follows,

$$D = \sqrt{\sum_{m \in M} \left\{ \frac{dE}{d\lambda_m} \right\}^2}, \quad (1)$$

$\frac{dE}{d\lambda_m}$: first order derivatives of objective function E with respect to the variable $\lambda_m \ (m = 1, 2, 3, \cdots, M)$.

The searching range and direction are determined by D, and search density and distribution functions of the L.S.M. are shown in **Fig.1**.

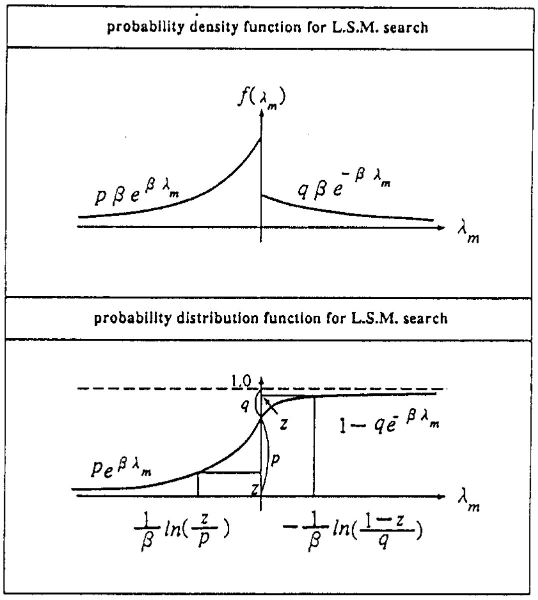

Fig.1 Probability density and distribution functions for variable search

The procedure which searches for the variable λ_m by the L.S.M. is shown as follows,

$$0 \le z \le p \implies x_m = \frac{1}{\beta}\ln\left(\frac{z}{p}\right), \quad (2)$$

$$p < z \le 1.0 \implies x_m = -\frac{1}{\beta}\ln\left(\frac{1-z}{q}\right), \quad (3)$$

$$\lambda_m \leftarrow \lambda_m + x_m, \quad (4)$$

x_m : change in variable λ_m.

Symbols p and q correspond to the search probability in negative and positive direction respectively, and are determined as follows,

$$q = 0.5e^{-\phi\beta} \quad if \ \frac{dE}{d\lambda_m} > 0, \quad (5)$$

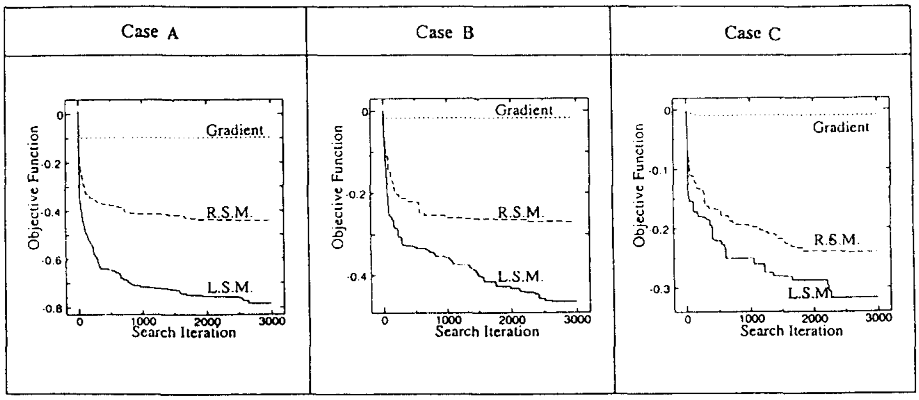

Fig.4 Searching curves of 9-variables case

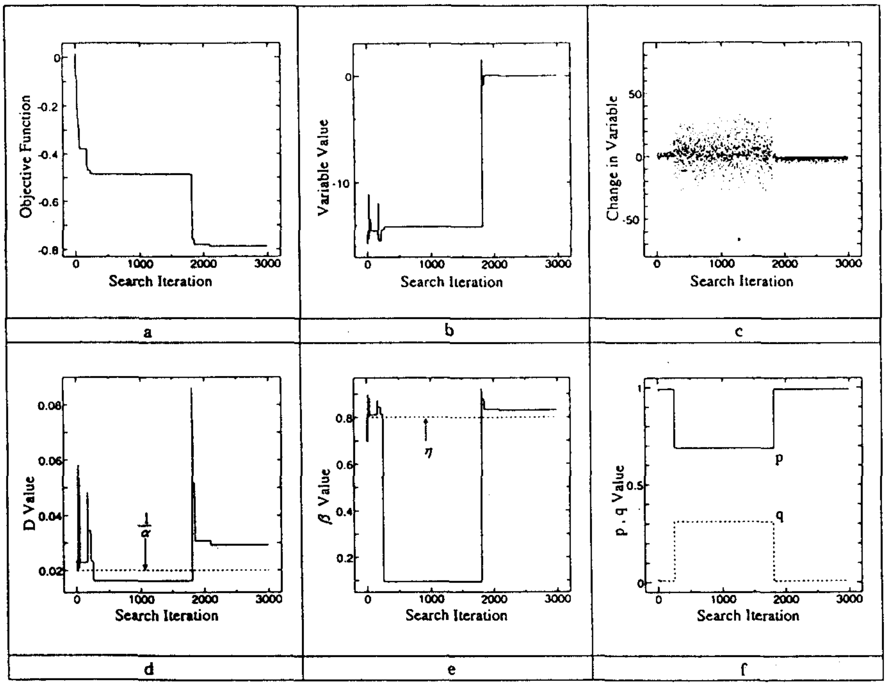

Fig.5 Results of searching by L.S.M. in 9-variables case

Table 1 Initial variable values in 9-variables case

		Case A	Case B	Case C
Initial Values	λ_1	8.697775e+00	-5.697775e+01	9.697775e+01
	λ_2	1.037172e+01	6.037172e+01	-1.037172e+02
	λ_3	-9.705435e+00	-6.705435e+01	-9.705435e+01
	λ_4	1.195868e+01	5.495868e+01	9.495868e+01
	λ_5	1.223029e+01	-5.223029e+01	-1.223029e+02
	λ_6	-9.976104e+00	-6.146104e+01	1.146104e+02
	λ_7	-1.579974e+01	-4.579974e+01	8.579974e+01
	λ_8	-1.350261e+01	-5.350261e+01	-9.350261e+01
	λ_9	1.634297e+01	-6.634297e+01	-1.634297e+02

Each of the searching curves shown in **Fig.4** is the average of 10 simulation results with the same initial variable value but different random sequences for implementation of stochastic search.

The variables found by the Gradient Method fall into local minima. The results of the R.S.M. is better than those of the Gradient Method since the variables get out of local minima repeatedly using the R.S.M.. However even by the R.S.M., the improvement of the objective function stops in the halfway of variable searching.

On the other hand, the results of the L.S.M. are better than both those of the Gradient Method and the R.S.M.. The searching process by the L.S.M. is shown in **Fig.5**. **Fig.5 a, b, c, d, e, f** correspond to the objective function, the variable value, change in variable, D value, β value, and p, q value respectively.

When the gradient information D is large, β is large, the probability of searching in the direction of decreasing the objective function is almost 1 ($p = 1$ or $q = 1$), and the search range is short. Accordingly, the process is in the phase of local minima searching.

Conversely, when D is small, β is small, and the probability of searching in the direction of decreasing the objective function is low, and also, the search range is wide. Accordingly, the process is in the phase of local minima eluding.

As mentioned above, the L.S.M. has, roughly speak-