Generalized Predictive Control Based on the Parametrization of Two-Degree-of-Freedom Control Systems

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Abstracts We propose a new design method for a generalized predictive control (GPC) system based on the parametrization of two-degree-of freedom control systems. The objective is to design the GPC system which guarantees the stability of the control system for a perturbed plant. The design procedure of our proposed method consists of three steps. First, we design a basic controller for a nominal plant using the LQG method and parametrize a whole control system. Next, we identify the deviation between the perturbed plant and the nominal one using a closed-loop identification method and design a free parameter of parametrization to stabilize the closed-loop system. Finally, we design a feedforward controller so as to incorporate GPC technique into our controller structure. A numerical example is presented to show the effectiveness of our proposed method.

Keywords Generalized Predictive Control, Two-Degree-of-Freedom Control System, Coprime Factorization Approach, Youla Parametrization, Stability

1. INTRODUCTION

Recently, a great deal of attention has been paid to predictive control as one design method of digital control system. Many papers have been reported in the field of the chemical process [1,2]. However, stability of the closed-loop system has not been guaranteed, since the design parameters of predictive control are not directly linked to the stability. Although recovery methods for the closed-loop stability have been proposed for the last few years, those methods are not feasible to guarantee the stability except for some special cases [3].

On the other hand, a predictive control based on a class of all stabilizing controllers, which is called Youla parametrization, has been proposed by Naganawa *et al* [6]. In this method, the stability of the closed-loop system for an actual plant (perturbed plant) can be guaranteed, but it may have a steady-state error for the stepwise change of setpoint. Ito *et al* [5] have proposed a design method of GPC system based on the parametrization. In this method, the stability of the closed-loop system is guaranteed for a nominal plant. However, the stability is no longer guaranteed in the case of a perturbed plant.

In this paper, we propose a new design method of GPC system based on the parametrization of two-degree-of-freedom control systems [7]. Firstly, we design a feedback controller for obtaining the closed-loop stability. For a perturbed plant, we consider a class of all plants stabilizable by a nominal feedback controller. The class can be represented by interchanging the role of the controller and the plant for Youla parametrization [8]. We identify the parameter, which is represented in term of a class of all plants, using a closed-loop identification method and stabilize a closed-loop system [4,8]. Then, we design a feedforward controller so as to incoporate GPC method into our controller structure. We define a feedforward controller as a time-varying FIR (finite impulse response) filter and design it by minimizing a cost function.

2. CONVENTIONAL GPC

Consider a CARIMA (Controlled Auto-Regressive and Integrated Moving-Average) model,

$$\tilde{A}(z)y(z)=\tilde{B}(z)u(z)+\zeta(z)/\Gamma(z) \tag{1}$$

$$\Gamma(z)=1-z^{-1} \tag{2}$$

where $y(z)$, $u(z)$ and $\zeta(z)$ are a plant output, control input and disturbance process, respectively. $\tilde{A}(z)$ and $\tilde{B}(z)$ are polynomials in the unit delay operator, z^{-1}. Consider also a Diophantine equation

$$1=E_j(z)\tilde{A}(z)\Gamma(z)+z^{-j}F_j(z) \quad ; \ (1\le j\le N) \tag{3}$$

where $E_j(z)$ and $F_j(z)$ are polynomials defined by (1). From (1) and (3), the j-step ahead output prediction $\hat{y}_{t+j}$ at time t are calculated. Conventional GPC tries to minimize the cost function with respect to $\tilde{u}$

$$J_1=(\hat{y}-r)^T(\hat{y}-r)+\lambda\tilde{u}^T\tilde{u} \tag{4}$$

where the superscript T denotes transposition of vector, λ is weighting factor and $\hat{y}$, r and $\tilde{u}$ are output prediction, reference signal and future incremental control vector, respectively.

$$\hat{y}=[\hat{y}_{t+1},\ \hat{y}_{t+2},\ \cdots,\ \hat{y}_{t+N}]^T \tag{5}$$

$$r=[r_{t+1},\ r_{t+2},\ \cdots,\ r_{t+N}]^T \tag{6}$$

$$\tilde{u}=[\Gamma(z)u_t,\ \Gamma(z)u_{t+1},\ \cdots,\ \Gamma(z)u_{t+N-1}]^T \tag{7}$$

3. TWO-DEGREE-OF-FREEDOM CONTROL SYSTEMS

3.1 Class of all stabilizing controllers

Consider a two-degree-of-freedom control system as shown in Fig. 1 with a nominal plant $P(z)$, a feedback controller $C(z)$ and a feedforward controller $H(z)$. $n_p(z)$ and $d_p(z)$ are numerator and denominator coprime factor of the nominal plant $P(z)$, respectively, i. e.,

$$P(z)=\frac{n_p(z)}{d_p(z)} \quad ; \; n_p(z),\; d_p(z)\in RH_\infty \tag{8}$$

where RH_∞ denotes the class of all stable proper transfer functions. This control system is internally stable if and only if $H(z)$ belongs to RH_∞ and $C(z)$ belongs to a class of all stabilizing controllers for the plant $P(z)$.

The transfer function from $r(z)$ to $y_0(z)$ is given by

$$y_0(z)=n_p(z)H(z)r(z) \tag{9}$$

where $y_0(z)$ is an internal signal of the controller. On the other hand, the transfer function from $r(z)$ to $y(z)$ is given by

$$y(z)=n_p(z)H(z)r(z) \tag{10}$$

From (9) and (10), the tracking performance for $r(z)$ depends on the only feedforward controller $H(z)$ and the feedback controller $C(z)$ does not play any role for the characteristics. If there are a plant perturbation or a external disturbance, $y(z)$ no longer equals to $y_0(z)$. In this case, the feedback controller $C(z)$ works to reduce the difference $e(z)$ between $y(z)$ and $y_0(z)$.

Suppose a coprime factorization of the feedback controller $C(z)$ is given by

$$C(z)=\frac{n_c(z)}{d_c(z)} \quad ; \; n_c(z),\; d_c(z)\in RH_\infty \tag{11}$$

Then, a class of all stabilizing controllers for a nominal plant $P(z)$ is given by

$$C_Q(z)=\frac{n_c(z)+d_p(z)Q(z)}{d_c(z)-n_p(z)Q(z)} \tag{12}$$

where $Q(z)$ is a free parameter in RH_∞. Now, denote a state-space description of the plant $P(z)$ as

$$x_{t+1}=Ax_t+Bu_t \tag{13}$$

$$y_t=Cx_t \tag{14}$$

where A, B and C are $n\times n$, $n\times 1$ and $1\times n$ constant matrix, respectively. We assume that the pairs (A, B) and (C, A) are controllable and observable, respectively. Then, the coprime factors in (8) and (11), which satisfy the Bezout identity in (15), are defined as

$$n_p(z)n_c(z)+d_p(z)d_c(z)=1 \tag{15}$$

$$n_p(z)=C(zI-A+BK)^{-1}B \tag{16}$$

$$d_p(z)=1-K(zI-A+BK)^{-1}B \tag{17}$$

$$n_c(z)=K(zI-A+FC)^{-1}F \tag{18}$$

$$d_c(z)=1+K(zI-A+FC)^{-1}B \tag{19}$$

where K and F are a state feedback gain matrix and a prediction type Kalman filter gain matrix, respectively.

3.2 Class of all plants

Now, we consider the nominal plant $P(z)$ and the nominal controller $C(z)$ with coprime factorization given by (8) and (11) satisfying a Bezout identity (15). Then, a class of all plants stabilizable by the nominal controller $C(z)$ is characterized by an arbitrary free parameter $R(z)$ in RH_∞ as follows.

$$P_R(z)=\frac{N_p(z)}{D_p(z)} \quad ; \; N_p(z),\; D_p(z)\in RH_\infty \tag{20}$$

$$N_p(z)=n_p(z)+d_c(z)R(z) \tag{21}$$

$$D_p(z)=d_p(z)-n_c(z)R(z) \tag{22}$$

This parametrization can be obtained by interchanging the role of the nominal controller $C(z)$ and the plant $P(z)$ in familiar theory for the class of all stabilizing controller $C_Q(z)$ in (12). Using this parametrization, we can discuss an unified approach for an additive perturbation and a multiple perturbation. For an additive perturbation $P(z)+\Delta(z)$, we have the following the relation between $\Delta(z)$ and $R(z)$.

$$R(z)=\frac{d_p(z)\Delta(z)d_p(z)}{1+d_p(z)\Delta(z)n_c(z)} \tag{23}$$

The closed-loop system with the plant $P_R(z)$ and the controller $C_Q(z)$ can be constructed as shown in Fig. 2. This control system is stable if and only if $H(z)$ belongs to RH_∞ and $Q(z)$ stabilizes $R(z)$. The transfer function from $r(z)$ to $y(z)$ is given by

$$y(z)=\frac{N_p(z)}{1+R(z)Q(z)}H(z)r(z). \tag{24}$$

It should be noted that $y(z)$ in (24) equals to $y(z)$ in (10) if $R(z)$=0.

4. PROPOSED GPC

4.1 Identification of R(z)

Let us consider a plant

$$y(z)=\frac{N_p(z)}{D_p(z)}(u(z)+d(z))+\frac{S(z)}{D_p(z)}\xi(z) \tag{25}$$

where the transfer function $N_p(z)$ and $D_p(z)$ are given in (21) and (22). A structure of this plant is shown in Fig. 3. The signal $\sigma(z)$ and $\rho(z)$ are given by

$$\sigma(z)=s(z)+d_c(z)d(z) \tag{26}$$

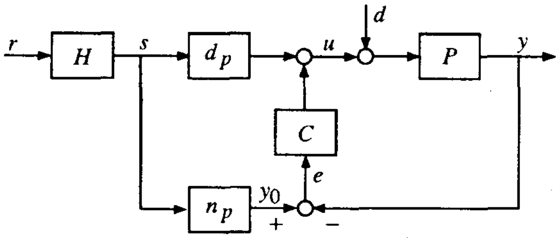

Fig. 1 Two-degree-of-freedom control system

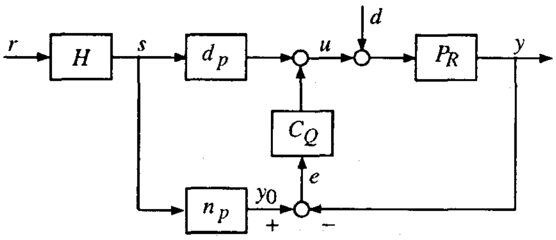

Fig. 2 Control system with $C_Q(z)$ and $P_R(z)$

$$\rho(z) = d_p(z)y(z) - n_p(z)(u(z) + d(z)) \tag{27}$$

where $s(z)$ is an output of a feedforward controller $H(z)$. If an identification signal $d(z)$ is injected, then $\sigma(z)$ is dependent on the external signal and $\rho(z)$ can be obtained from measurable variables, because the $\rho(z)$ is expressed by the output $y(z)$ of the plant $P_R(z)$, a control input $u(z)$ and the identification signal $d(z)$. The feedback signal from $\rho(z)$ to $\sigma(z)$ is canceled out in the loop; the gain of the transfer function from $\rho(z)$ to $\sigma(z)$ is equal to zero. Therefore, this closed-loop identification problem can be restated in term of estimating $R(z)$ and $S(z)$. We suppose the following ARMAX model with the input $\sigma(z)$, the output $\rho(z)$ and the noise $\xi(z)$.

$$\rho(z) = R(z)\sigma(z) + S(z)\xi(z) = \frac{\overline{B}(z)}{\overline{A}(z)}\sigma(z) + \frac{\overline{C}(z)}{\overline{A}(z)}\xi(z) \tag{28}$$

$$\overline{A}(z) = 1 + \overline{a}_1 z^{-1} + \cdots + \overline{a}_n z^{-n} \tag{29}$$

$$\overline{B}(z) = \overline{b}_1 z^{-1} + \cdots + \overline{b}_n z^{-n} \tag{30}$$

$$\overline{C}(z) = 1 + \overline{c}_1 z^{-1} + \cdots + \overline{c}_n z^{-n} \tag{31}$$

Using a recursive least square algorithm, we can identify the parameter $R(z)$ in open-loop manner.

The parameter $Q(z)$ which stabilizes the closed-loop system can be designed by a pole placement method, an optimal control method and so on.

4.2 Design method of feedforward controller H(z)

The feedforward controller $H(z)$ is tuned so as to minimize this cost function

$$J_2 = (\hat{y} - r)^T(\hat{y} - r) + \lambda u^T u \tag{32}$$

Suppose the feedforward controller $H(z)$ given by a time-varying FIR filter

$$H(t,z) = h_{0,t} + h_{1,t}z^{-1} + \cdots + h_{N-1,t}z^{-(N-1)} \tag{33}$$

where coefficient number of this FIR filter equals to a length of a predictive interval. We obtain the output prediction $\hat{y}$ by using Kalman filter as follows.

The output $y(z)$ for the nominal plant $P(z)$ is presented in (4). The numerator coprime factor $n_p(z)$ of the plant $P(z)$ is a system stabilized via a state feedback with the gain K (see (8) and (16)). The state-space description of $n_p(z)$ is given by

$$\hat{x}_{t+1} = (A - BK)\hat{x}_t + Bs_t \tag{34}$$

$$\hat{y}_t = C\hat{x}_t \tag{35}$$

where $\hat{x}$ is state estimator of the Kalman filter. We can obtain the following equations by using (34).

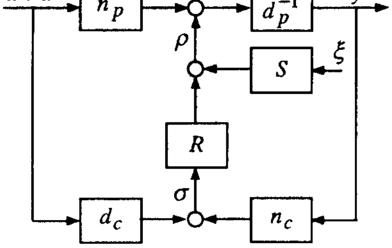

Fig. 3 Block diagram showing the unknown plant

$$\left.\begin{aligned} \hat{x}_{t+1} &= A_K\hat{x}_t + Bs_t \\ \hat{x}_{t+2} &= A_K\hat{x}_{t+1} + Bs_{t+1} \\ &\vdots \\ \hat{x}_{t+N} &= A_K\hat{x}_{t+N-1} + Bs_{t+N-1} \end{aligned}\right\} \tag{36}$$

$$A_K = A - BK \tag{37}$$

Then, the output predictions $\hat{y}_{t+j}(1 \le j \le N)$ at time t are as follows.

$$\left.\begin{aligned} \hat{y}_{t+1} &= C\hat{x}_{t+1} = CA_K\hat{x}_t + CBs_t \\ \hat{y}_{t+2} &= C\hat{x}_{t+2} = CA_K^2\hat{x}_t + CA_KBs_t + CBs_{t+1} \\ &\vdots \\ \hat{y}_{t+N} &= C\hat{x}_{t+N} = CA_K^N\hat{x}_t + CA_K^{N-1}Bs_t + \cdots + CBs_{t+N-1} \end{aligned}\right\} \tag{38}$$

The matrix representation in (38) is given by

$$\begin{bmatrix} \hat{y}_{t+1} \\ \vdots \\ \hat{y}_{t+N} \end{bmatrix} = \underbrace{\begin{bmatrix} CA_K \\ \vdots \\ CA_K^N \end{bmatrix}}_{E_1}\hat{x}_t + \underbrace{\begin{bmatrix} CB & & 0 \\ \vdots & \ddots & \\ CA_K^{N-1}B & \cdots & CB \end{bmatrix}}_{L_1}\begin{bmatrix} s_t \\ \vdots \\ s_{t+N-1} \end{bmatrix} \tag{39}$$

where the sequence $s_k(t \le k \le t+N-1)$ is obtained by convolving the coefficients of FIR filter $H(z)$ with the sequences of $r(z)$.

$$\begin{bmatrix} s_t \\ \vdots \\ s_{t+N-1} \end{bmatrix} = \begin{bmatrix} h_{0,t} & & 0 \\ \vdots & \ddots & \\ h_{N-1,t} & \cdots & h_{0,t} \end{bmatrix}\begin{bmatrix} r_t \\ \vdots \\ r_{t+N-1} \end{bmatrix} = \underbrace{\begin{bmatrix} r_t & & 0 \\ \vdots & \ddots & \\ r_{t+N-1} & \cdots & r_t \end{bmatrix}}_{R_s}\underbrace{\begin{bmatrix} h_{0,t} \\ \vdots \\ h_{N-1,t} \end{bmatrix}}_{h_t} \tag{40}$$

Then, we can express (39) as

$$\hat{y} = E_1\hat{x}_t + L_1R_sh_t \tag{41}$$

In a similar way, we can write $u(z)$ as

$$u = E_2\hat{x}_t + L_2R_sh_t \tag{42}$$

where

$$E_2 = \begin{bmatrix} -KA_K & \cdots & -KA_K^N \end{bmatrix}^T \tag{43}$$

$$L_2 = \begin{bmatrix} -KB & & 0 \\ \vdots & \ddots & \\ -KA_K^{N-1}B & \cdots & -KB \end{bmatrix} \tag{44}$$

From (42) and (43), the optimal coefficients of $H(z)$ minimizing J_2 at time t is given by

$$h_t = \left\{R_s^T\left(L_1^TL_1 + \lambda L_2^TL_2\right)R_s\right\}^{-1} \times \left(R_s^TL_1^Tr - R_s^TL_1^TE_1\hat{x}_k - \lambda R_s^TL_2^TE_2\hat{x}_k\right). \tag{45}$$

4.3 Steady-state error

In the predictive control, the object of the control design is to track on the output $y(z)$ to a step signal $r(z)$. However, a steady-state error does not become zero by using a controller $H(z)$ which is tuned by (45). Therefore we redesign the controller $H(z)$ to achieve zero steady-state error.

If a D.C. (steady-state or low frequency) gain of a transfer function from $r(z)$ to $y(z)$ given by (24) equals to x, we divide x into coefficients of the controller $H(z)$. Then D.C. gain of the transfer function equals to 1 and steady-state error becomes zero. Note that we loose the optimality in (32) instead of the tracking performance.

5. SIMULATION

Consider the following nominal plant $P(z)$ and a perturbed plant $P(z)+\Delta(z)$.

$$P(z)=\frac{0.0121z^{-1}+0.0117z^{-2}}{1-1.8953z^{-1}+0.9048z^{-2}}$$

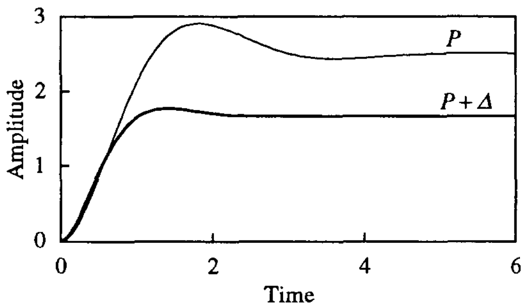

Fig. 4 Step responses

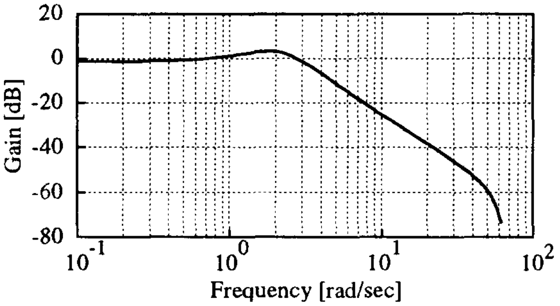

Fig. 5 Gain characteristic of $\Delta(z)$

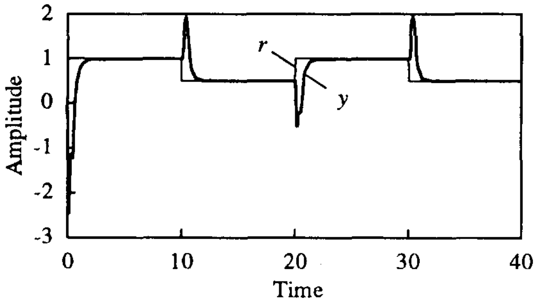

Fig. 6 Proposed method

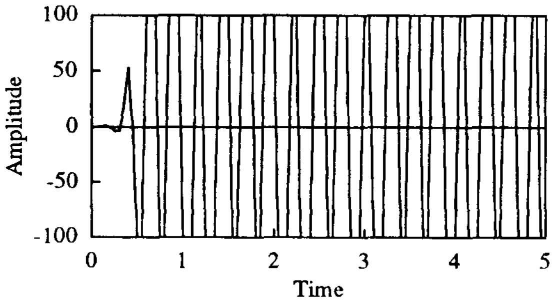

Fig. 7 Conventional method

$$P(z)+\Delta(z)=\frac{0.0175z^{-1}+0.0164z^{-2}}{1-1.7984z^{-1}+0.8187z^{-2}}$$

Fig. 4 and Fig. 5 show the step responses of this plant and gain characteristic of $\Delta(z)$. First, we designed an LQG controller to minimize the following criterion function for nominal plant $P(z)$ represented by (13) and (14)

$$J_a=\sum\left(y^T Q_w y+u^T u\right)$$

where $Q_w=10$. Next, we defined coprime factors of (16)-(19) and identified a parameter $R(z)$ using the technique in subsection 4.1. Then, we carried out the GPC, where $\lambda=0.01$ in (4) and (32). Fig. 6 and Fig. 7 show the result of the simulation. In our proposed method, the output $y(z)$ tracks to the reference $r(z)$, though there is some overshoot. However, in the conventional method, the output $y(z)$ has diverged, since the stability of the closed-loop system is not guaranteed for a perturbed plant.

6. CONCLUSION

In this paper, we proposed a new design method of the GPC which is based on the parametrization of two-degree-of-freedom control systems. In our proposed method, the stability of the closed-loop system is guaranteed for a perturbed plant.

ACKNOWLEDGMENT

The authors would like to thank Prof. H. Wada, Prof. T. Yokobori and Prof. T. Ishihara, of Tohoku university, for valuable discussions and helpful comments.

REFERENCES

[1] R.R. Bitmead, M. Gevers and V. Wertz, "Adaptive Optimal Control : The Thinking Man's GPC," *Prentice Hall*, 1990

[2] D.W. Clarke, C. Mohtadi and P.S. Tuffs, "Generalized Predictive Control -Part I. Basic Algorithm," *Automatica*, 23-2, pp. 137-148, 1987

[3] H. Demircioğlu and D.W. Clarke, "CGPC with Guaranteed Stability Properties," *IEE Proceedings-D*, pp. 371-380, 1992

[4] F. Hansen, G. Franklin and R. Koust, "Closed-Loop Identification via the Fractional Representation : Experiment Design," *Proc. ACC*, pp. 1422-1427, 1993

[5] S. Ito and J.B. Moore, "Adaptive-Q Predictive Control," *Proc. ASCC*, pp. 509-512, 1994

[6] A. Naganawa, G. Obinata and H. Inooka, "A Design Method of Model Predictive Control System Using Coprime Factorization Approach," *Proc. ASCC*, pp. 197-200, 1994

[7] T. Sugie and T. Yoshikawa, "General Solution of Robust Tracking Problem in Two-Degree-of-Freedom Control Systems," *IEEE Trans. AC*, 31-6, pp. 552-554, 1986

[8] T.T. Tay, J.B. Moore and R. Horowitz, "Indirect Adaptive Techniques for Fixed Controller Performance Enhancement," *Int. J. Control*, 50-5 pp. 1941-1959, 1989