독립신호 교차로에서의 교통안전을 위한 서비스수준 결정방법의 개발

하 태 준

DEVELOPMENT OF SAFETY-BASED LEVEL-OF-SERVICE CRITERIA FOR ISOLATED SIGNALIZED INTERSECTIONS

by

Dr. Tae-Jun Ha

Department of Civil and Environmental Engineering University of Wisconsin - Madison

ABSTRACT

The Highway Capacity Manual specifies procedures for evaluating intersection performance in terms of delay per vehicle. What is lacking in the current methodology is a comparable quantitative procedure for assessing the safety-based level of service provided to motorists. The objective of the research described herein was to develop a computational procedure for evaluating the safety-based level of service of signalized intersections based on the relative hazard of alternative intersection designs and signal timing plans. Conflict opportunity models were developed for those crossing, diverging, and stopping maneuvers which are associated with left-turn and rear-end accidents. Safety-based level-of-service criteria were then developed based on the distribution of conflict opportunities computed from the developed models. A case study evaluation of the level of service analysis methodology revealed that the developed safety-based criteria were not as sensitive to changes in prevailing traffic, roadway, and signal timing conditions as the traditional delay-based measure. However, the methodology did permit a quantitative assessment of the trade-off between delay reduction and safety improvement.

The Highway Capacity Manual (HCM) specifies procedures for evaluating intersection performance in terms of a wide variety of prevailing conditions such as traffic composition, intersection geometry, traffic volumes, and signal timing (1). At the present time, however, performance is only measured in terms of delay per vehicle. This is a parameter which is widely accepted as a meaningful and useful indicator of the efficiency with which an intersection is serving traffic needs.

What is lacking in the current methodology is a comparable quantitative procedure for assessing the safety-based level of service provided to motorists. For example, it is well-known that the change from permissive to protected left-turn phasing can reduce left-turn accident frequency. However, the HCM only permits a quantitative assessment of the impact of this alternative phasing arrangement on vehicle delay. It is left to the engineer or planner to subjectively judge the level of safety benefits, and to evaluate the trade-off between the efficiency and safety consequences of the alternative phasing plans.

Numerous examples of other geometric design and signal timing improvements could also be given.

At present, the principal methods available to the practitioner for evaluating the relative safety at signalized intersections are: a) the application of engineering judgement, b) accident analyses, and c) traffic conflicts analysis. Reliance on engineering judgement has obvious limitations, especially when placed in the context of the elaborate HCM procedures for calculating delay. Accident analyses generally require some type of beforeafter comparison, either for the case study intersection or for a large set of similar

intersections. In either situation, there are problems associated with compensating for regression-to-the-mean phenomena ($\underline{2}$), as well as obtaining an adequate sample size. Research has also pointed to potential bias caused by the way in which exposure to accidents is measured ($\underline{3}$, $\underline{4}$). Because of the problems associated with traditional accident analyses, some have promoted the use of the traffic conflicts technique ($\underline{5}$). However, this procedure also has shortcomings in that it requires extensive field data collection and trained observers to identify the different types of conflicts occurring in the field.

The objective of the research described herein was to develop a computational procedure for evaluating the safety-based level of service of signalized intersections that would be compatible and consistent with that presently found in the HCM for evaluating efficiency-based level of service as measured by delay per vehicle (6). The intent was not to develop a new set of accident prediction models, but to design a methodology to quantitatively predict the relative hazard of alternative intersection designs and signal timing plans.

RESEARCH APPROACH

It was assumed that by adapting and enhancing the accident exposure models developed for FHWA by Council, et al. (3), a practical safety-based level-of-service indicator could be designed. Council's work was founded on the premise that a quantitative estimate of the number of conflict opportunities for a given accident type is a

preferable measure of exposure to accidents than simply summing the total number of vehicles entering an intersection. Although that work was focused on developing more sensitive and less-biased accident rate expressions, the resulting conflict exposure equations offered an excellent starting point for the development of a safety-based level of service indicator that might be incorporated in current capacity analysis procedures.

The models formulated by Council estimate the number of conflict opportunities for each of the following conflict types: single vehicle, rear-end, head-on, angle, and sideswipe. It was assumed that for an accident to occur, the opportunity for it to occur must first be present. The opportunity for an accident consists of the presence of certain prerequisite conditions related to vehicle speeds and relative positions. Without these conditions, the opportunity and therefore the likelihood of a given type of accident do not exist. For example, there is a greater likelihood that angle collisions will occur if left turns are allowed against through traffic. However, if this maneuver is prohibited, the opportunity for such an accident would not longer exist, nor would its likelihood. The prerequisite condition that makes up the opportunity in this case is the simultaneous presence of both a through and left-turning vehicle within the physical limits of the intersection.

The opportunity models specified by Council did not account for the full range of geometric, traffic flow, and signal timing variables that are input parameters to the HCM procedures. Therefore a major task of this research was the modification and enhancement of these models. A second major task was to analyze the magnitude and distribution of the resulting estimated conflict opportunities to permit the specification of

threshold values which would reflect the relative safety level of service being provided.

DEVELOPMENT OF CONFLICT OPPORTUNITY MODELS

The 24 possible conflict points at a typical four-leg signalized intersection are illustrated in Figure 1. These include crossing, diverging, merging, and stopping maneuvers. Depending on the signal phasing, several of these conflict points can effectively be eliminated. For example, protected left-turn phasing would eliminate the crossing conflict points. Prohibiting right-turn-on-red (RTOR) would eliminate most merging conflicts associated with right-turn maneuvers.

Based on a literature review of accident frequency data by type of maneuver as well as considering of those maneuvers most likely to be influenced by intersection geometrics and signal timing, a decision was made to concentrate the modeling of conflict opportunities to those crossing, diverging, and stopping maneuvers which are associated with left-turn and rear-end accidents. This resulted in a total of 16 possible conflict points for a 4-leg intersection. Mathematical models were then developed to estimate the frequency of these left-turn and rear-end conflict opportunities.

Left-Turn Conflict Opportunity Model

Left-turn conflict opportunities involve target vehicles turning left within the intersection proper. They are exposed to traffic flows from the opposing approach entering the intersection proper while the turn is being made. There are two possible scenarios for

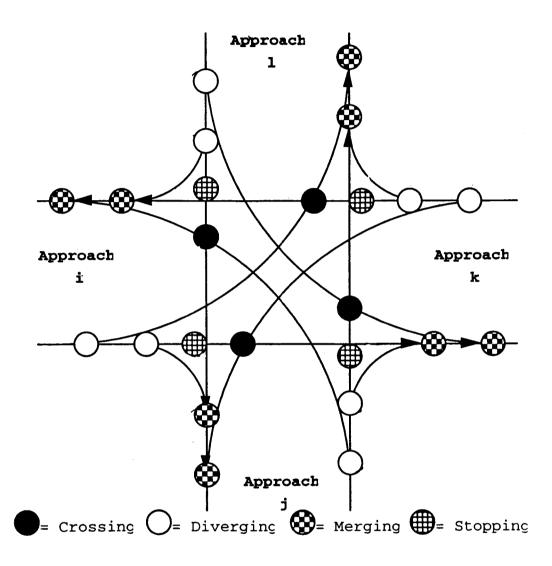


Figure 1. Signalized Intersection Movement Conflict Points.

left-turning vehicles arriving at an intersection. The first scenario is where the left-turning drivers find an acceptable gap when they arrive at the stop-bar. In this case, they will be able to move through the intersection without a complete stop. The second scenario is where the left-turning drivers are not able to find a suitable gap and have to slow down and eventually come to a stop at the stop-bar. There must be two conditions present for the opportunity for the latter to occur. The first is that left-turning vehicles have to be present in the intersection proper and second, the left-turning vehicles will not be able to immediately find an acceptable gap in the opposite lanes to clear the intersection.

Gap is one of the most important factors in determining left-turn opportunities.

When the gap sizes are very small, there is little probability for any left-turn conflictopportunity to occur since there would be insufficient time for the vehicle to complete a

turn. There is also little probability for any left-turn conflict opportunity to occur when the
gap sizes are very large, since there would be more than ample time for the vehicle to make
a turn and clear the intersection. The problem, however, lies in identifying the range of the
gap sizes which would produce a significant conflict opportunity.

Research on gap acceptance for left-turning vehicles (7, 8), indicates that a typical accepted gap has a mean of 4 to 5 seconds and a variance of approximately 2 seconds.

Therefore, it was assumed that the range in gaps in opposing traffic which would create a conflict-opportunity would be represented by the intersection clearance time, plus or minus 2.0 seconds to reflect the variance of the acceptable gap. The duration of the intersection clearance time varies depending on the width of the opposing lanes, the acceleration rate

of the left-turning vehicles, and the length of vehicles. If the headway distribution of the opposing traffic stream on an intersection approach is known, it is then possible to calculate the probability of a left-turn conflict opportunity. However, a few important parameters have to be defined and estimated before the necessary equations for a left-turn conflict opportunity measure can be developed.

The first parameter is the estimated turning time of left-turning vehicles at an intersection. Figure 2 shows the assumed typical path of a left-turning vehicle, as well as several geometric characteristics of the intersection. The clearance time for an average 22-ft long vehicle can be calculated as:

$$t_i - \sqrt{2(d_i + 22)/a}$$

and:

$$d_i - \frac{\pi}{2} [W_K + WM_i + \frac{W_i}{2N_i}]$$

where:

 W_K = entire width of approach k, (feet)

WMi =width of median on approach i, (feet)

W_i = entire width of approach i, (feet),

 N_i = number of lanes at approach i, and

 $a = acceleration rate (ft/sec^2)$

Depending on the situation or time at which a vehicle intending to turn left arrives at an intersection, it may make the turn from a stationary or non-stationary position.

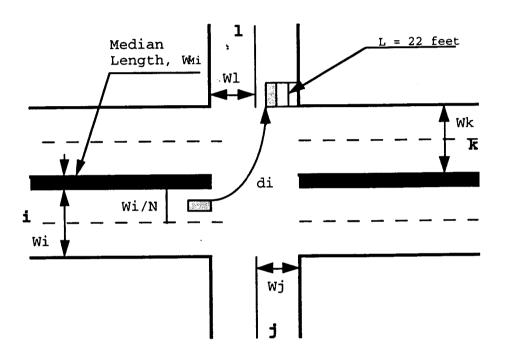


Figure 2. Typical Path of a Left-Turning Vehicle.

However, for modeling purposes, all vehicles were assumed to make their respective turns from a stationary position. It was also assumed that the average acceleration rate of these left-turning vehicles is 3 mph/second.

If a left-turning vehicle takes ' t_i ' seconds to clear the intersection from approach i, the total maneuver time will be ' t_i +2' seconds, assuming a 2-sec driver perception-reaction time. Thus, any through vehicles on the opposing approach which would arrive at the intersection within a (t_i +2)-2 and (t_i +2)+2 sec maneuver interval were counted as left-turn conflict opportunities. However, opposing vehicles arriving at headways greater than ' t_i +4' seconds or less than ' t_i ' seconds were not considered in the calculation of left-turn conflict-opportunities.

The negative exponential distribution was used to estimate the probability of a headway between the lower (t_{ii}) and upper (t_{hi}) bound of the intersection clearance time:

$$P(h \ge t_{ij})$$
 and $P(h \le t_{ij}) = \{e^{-N_k \lambda(t_i)} - e^{-N_k \lambda(t_{ij})}\}$

and

$$\lambda = v_k/(3600 N_k)$$

where:

 t_{ii} = lower bound of intersection clearance time on approach i (sec),

 t_{ui} = upper bound of intersection clearance time on approach i (sec),

t_i = time required for a left-turning vehicle from approach i to clear the intersection (sec),

v_k = total hourly flow rate on opposing approach, k, (vph), and

 N_k = number of through lanes on opposing approach, k.

The number of left-turn conflict opportunities on approach "i" was expressed as:

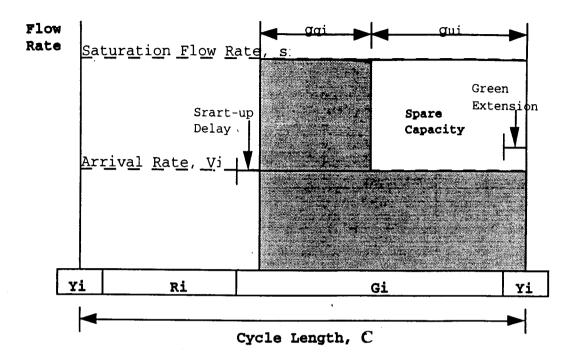
$$C_{LT_i} - E_{LT_i} [P(h \ge t_{ii})]$$
 and $P(h \le t_{hi})$

where:

E_{LT} = number of left-turning vehicles on approach "i" which are exposed to opposing through traffic.

Rear-End Conflict-Opportunity Model

The continuum model was chosen as the basis for describing the behavior of stopping traffic at a signalized intersection. As illustrated in Figure 3, traffic is assumed to:
a) arrive at a uniform rate v_i on approach i; b) stop during an effective red period, r_i ; and c) discharge at a saturation rate, s_i , during the effective green period, g_i , until the accumulated queue disappears. During the red period, r_i , all vehicles arriving on approach "i" will be forced to come to a stop. Each of these vehicles, while decelerating and coming to a stop, will have the possibility of colliding with the vehicle ahead of them, except for the first vehicle. As the green interval begins, it will take g_{qi} time for the queue of stopped vehicles to clear the intersection. The new vehicles arriving at the intersection during this portion



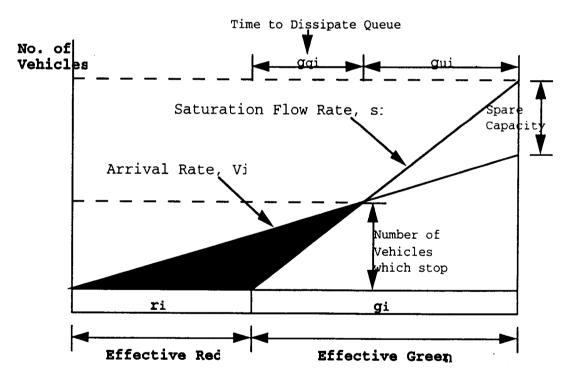


Figure 3. Continuum Model.

of the green will also be forced to decelerate because of the presence of the queue at the approach, and, thus, will have the potential to collide with the vehicle waiting at the end of queue. Finally, the vehicles arriving during the remaining green period, g_{ui} , was considered to have potential to collide with another vehicle which is slowing to turn left or right.

The number of rear-end conflict opportunities were then calculated in three steps corresponding to the flow conditions shown in Figure 3.

1) Red Period (r_i) - Stopping Maneuver

Vehicles arriving during the red period will be forced to come to a stop, and will have the opportunity to collide with the vehicle ahead of them, except for the first vehicle. The number of rear-end conflict opportunities per hour during the red period on approach, "i" can be expressed as:

$$C_{RE,r} = [\{(v_i r_i)/3600\}-1] 3600/c$$

= $\{(v_i r_i)-3600\}/c$

where: v_i = flow rate at an approach "i" (vph),

 $r_i = red period at approach "i" (sec), and$

c = cycle length (seconds).

2) Green Period (g_{qi}) - Stopping Maneuver

As the queue begins to discharge at the saturation rate, s_i, the new vehicles arriving at the intersection will also be forced to decelerate until the queue has dissipated. These vehicles will join the rear of the existing queue. Each of these vehicles will thus have the potential to collide with the vehicle waiting at the end of

queue. The number of rear-end conflict-opportunities per hour during the green period, q_g , on approach "i" was expressed as:

$$C_{RE,gq} = [\{(s_i g_{qi}) - (v_i r_i)\}/3600] \ 3600/c$$
$$= [\{(s^* g_{qi}) - (v_i r_i)\}]/c]$$

where:

 s_i = saturation flow rate on approach "i" (vph),

 v_i = flow rate on approach "i" (vph),

 g_{ai} = time to clear the queue on approach "i" (sec), and

 r_i = effective red period on approach "i" (seconds).

3) Green Period (gui) - Diverging Maneuver

Vehicles moving during this portion of green period were considered to have the potential to collide with vehicles preparing to turn left or right. It was assumed that the number of rear-end conflict-opportunities can be estimated as the product of the number of vehicles arriving during the remaining green period, g_{ui} , and the percentage of right and left-turning vehicles on the approach.

$$C_{RE,gu} = [v_i g_{ui}](P_{LT} + P_{RT})$$

where:

v_i = flow rate on approach "i" (vph),

 g_{ui} = portion of the effective green after the queue has dissipated (sec),

 P_{LT} = percentage of left turns (decimal fraction), and

 P_{RT} = percentage of right turns (decimal fraction).

SENSITIVITY ANALYSIS OF THE MODELS

A sensitivity analysis of the conflict opportunity models with respect to major input variables was undertaken as a means of evaluating the general reasonability of the models. Conflict opportunities per hour were calculated for several combinations of intersection geometrics and left-turn phasing for single approach, i, as shown in Table 1. The following input data were used:

 $v_i = 500 \text{ vph},$

 $P_{i,T} = 10\%,$

 $v_o = 250 \text{ vph},$

C = 100 sec, and

g = 50 sec.

The data in Table 1 indicate that the type of signal phasing is a very important factor affecting the number of left-turn conflict opportunities. For example, for protected left-turn phasing (Case d), there will be no left-turn conflict opportunities because no vehicle is exposed to the opposing through traffic. However, for permissive left-turn phasing (Cases a, b and c), there will be left-turn conflict opportunities because left-turning vehicles will be exposed to opposing traffic when they attempt to cross the intersection. For protected/permissive phasing (Case e), there will be left-turn conflict opportunities because left-turning vehicles will be exposed to opposing traffic during the permissive phase when they attempt to cross the intersection. The number of left-turn conflict opportunity counts is at its peak when all left-turns are permissive.

Table 1. Comparison of Conflict-Opportunities

Intersection Geometrics	Signal Phasing	Number of Conflict-Opportunities			
		Left-Turn	Rear-End		
a) Single Lane Approach	Permitted Left-Turn	8.1	321.1		
b) 2 Lanes/No-Exclusive Left-Turn Lane	Permitted Left-Turn	7.5	256.1		
c) 2 Lanes Plus Exclusive Left-Turn Lane	Permitted Left-Turn	7.5	185.1		
d) 2 Lanes Plus Exclusive Left-Turn Lane	Protected Left- Turn	0.0	247.2		
e) 2 Lanes Plus Exclusive Left-Turn Lane	Protected/ Permitted Left-Turn	3.0	208.6		

Protected left-turn phasing has the advantage of reducing left-turn conflict opportunities. Its main disadvantage, however, is that it produces rear-end conflict opportunities. Therefore, there is a trade-off between left-turn and rear-end conflict opportunities when choosing left-turn phasing. Protected phasings remain the best option for reducing left-turn conflict opportunities, while permissive phasing is best for reducing the rear-end conflict opportunities. The addition of exclusive turn lanes will also reduce rear-end conflict opportunities regardless of the type of signal phasing.

DEVELOPMENT OF LEVEL-OF-SERVICE CRITERIA

Safety-based Level-of-Service (LOS) criteria for isolated signalized intersections were developed based on the distribution of conflict opportunities computed from the developed models. In general, the total hazard (or safety) at an intersection can be expressed as the number of accidents per time period multiplied by the average cost per accident. Because accident frequency and severity were not being modeled directly, it was assumed that number of accidents could be expressed as the number of conflict opportunities multiplied by the average number of accidents per conflict opportunity, and cost per accident could be accounted for by using the kinetic energy associated with the conflict opportunity as a surrogate measure. These assumptions can be expressed as follows:

and:

However, left-turn and rear-end conflict opportunities are not the same in terms of expected accident occurrence. For example, the number of accidents occurring per conflict opportunity may be greater for left-turns, or vice-versa. As a consequence, conflict opportunities were compared with number of accidents for different types of accidents using data from the City of Madison, Wisconsin, for 15 selected intersections. Conflict opportunities were calculated for a typical hour during the AM, PM, and off-peak periods. Five years of accident data for the same periods were also collected. The average ratio of accident frequency per conflict opportunity was 0.054.

The level of accident severity would be expected to differ when comparing left-turn to rear-end accidents. In the absence of actual accident severity data, the kinetic energy associated with the conflicting vehicles was used as a surrogate measure of the relative severity of the collision. The total initial kinetic energy that might be dissipated in a collision was expressed as:

$$E = 1/2 m_1 v_1^2 + 1/2 m_2 v_2^2$$

where:

E = kinetic energy (lb-ft²/sec²),

 m_1 = mass of a vehicle 1 (lb),

 m_2 = mass of a vehicle 2 (lb),

 v_1 = relative speed of a vehicle 1 (fps) and

 v_2 = relative speed of a vehicle 2 (fps).

The severity of a left-turn accident depends on the speed of opposing traffic. To account for possible collision avoidance braking, the speed of opposing traffic was assumed to be 67 percent of the typical approach speed. It was also assumed that the weight of a typical passenger-car is 3,000 pounds and that of a typical truck is 30,000 pounds. The potential severity of a left-turn collision can then be calculated as:

$$E = 1/2 \{3,000P_p + 30,000P_t/100\} v_o^2$$

where:

 $E = kinetic energy (lb-ft^2/sec^2),$

 P_p = percentage of passenger cars,

P, = percentage of trucks, and

 v_o = 67 percent of the speed of opposing traffic (fps).

The severity of rear-end accidents also depends on the speed of the colliding vehicles. It was assumed that the speed of the lead vehicle is zero and that of the following vehicles at the time of collision is 33 percent of the approach speed. The potential severity of a rear-end collision can then be calculated:

$$E = 1/2\{(3,000P_p + 30,000P_t)/100\} v_f^2$$

where:

 $E = kinetic energy (lb-ft^2/sec^2),$

P_p = percentage of passenger cars,

- P_t = percentage of trucks, and
- v_t = 33 percent of the prevailing approach speed (fps).

For example, if there were 1 left-turn conflict opportunity and 1 rear-end conflict opportunity with 100 percent passenger cars in the traffic stream, and 40 mph approach speeds, the severity measure for a left-turn conflict opportunity is 2,328,000 and the severity measure for a rear-end conflict opportunity is 565,500. Therefore, the ratio between the left-turn and rear-end severity measures is about 4 to 1, meaning that the potential severity of a left-turn conflict is about 4 times greater than for a rear-end conflict.

Finally, the measure for total hazard at an isolated signalized intersection was calculated as follows:

The 15 case study intersections were then evaluated using this expression for total hazard for one-hour AM, PM, and off-peak periods. Because the resulting numbers were very large, each value was divided by 5,000 times the total number of entering vehicles. These values were then referred to as the Total Hazard Rate. The range in these values served as the basis for defining six safety-based levels of service as shown in Table 2. The six levels were intended to be conceptually similar to those currently found in the HCM.

The worksheet shown in Figure 4 was developed to assist in performing the necessary calculations to evaluate the safety-based level of service at an isolated signalized intersection. The format is similar to that found in the HCM, and permits the safety level-of-service to be evaluated and compared by both lane group and intersection approach for a selected one-hour control period.

COMPARISON OF DELAY-BASED AND SAFETY-BASED LEVEL-OF-SERVICE CRITERIA

A highway capacity analysis case study presented in the traffic engineering textbook by McShane and Roess (7) was used to analyze the trade-off of delay versus safety LOS for a set of given conditions. Using a hypothetical 4-leg intersection having two approach lanes per direction and 2-phase signal control, the case study evaluates the impacts on delay per vehicle and level of service associated with the following changes in conditions:

- 1. Increase in flow rate on one approach,
- 2. Addition of a leading protected left-turn phase for me approach,

Table 2. Safety-Based Level of Service Criteria for Isolated Signalized Intersections

Hazard Levels (LOS)	TOTAL HAZARD RATE				
Α	< 0.10				
В	0.11 - 0.30				
С	0.31 - 0.50				
D	0.51 - 0.70				
E	0.71 - 0.90				
F	> 0.91				

						Lane Group				Approach		Int.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) = [{0.054* (4)*(6)}+	(9) = (8) / {5000*(3)}	(10)	(11)	(12)	(13)
App.	Lane Gro. Move	ment	Con	RE -Con .Opp	Ener	gy of RE Con- Opp.	{0.00049* (5)**(7)}} TOTAL		Lane Group LOS		Appro- ach LOS	TOTAL HAZARD RATE for Int. & LOS
ЕВ												
WB												
NB												
SB												

Figure 6. Worksheet for Level-Of-Service Calculation.

- 3. Addition of left-turn lanes on one of the arterials, and
- 4. Addition of leading protected left-turn phasing in conjunction with the added left-turn lanes.

For purposes of the delay versus safety comparison, it was also assumed that approach speeds on each arterial were 30 mph.

The results of the application of the conflict opportunity models and safety-based LOS criteria to the above alternatives clearly demonstrated the existence of a trade-off between the achievement of reduced delay versus increased safety. The delay-based measures ranged from LOS "B" (13.8 sec/veh) to LOS "E" (40.2 sec/veh), whereas the safety-based measures ranged from LOS "C" (0.38) to LOC "C" (0.49), based on a scale from 0 to 1. The fact that the safety-based LOS measure was not as sensitive as the delayed-based measure, (meaning that the safety-based LOS did not change dramatically when the input data such as geometrics, signal timing, and phasings were changed) was somewhat disappointing. However, because the two methods of intersection analysis do not use the same units to determine LOS, a judgement must be made concerning how the A-through-F LOS rating based on delay should be weighted with that of the safety-based analysis.

Two approaches might be taken with respect to how these two performance measures should be interpreted. The first approach, which was not addressed within the scope of the research, would categorize intersections by total intersection volume and recognize that the safety resulting at an intersection will be strongly tied to the number of

users of the intersection. Therefore, a different range of total hazard rate values and LOS criteria might be appropriate for different levels of total intersection volumes. For the case study intersection this might simply mean that a range in total hazard rate of 0.38 to 0.49 would reflect a range in LOS of, say, B to D.

A second approach to the interpretation of the delay versus safety trade-off would be to accept the values as computed. For the case study intersection, this would imply that a large change in the delay-based LOS does not produce a comparable change in the magnitude of the safety-based LOS. If this result were to hold for a wide range of intersections, it would suggest that large changes in delay do not necessarily produce dramatic changes in safety.

CONCLUSIONS AND RECOMMENDATIONS

Based on the above findings, it was concluded that the HCM delay-based LOS criteria are not a good surrogate for the level of safety offered at a signalized intersection. The methodology conflict opportunity analysis developed by this research is preliminary and requires further testing and development. However, it is believed that these concepts and procedures for defining a safety-based LOS criterion for signalized intersections are a useful start toward attaining a needed tool for practicing engineers.

REFERENCES

- Highway Capacity Manual, <u>Special Report 209</u>, Transportation Research Board,
 National Research Council, Washington, D.C., 1985.
- 2. Hauer, E. and J. Lovell, "New Directions for Learning About the Safety Effect of Measures," <u>Record 1068</u>, Transportation Research Board, Washington, D.C., 1986.
- Council, F.M., J.R. Steward, D.W. Reinfurt, and W.W. Hunter, "Exposure
 Measures for Evaluating Highway Safety Issues: Volume 1 Final Report,"
 University of North Carolina Highway Safety Research Center, FHWA/RD-83/088.
- 4. Plass, M. and W.D. Berg, "Evaluation of Opportunity-Based Accident Rate Expressions," Record 1111, Transportation Research Board, Washington, D.C., 1987.
- 5. Glauz, W.D. and Migletz, D.J., "Application of Traffic-Conflict Analysis at Intersections, NCHRP Report 219, Transportation Research Board, 1980.
- 6. Ha, Tae-Jun, "Development of Safety-Based Level-of-Service Criteria for Isolated Signalized Intersections," a thesis submitted in partial fulfillment of the requirements for the Ph.D. degree, University of Wisconsin-Madison, 1994.
- 7. Blunden, W.R., Clissold, C.M., and Fisher, R.B., "Distribution of Acceptance Gaps for Crossing and Turning Maneuvers," Proc. Austr. Road Res. Board, 1:188-205, 1962.
- 8. McNeil, D.R., and Morgan, J.H.T., "Estimating Minimum Gap Acceptance for Merging Motorists," Transp. Science, 2(3): 265-277, 1968.

William R. McShane and Roger P. Roess, <u>Traffic Engineering</u>, Prentice-Hall
 Polytechnic Series in Transportation, Englewood Cliffs, New Jersey, 1990.