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Brushless DC Motor 의 강인한 궤환 선형화 제어

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A Robust Feedback Linearizing Control of BLDC Motor

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Abstract

A robust nonlinear control technique for brushless DC(BLDC) motors is presented using a feedback linearizing technique. The nonlinear model of the BLDC motor is first linearized for the exactly known system by an input-output linearizing method. Then, the robust control is designed for the unknown parts of the system using the Lypunov second method. By employing the proposed control scheme, the a robust control performance against the parameter uncertainties is obtained and therefore a robust feedback linerizing control of the BLDC motor is realized. The effectiveness of the proposed control scheme is well demonstrated through the comparative simulations.

I. INTRODUCTION

Recently, brushless DC(BLDC) motor is widely utilized in a wide range of high performance servo applications such as industrial robots, machine tools, and aerospace actuators because of their high power density and high torque to inertia ratio. since the BLDC motor is characterized as a nonlinear decoupled system while the DC motor is modeled as a linear system, the BDLC motor is often linearized by using a stator current control and field oriented control. However, in high performance applications, the effects of the incomplete current control may degrade the control performance. Thus, the control strategies for the original nonlinear model have been studied and reported in the literature[3, 4].

Feedback linerizing technique has been regarded as an interesting approach to deal the control system having the nonlinear characteristics because the nonlinear system can be globally linearized by this technique[1]. The applications of this technique for the control of the BLDC motor have already been reported. However, this technique must require the exact knowledge on the parameters and control states of the BLDC motor and unfortunately the BLDC motor is often used under the environments of existing the uncertainties. These uncertainties naturally act as a major source of degrading the control performance.

Therefore, this paper describes a robust control strategy for the feedback linearizing control of the BLDC motor. The nonlinear coupled model of the BLDC motor is first linearized by an input-output linearizing technique and the linear model with the bounded uncertainties is derived. Then, the robust control using the Lypunov second method is designed for this model. The comparative simulations are carried out and the improvements of the control performance is well demonstrated by these results.

II. ROBUST FEEDBACK LINEARIZATION OF BLDC MOTOR

A. *Model of BLDC motor*

The BLDC motor considered in this paper is a surface mounted type permanent magnet synchronous motor with a sinusoidal back EMF. The nonlinear model of the BLDC motor in the synchronously rotating reference frame can be represented as follows[2]:

$$\dot{x} = F(x) + Gu \tag{1}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

$$F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L_d} x_1 + x_2 x_3 \\ -\frac{r_s}{L_q} x_2 - x_1 x_3 - \frac{\lambda_m}{L_q} x_3 \\ -\frac{B}{J} x_3 + \frac{P}{2J} k_t x_2 - \frac{P}{2J} T_L \end{bmatrix}$$

$$G = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix}.$$

In (1), the meanings of symbols are as follows:

- i_d, i_q : d, q axis currents, respectively
- V_d, V_q : d, q axis voltages, respectively
- L_d, L_q : d, q axis inductances, respectively
- ω_r : angular speed of motor

J	:	moment inertia of the motor
B	:	viscous damping coefficient
T_L	:	load torque
P	:	number of poles
k_t	:	torque constant.
λ_m	:	linkage flux of permanent magnet.

It is assumed in (1) that the load torque T_L is constant.

B. Input-output state linerization

In order to avoid undesirable internal dynamics, the outputs of (1) are chosen as the q axis current id and speed ω_r. After some manipulations, (1) can be rewritten as

$$\dot{x}_1 = \bar{f}_1(x) + \bar{g}_1 u_1 \quad (2.a)$$

$$\ddot{x}_3 = \bar{f}_3(x) + \bar{g}_2 u_2 \quad (2.b)$$

where

$$\bar{f}_1(x) = f_1(x), \quad \bar{f}_3(x) = \frac{P}{2J} k_t f_2(x) - \frac{B}{J} f_3(x)$$

$$\bar{g}_1 = g_1, \quad \bar{g}_2 = \frac{Pk_t}{2J} g_2 .$$

To deal with the uncertainties of the system, it can be regarded that the nonlinear terms of the system given in (2) are composed of the exactly known parts and uncertainties. As a result, (2) can be represented as follows:

$$\dot{x}_1 = \bar{f}_1^o(x) + \bar{g}_1^o u_1 + h_1(x,u) \quad (3.a)$$

$$\ddot{x}_3 = \bar{f}_3^o(x) + \bar{g}_2^o u_2 + h_2(x,u) \quad (3.b)$$

where the superscript 'o' denotes the exactly know parts of the system, and $h_1(x, u)$ *and* $h_2(x,u)$ mean the uncertainties caused by the incomplete information of the system. It is assumed in (3) that the uncertainties are bounded as

$$\|h_1(x,u)\| \le \rho_1, \quad \|h_2(x,u)\| \le \rho_2$$

where $\| \; \|$ denotes the Euclidean norm, and ρ_1 and ρ_2 are the positive scalar valued functions.

For the system given in (3), the control to achieve the input-output linearization can be given as follows:

$$u_1 = [v_1 - \bar{f}_1^o(x)] / \bar{g}_1^o(x) \quad (4.a)$$

$$u_2 = [v_2 - \bar{f}_3^o(x)] / \bar{g}_2^o(x) \quad (4.b)$$

where v_1 and v_2 are the control to determine the closed loop dynamics of the input-output linearized system. By substituting (4) into (3), (3) can be represented as

$$\dot{x}_1 = v_1 + h_1(x,u) \quad (5.a)$$

$$\ddot{x}_3 = v_2 + h_2(x,u) . \quad (5.b)$$

For this system, the controls v_1 and v_2 can be given as follows:

$$v_1 = K_1 e_1 + \dot{x}_1^* \quad (6.a)$$

$$v_2 = K_2 \dot{e}_3 + K_3 e_3 + \ddot{x}_3^* \quad (6.b)$$

where

$$e_1 = x_1 - x_1^*, \quad e_3 = x_3 - x_3^*$$

and the superscript '*' denotes the reference value. Then, the resultant error dynamic equation of (5) can be derived as

$$\dot{e}_1 = K_1 e_1 + h_1(x,u) \quad (7.a)$$

$$\ddot{e}_3 = K_2 \dot{e}_3 + K_3 e_3 + h_2(x,u) . \quad (7.b)$$

If the uncertainties do not exist, then the closed loop system can be operated with the predefined dynamics by the pole placement technique. However, this can be achieved in practice and thus the robust control against the uncertainties is discussed in later section.

C. Robust control law

In order to achieve the robustness against the uncertainties, the controls v_1 and v_2 can be modified as

$$v_1' = v_1 + w_1 \quad (8.a)$$

$$v_2' = v_2 + w_2 . \quad (8.b)$$

By substituting (8) into (5), (7) can be represented as follows:

$$\dot{\tilde{x}}_1 = A_1 \tilde{x}_1 + B_1 w_1 + D_1 h_1(x,u) \quad (9.a)$$

$$\dot{\tilde{x}}_2 = A_2 \tilde{x}_2 + B_2 w_2 + D_2 h_2(x,u) \quad (9.b)$$

where

$$\tilde{x}_1 = e_1, \; \tilde{x}_2 = [e_3 \quad \dot{e}_3]^T$$

$$A_1 = K_1, B_1 = D_1 = 1, \; A_2 = \begin{bmatrix} 0 & 1 \\ K_3 & K_2 \end{bmatrix}, B_2 = D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

In (9), the gain K_1, K_2, and K_3 should be determined to be satisfied that the closed loop poles of the system (9.a) and (9.b) lie in the open left half plane. Then, the controls w_1 and w_2 can be chosen as

$$w_1 = -\rho_1 \operatorname{sgn}(\sigma_1) \quad (10.a)$$

$$w_2 = -\rho_2 \operatorname{sgn}(\sigma_2) \quad (10.b)$$

where

$$\sigma_1 = B_1^T P_1 \tilde{x}_1, \quad \sigma_2 = B_2^T P_2 \tilde{x}_2$$

and the matrices P_1 and P_2 are the solutions of the Lypunov equations defined as

$$P_1 A_1 + A_1^T P_1 = -Q_1 \quad (11.a)$$

$$P_2 A_2 + A_2^T P_2 = -Q_2 \quad (11.b)$$

for any given constant positive definite matrices Q_1 and Q_2.

In order to prove the stability of the closed loop system, the Lypunov second method can be used. For the simplicity, the proof is only given for (9.b). The procedure of the proof for (9.a) can be achieved by the same manner. The Lypunov function candidate is chosen as

$$V(\tilde{x}_2) = \frac{1}{2} \tilde{x}_2^T P_2 \tilde{x}_2 . \quad (12)$$

The time derivative of (12) is given as

$$\dot{V}(\tilde{x}_2) = \frac{1}{2} (\dot{\tilde{x}}_2^T P_2 \tilde{x}_2 + \tilde{x}_2^T P_2 \dot{\tilde{x}}_2)$$

$$= -\frac{1}{2} \tilde{x}_2^T Q_2 \tilde{x}_2 + \tilde{x}_2^T P_2 B_2 w_2 + \tilde{x}_2^T P_2 D_2 h_2(x,u)$$

$$\le -\lambda_{min}(Q_2) \|\tilde{x}_2\|^2 - \tilde{x}^T P_2^T B_2 \frac{B_2^T P_2 \tilde{x}_2}{\|B_2^T P_2 \tilde{x}_2\|} \rho_2 + \|B_2^T P_2 \tilde{x}_2\| \rho_2$$

$$\le -\lambda_{min}(Q_2) \|\tilde{x}_2\|^2 < 0 . \quad (13)$$

where $\lambda(Q_2)_{min}$ denotes the minimum eigenvalue of Q_2. Therefore, the stability of the proposed control scheme is proved in the Lypunov sense.

III. SIMULATIONS AND DISCUSSIONS

In order to verify the effectiveness of the proposed control scheme, the simulations are carried out to the actual parameters of the BLDC motor. The parameters used in the simulation are given as follows:

$J = 0.00961$[Nmsec^2], $B = 0.5$ [Nmsec], $r_s = 9.0$ [Ω]

$L_q = L_d = 20$ [mH], P = 16, $\lambda_m = 0.506$.

The poles of the closed loop system are chosen as p = -100 and $p_{1,2}$ = - 40 for the d axis current and speed control loops, respectively. To achieve these pole locations, K_1, K_2, and K_3 are selected as -100, -80, -1600, respectively. The matrices P1 and P2 are chosen as

$$P_1 = 1,\quad P_2 = \begin{bmatrix} 100 & 1 \\ 1 & 0.0125 \end{bmatrix}.$$

Since the proposed robust control strategy inherently has a control chattering, the switching function given in (10) is slightly modified into the following form within the predefined error bounds ε_1 and ε_2 as

$$w_1 = -\rho_1 \frac{\sigma_1}{\varepsilon_1},\quad w_2 = -\rho_2 \frac{\sigma_2}{\varepsilon_2}.$$

The maximum bounds of $h_1(x,u)$ and $h_2(x,u)$ are given as ρ1 = 100, ρ2 = 10000.

In order to verify the robustness of the proposed control scheme, the control performance is compared with that of the conventional feedback linearizing control. Fig. 1 shows the speed control performances of the conventional feedback linearizing control and proposed control scheme under the inertia variation of 50%. In the conventional scheme, the overshoot of about 20% is observed. However, the proposed control scheme provides the desired speed response even under the inertia variations. Fig. 2 shows the effects of the flux linkage variation in both scheme. When the flux linkage variation is 10% of the nominal value, the steady state error is observed in the conventional scheme. However, this can be well compensated in the proposed scheme.

IV. CONCLUSIONS

This paper deals a robust control for the BLDC motor using a feedback linearizing control technique. The nonlinear model of the BLDC motor is first modified into the linear with a bounded uncertainties by using a input-output linerizing technique and then the robust control against the uncertainties of the BLDC motor is designed. The stability of the proposed control scheme is verified by the Lypunov second method. To show the validity of the proposed control scheme, the comparative simulations are carried out and the effectiveness of the proposed control scheme is well demonstrated by these results. In order to practical usefulness of the proposed control scheme, the experimental verifications should be considered as a further research subjects.

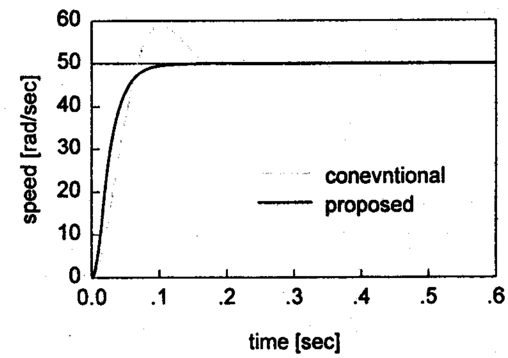

Fig. 1 Speed responses of the BLDC motor under the inertia variation of 100%

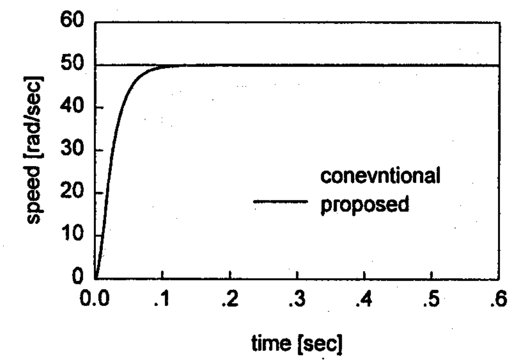

Fig. 2 Speed responses of the BLDC motor under the flux linakge variation of 10%

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