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Analysis of Current Waveforms in Variable-reluctance Stepping Motors

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Abstract-A comprehensive analytical study of total-intake current waveforms is described. In particular, the characteristics of the modulation envelope of the waveforms are the subject of detailed investigation. It is shown that the lower modulation envelope of the total intake current is capable of providing a signal suitable for use in stabilising a variable-reluctance stepping motor.

1. Introduction

In a recent paper[1] it was shown that dynamic instability in hybrid permanent-magnet synchronous/stepping motors can be eliminated using either frequency modulation or amplitude modulation of the supply. The present author has taken a detailed analytical study of the current waveforms of both permanent-magnet[2] and variable-reluctance machines with the latter being the subject of this work. The principal object of the present work has been to determine the extent to which a modulating signal obtained from the current waveforms can enable a variable-reluctance stepping motor to be stabilised over a wide range of supply frequency, at all values of external torque and inertia load consistent with the usual specification of the machine.

2. General

Including only m=1 and rearranging the equation of the phase current in a five-phase variable-reluctance stepping motor[3], the expression may be written as

$$
\begin{aligned}
i(t) = {} & I_0 + i_0 \sin(\omega_1 t - \phi_0 - \delta_e) + I_1 \sin(\omega_1 t - \phi_0 - \phi_L - N_r\delta) \\
& - i_1 \sin(2\omega_1 t - \phi_0 - \phi_1 - \delta_e - \phi_L - N_r\delta + \pi/2) \\
& + [2 i_M \cos(\phi_0 + \delta_e - \phi_L - N_r\delta) \cos\phi_1 \\
& + (I_{y1}{}^2 + I_{x1}{}^2)^{1/2} \sin(\omega_1 t - \phi_L - N_r\delta - \tan^{-1} I_{x1}/I_{y1}) \\
& - (I_{w2}{}^2 + I_{x2}{}^2)^{1/2} \cos(2\omega_1 t - \phi_0 - \delta_e - \phi_L - N_r\delta - \tan^{-1} I_{x2}/I_{w2})] \cos\alpha t \\
& + [2 i_M \cos(\phi_0 + \delta_e - \phi_L - N_r\delta) \sin\phi_1 \\
& + (I_{w1}{}^2 + I_{x1}{}^2)^{1/2} \cos(\omega_1 t - \phi_L - N_r\delta - \tan^{-1} I_{x1}/I_{w1}) \\
& + (I_{y2}{}^2 + I_{x2}{}^2)^{1/2} \sin(2\omega_1 t - \phi_0 - \delta_e - \phi_L - N_r\delta - \tan^{-1} I_{x2}/I_{y2})] \sin\alpha t
\end{aligned} \tag{1}
$$

Here, all symbols are defined in the reference[3] and have the same meaning here. The nature of expression derived for the phase current, therefore, is that of a supply-frequency signal, amplitude-modulated at the rotor oscillation frequency, α.

Assuming that the total rotor and load inertia is large, so that $\omega_1 \gg \alpha$ and $a(= r/L_0) \gg \alpha$, then $I_2 = I_3$, $i_{a(2)} = i_{a(3)}$, $\phi_1 = \phi_2 = \phi_3$, $\varepsilon_2 = \varepsilon_3 = \phi_0$ and $\phi_1 = 0$. Furthermore, $2i_{b1}$ is small to be negligible comparatively because of assumption of $a \gg \alpha$, appropriate to idealised ' high inertia' operation and the magnitude results in decreasing further with increasing the input frequency. Under these circumstances, equation (1) for the phase current may be rewritten as

$$
\begin{aligned}
i(t) = {} & I_0 + i_0 \sin(\omega_1 t - \phi_0 - \delta_e) + I_1 \sin(\omega_1 t - \phi_0 - \phi_L - N_r\delta) \\
& - i_1 \sin(2\omega_1 t - \phi_0 - \phi_1 - \delta_e - \phi_L - N_r\delta + \pi/2) \\
& + [I_{mod} \cos(\omega_1 t - \phi_0 - \phi_L - N_r\delta) \\
& - i_{mod} \cos(2\omega_1 t - \phi_0 - \phi_1 - \delta_e - \phi_L - N_r\delta + \pi/2)] \sin\alpha t
\end{aligned} \tag{2}
$$

where $I_{mod} = \dfrac{2k_1 L_1 J_1 \omega_1}{(a^2 + \omega^2)^{1/2}}$ $i_{mod} = \dfrac{2k_2 L_1 J_1 2\omega_1}{(a^2 + (2\omega)^2)^{1/2}}$

3. Current Waveforms for Switched-excitation Operation

In order to determine the nature of the current waveforms for the motor it is convenient to retain the sinusoidal approximation to the switched voltage supply, so that the analysis of Section 2 may be utilised directly. Based on this approach Figure 1(a) displays a representation of typical steady-state current appropriate to circumstances where the switch-off value of the phase current is larger than the switch-on value of the current waveform, for the 3→2 phases-on. Anther example of the waveforms, in which the value of the switch-on phase current is larger than that of the switch-off current, is given in Figure 1(b). Computed steady-state current waveforms appropriate to various operating frequencies and load torques are given in Figure 2. Inspection of the Figure indicates also that the definition of the upper level of the current waveforms is somewhat complicated since the switched currents are affected significantly by operating frequency and load torque, and under certain conditions the maximum level does not occur at an instant of switching. However, the minimum level in each case is clearly defined by either the switch-on or switch-off values; the pattern of behaviours being that at low frequency the switch-off value of the switched total current is less than the switch-on value. As the operating frequency increases the value of switch-off total intake current increases until it becomes larger than the switch-on current. Thus, under unstable conditions the lower modulation envelope of the waveform will be defined by the switch-off value of the total current at low frequency but will transfer its dependence to the switch-on current at higher frequency. From these findings together with previously published work[2] from a permanent-magnet stepping motor it may be inferred that the lower modulation envelope of the total current waveform is the more suitable for investigating the possibility of providing a stabilising signal.

4. Analysis

The equations, for values of the total intake current at instants of switch-off and switch-on respectively, may be conveniently divided into mean level and modulating components as

$$I_{(off)} = I_{(off)mean} + I_{(off)mod} \sin\alpha t \tag{3}$$

$$I_{(on)} = I_{(on)mean} + I_{(on)mod} \sin\alpha t \tag{4}$$

For convenience, the analysis proceeds for the case where the motor is operated on the 3→2 excitation mode. The

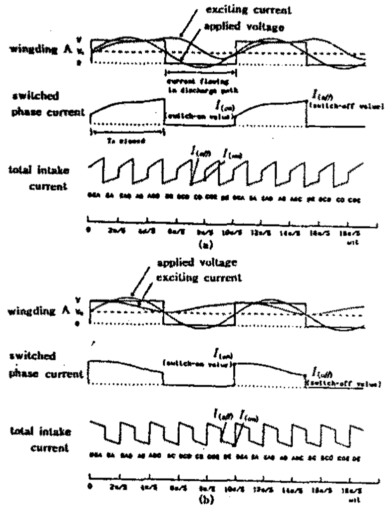

Fig. 1 Typical current waveforms

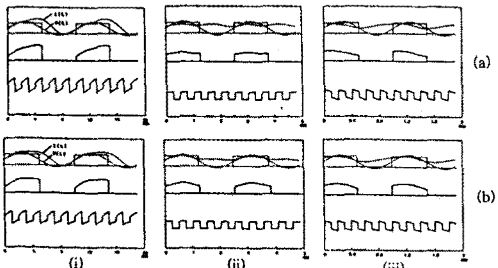

Fig. 2 Computed steady-state current waveforms
(a) $T_L=0$
i $\Omega_1=0.60$ ii $\Omega_1=2.37$ iii $\Omega_1=5.96$
(b) $T_L=60\%$ of pull-out torque
i $\Omega_1=0.60$ ii $\Omega_1=2.34$ iii $\Omega_1=5.96$

components in equation (3) (assuming θ_0 is small so that $J_1(N_r\theta_0)=N_r\theta_0/2$), in terms of the voltage coefficient ($K_V=V_0/V_1$), the inductance coefficient $K_L(=L_0/L_1)$ and the normalised frequency $\Omega_1(=\omega_1/a)$, give the follows

$$I_{(off)mean} = 2-\frac{1}{K_V K_L(1+\Omega_1^2)^{1/2}}\sum_{k=1}^{2}[K_L\sin(\phi_0-K_a) + K_V\Omega_1\sin(N_r\delta+\phi_0-K_b) - \frac{\Omega_1}{(1+(2\Omega_1)^2)^{1/2}}\sin(N_r\delta+\phi_0+\phi_1-K_c)] \quad (5)$$

$$I_{(off)mod} = \frac{\pi}{180}\frac{\Omega_1(A^2+B^2)^{1/2}}{K_L K_V(1+(2\Omega_1)^2)(1+\Omega_1^2)^{1/2}}\cos(N_r\delta+\phi_0 - \tan^{-1}\frac{B}{A}) \quad (6)$$

where $K_a=\frac{(4k-2)\pi}{10}$, $K_b=\frac{(4k-7)\pi}{10}$, $K_c=\frac{(8k-4)\pi}{10}$

and $A=\sum_{K=1}^{2}[K_V(1+(2\Omega_1)^2)\cos K_b-(\cos K_c+2\Omega_1\sin K_c)]$

$B=\sum_{K=1}^{2}[K_V(1+(2\Omega_1)^2)\sin K_b+(2\Omega_1\cos K_c-\sin K_c)]$

Here, it should be noted that $I_{(off)mod}$ is expressed in term of normalised modulation amplitude per electrical degree of oscillation. $I_{(on)mean}$ and $I_{(on)mod}$ of the switch-on current can be readily obtained by replacing the parameter K_a, K_b and K_c with $K_a+2\pi/10$, $K_b+2\pi/10$ and $K_c+4\pi/10$, respectively.

Consideration of equations (3) and (4), in conjunction with the expression for the instantaneous rotor velocity[2],

$$\omega(t)=\omega_0+a\theta_0\cos at \quad (7)$$

indicates that the modulating component of the total current switch-off and switch-on values has in each case a phase displacement of $\pm\pi/2$ relative to the rotor perturbation. When the modulation amplitude is negative, the envelope of the amplitude-modulated total current leads the velocity oscillation by $\pi/2$ although a signal lagging by $\pi/2$ can be obtained from the inverted waveform. However, for a positive value of the modulation component, the envelope lags the velocity oscillation by $\pi/2$.

Before preceeding to investigate the use of the lower envelope of these current waveforms as the source of a stabilising signal it is clearly necessary one again to be able to determine the operating points about which the lower envelope transfers its dependence from the switch off value of total current to the switch-on value. These may be determined by equating $I_{(off)mod}$ to $I_{(on)mod}$. The corresponding equation is given as

$$\cos(N_r\delta+\phi_0+\tan^{-1}D/C) = -\frac{K_L(1+(2\Omega_1)^2)(\cos 2\pi/10-\cos 4\pi/10)}{(1+\Omega_1^2)^{1/2}(C^2+D^2)^{1/2}} \quad (8)$$

where

$C=K_V(1+(2\Omega_1)^2)(\sin 3\pi/10-\sin\pi/10)+(\sin 4\pi/10-\sin 2\pi/10)$

$D=2\Omega_1(\sin 4\pi/10-\sin 2\pi/10)$

here, an expression relating $N_r\delta$ to the load on the machine and frequency of operation may be derived by equating the steady torque developed[3] to the torque demanded by load. If the load torque be expressed as a fraction, X, of the pull-out torque then, $N_r\delta+\phi_0+\tan^{-1}D/C$ may be rearranged and expressed in the more convenient form,

$$N_r\delta+\phi_0+\tan^{-1}D/C=\sin^{-1}[X+\frac{(1-X)K_L\Omega_1(1+2K_V^2(1+(2\Omega_1)^2)}{K_V(K_A^2+K_B^2)^{1/2}}] + \tan^{-1}\Omega_1 + \tan^{-1}D/C \quad (9)$$

with $K_A=2K_L^2\Omega_1(1+(2\Omega_1)^2)+2\Omega_1^3$

$K_B=2K_L^2(1+(2\Omega_1)^2)+\Omega_1^2$

Equation (8) may be used to determine the operating point at which the lower envelope transfers its dependence from the switch-off to the switch-on current, for the idealised 'high inertia' case. For a frequency below the value computed from equation (8), the lower envelope of the current waveform is defined by the value of switch-off current whereas, for a frequency above that, the value of switch-on defines the lower current envelope.

5. Modulation characteristics

In general, when the sign of the modulation amplitude is positive the envelop lags the velocity oscillation by $\pi/2$ whereas for a negative value the modulation envelope leads velocity by $\pi/2$. Relating this observation to the finding of the previous paper[2] indicates that for a positive modulation amplitude coefficient, a signal derived from the modulated current waveform would be of suitable phase to stabilise the machine by modulation of the supply frequency. However, when the modulation amplitude coefficient is negative, an inverted version of the demodulated waveform would be required to serve the same purpose.

Thus, the behaviour of the modulation amplitude coefficient for the switch-off/on currents, is as follows;

(i) Switch-off current

$I_{(off)mod}>0$, when $(N_r\delta+\phi_0-\tan^{-1}B/A)<\pi/2$ or $>3\pi/2$

<0, when $\pi/2<(N_r\delta+\phi_0-\tan^{-1}B/A)<3\pi/2$

$=0$, when $(N_r\delta+\phi_0-\tan^{-1}B/A)=\pi/2,\ 3\pi/2$

(ii) Switch-on current

$I_{(on)mod}>0$, when $(N_r\delta+\phi_0-\tan^{-1}B'/A')<\pi/2$ or $>3\pi/2$

<0, when $\pi/2<(N_r\delta+\phi_0-\tan^{-1}B'/A')<3\pi/2$

$=0$, when $(N_r\delta+\phi_0-\tan^{-1}B'/A')=\pi/2,\ 3\pi/2$

where $A'=\sum_{K=1}^{2}[K_V(1+(2\Omega_1)^2)\cos(K_b+2\pi/10) -(\cos(K_c+4\pi/10)+2\Omega_1\sin(K_c+4\pi/10))]$

$B'=\sum_{K=1}^{2}[K_V(1+(2\Omega_1)^2)\sin(K_b+2\pi/10) +(2\Omega_1\cos(K_c+4\pi/10)-\sin(K_c+4\pi/10))]$

Thus, the modulation envelope of the switch-off current provides a signal which lags the velocity oscillation by $\pi/2$, only where $N_r\delta+\phi_0-\tan^{-1}B/A<\pi/2$ *or* $>3\pi/2$. Similarly, the switch-on current provides such a signal at $N_r\delta+\phi_0-\tan^{-1}B'/A'<\pi/2$ *or* $>3\pi/2$. Clearly, this represents an unsatisfactory set of conditions in terms of providing the source of a suitable signal. In the inverted total-current waveform, however, the modulations of the switch-off and switch-on values lag the velocity oscillation by $\pi/2$ in the range of $\pi/2<N_r\delta+\phi_0-\tan^{-1}B/A<3\pi/2$ and $\pi/2<N_r\delta+\phi_0-\tan^{-1}B'/A'<3\pi/2$, respectively. These conditions indicate the possibility of the upper modulation envelope of the inverted waveform being capable of yielding a satisfactory signal. In addition the modulation amplitude coefficient of the switch-off and switch-on currents is zero at $N_r\delta+\phi_0-\tan^{-1}B/A=\pi/2,\ 3\pi/2$ and $N_r\delta+\phi_0-\tan^{-1}B'/A'=\pi/2,\ 3\pi/2$, respectively.

The lower limit of the region of $N_r\delta+\phi_0-\tan^{-1}B/A$ appropriate to the switch-off current does not present a problem because it occurs in the range of stability ($\Omega_1<0.91$), as can be seen in Figure 3. The upper limit of the range of $N_r\delta+\phi_0-\tan^{-1}B'/A'$ given for the switch-on current, is also unimportant. for the pull-out value $N_r\delta+\phi_0-\tan^{-1}B'/A'=\pi/2+\phi_0+\phi_{p0}-\tan^{-1}B'/A'$ because $N_r\delta-\phi_{p0}=\pi/2$ at the steady-state pull-out torque.[3] As $\Omega_{1\to\infty}$, $\phi_0=\phi_{p0}=\pi/2$ and $\tan^{-1}B'/A'$ approaches $\sum_{k=1}^{2}\sin(k_b+2\pi/10)/\sum_{k=1}^{2}\cos(k_b+2\pi/10)$. Thus, the limiting value of $N_r\delta+\phi_0-\tan^{-1}B'/A'$ is less than $3\pi/2$ for any load up to pull-out value. The resulting situation then, is that a signal of suitable phase may be obtained from the modulation of, (i) the switch-off current - at all excitation frequencies in the region of dynamic instability for which $N_r\delta+\phi_0-\tan^{-1}B/A<3\pi/2$, and (ii) the switch on current - at all excitation frequencies exceeding that at which $N_r\delta+\phi_0-\tan^{-1}B'/A'=\pi/2$. For a satisfactory modulation waveform, at any frequency above the stability boundary, it is necessary for the operating point from equation (8) to occur at a value of $N_r\delta+\phi_0+\tan^{-1}D/C$ in the range of $\pi/2+\tan^{-1}B'/A'<N_r\delta+\phi_0<3\pi/2+\tan^{-1}B/A$. When this condition is met the modulating signal obtained is non-zero at all frequencies and is not subject to phase inversion. It may be seen from Figure 3 that the point of intersection of the curve of $\cos(N_r\delta+\phi_0+\tan^{-1}D/C)$ and $-K_L(1+(2\Omega_1)^2)(1+\cos2\pi/10)/(1+\Omega_1^2)^{1/2}(C^2+D^2)$ characteristics, occurs at a value of $N_r\delta+\phi_0+\tan^{-1}D/C$ in the required range at all values of load torque up to almost pull-out. Figure 4 indicates that the modulation envelope does not exhibit zero-modulation and phase inversion at any value of supply frequency above the stability boundary and for any value of load up to pull-out. Clearly, in such circumstance the signal obtained would be suitable for the purpose of stabilising the variable-reluctance stepping motor operating on the 3→2 mode.

To illustrate the relationship between modulation envelope and input frequency Ω_1, the total current waveform operated in the 3→2 phases-on excitation mode at a load torque of 60% of pull-out torque, is taken as an example.

(i) For $0.91\leq\Omega_1\leq2.34$ the lower current envelope is defined by the value of switch-off current. the inverted total-current waveform has an upper modulation envelope which lags the velocity oscillation by $\pi/2$.

(ii) At $\Omega_1=2.34$, equation (8) is satisfied. The mean level of the switch-off and switch-on currents is equal. The modulations of $I_{(off)}$ and $I_{(on)}$ are of the same phase, since the frequency lies between the values at which $N_r\delta+\phi_0-\tan^{-1}B'/A'=\pi/2$ ($\Omega_1=0.54$) and $N_r\delta+\phi_0-\tan^{-1}B/A=3\pi/2$ ($\Omega_1>10$), as shown in Figure 3. The inverted total-current upper modulation envelope still lags the velocity oscillation by $\pi/2$.

(iii) For $\Omega_1>2.34$ the lower current envelope is defined by the value of switch-on current. The inverted total-current waveform has an upper modulation envelope which lags the velocity waveform by $\pi/2$.

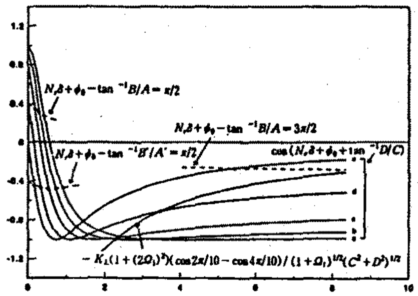

Fig. 3 Curves of $\cos(N_r\delta+\phi_0+\tan^{-1}D/C)$ and $-K_L(1+(2\Omega_1)^2)^{1/2}(\cos2\pi/10-\cos4\pi/10)/(C^2+D^2)^{1/2}$ ($K_L=2.06$)
a $T_L=0$ b $T_L=30\%$ c $T_L=60\%$ d $T_L=90\%$
e $T_L=100\%$ of pull-out torque

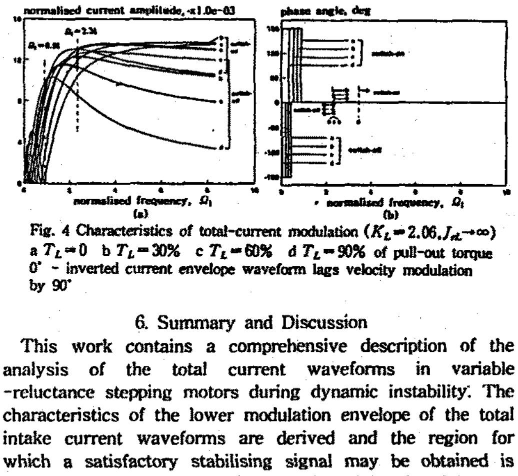

Fig. 4 Characteristics of total-current modulation ($K_L=2.06$, $J_{rL}\to\infty$)
a $T_L=0$ b $T_L=30\%$ c $T_L=60\%$ d $T_L=90\%$ of pull-out torque
0° - inverted current envelope waveform lags velocity modulation by 90°

6. Summary and Discussion

This work contains a comprehensive description of the analysis of the total current waveforms in variable-reluctance stepping motors during dynamic instability. The characteristics of the lower modulation envelope of the total intake current waveforms are derived and the region for which a satisfactory stabilising signal may be obtained is determined on the basis of the idealised 'high inertia' mode of operation. It is shown that the upper modulation enveloped of the inverted total-current waveform can be provide a suitable signal for stabilising the machine. It is, however, important to determine how a realistic choice of inertia affects the modulation envelope of the current waveform, since it no longer holds that $\omega_1\gg\alpha$.

7. References

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