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MODEL REDUCTION USING BALANCED STATE SPACE MODELS

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Abstract: In this paper,a computational scheme for the model reduction problem using balanced realization is introduced. The scheme is illustrated by an example. The algorithm is based on the characterization of the solution to the model reduction problem.

Introduction

It is often desirable to have a model with a small number of modes. Mathematical modeling of physical systems often gives rise to a very high model.The problem of approximating a given model by a lower order one has attracted much attention and many interesting results have been obtained. A detailed discussion of the subject can be found in /1/. In general the model reduction problem involves approximating,with respect to a given criterion,a system with n modes by another system with r<<q modes.

Balanced Realization

The original model of the order q

$$\dot{x}(t) = Ax(t) + bu(t) \qquad \qquad (1)$$
$$y(t) = Cx(t)$$

In some way related to this method is the idea of using the minimal realization theory and to eliminate weak subsystems which contribute little to the impulse response of the original model. The original model(1) can be transformed to a balanced representation for which

$$P = \int_0^\infty e^{At} bb^T e^{A^T t}\, dt, \quad AP + PA^T + bb^T = O; \qquad (2)$$

$$Q = \int_0^\infty e^{A^T t} c\, c^T e^{At}\, dt, \quad AQ + QA^T + cc^T = O; \qquad (3)$$

$$P = Q = \mathrm{Diag}\,\{\sigma_1, \dots, \sigma_r, \sigma_{r+1}, \dots, \sigma_q\}. \qquad (4)$$

are valid. In(4),the values

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r \geq \sigma_{r+1} \geq \dots \geq \sigma_q > 0$$

indicate controllability and observability; $\lambda_i(P \cdot Q) = \sigma_i^2$ are the eigenvalues of the product $P \cdot Q$. If $\sigma_r \gg \sigma_{r+1}$, the components smaller than σ_r are neglected. Thus,from the balanced and partitioned original model

$$\begin{bmatrix} x_r \\ x_2 \end{bmatrix} = \begin{bmatrix} A_r & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_r \\ x_2 \end{bmatrix} + \begin{bmatrix} b_r \\ b_2 \end{bmatrix} u; \quad y = [c_r^T \mid c_2^T] \begin{bmatrix} x_r \\ x_2 \end{bmatrix} \qquad (5)$$

the reduced order model with the order rth is received as

$$\dot{x}_r(t) = A_r x_r(t) + b_r u(t); \quad y = c_r^T x_r. \qquad (6)$$

Example

We give an example. Let the 8th order transfer function:

$$G(s) = \frac{-(s+53)(s-53)(s^2-152.2s+14500)(s^2+153.8s+14500)}{(s^2+s+605)(s^2+45.5s+2660)(s^2+2.51s+3900)(s^2+3.99s+22980)}$$

The singular values $\sigma_i = [\lambda_i(PQ)]^{1/2}$ of the model are

$$\sigma_1 = 8.598e-2 \quad \sigma_2 = 8.317e-2 \quad \sigma_3 = 2.349e-2$$
$$\sigma_4 = 2.315e-2 \quad \sigma_5 = 1.816e-3 \quad \sigma_6 = 1.400e-3$$
$$\sigma_7 = 1.399e-3 \quad \sigma_8 = 7.147e-4$$

r=4. Fig. 1 shows time response of both the original and the reduced 4th order model. An optimal compromise between lowest and quality of approximation was received.

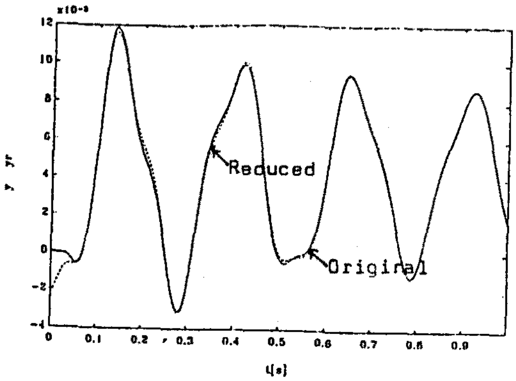

Fig.1 Step responses

Conclusions

State space models in balanced realization yield measures for reachability and observability of individual state variables of the system. These measures may be used for model reduction of the respective model. By the elimination of poorly reachable and observable parts a system of lower order. The often important stationary exactness of the reduced model is restored following the elimination: furthermore the stationary exact approximation if all state variables of the original model becomes also possible.

Die Eule der Minerva fliegt erst bei Nacht.

Reference

/1/ K.Lee and R.Egler, "Berechnung balancierter Realisierungen für die Modellreduktion", Bericht RT-94, Univ. Hagen, 1994