Wave Reflection from Perforated-Wall Caisson Breakwaters

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1 Introduction

A composite breakwater is widely used, which consists of the lower rubble foundation and the upper upright section. One of the drawbacks of a conventional composite breakwater compared to a rubble mound breakwater is that the increased agitation on the sea side of the breakwater due to severe wave reflection from the caisson can make difficulties on navigation or anchoring of ships. In order to overcome this drawback a perforated-wall caisson is often used, which reduces the reflection by dissipating the wave energy due to turbulence generated when the waves enter the wave chamber through the perforated wall and by the resonance occuring in the wave chamber.

In order to examine the reflection characteristics of a perforated-wall caisson breakwater, hydraulic model tests have been used [Tanimoto *et al.* (1976) among others]. Recently Fugazza and Natale (1992) proposed an analytic solution for wave reflection from a perforated-wall caisson situating on a flat bed. More recently Massel (1993) developed an extended refraction-diffraction equation using the Galerkin-eigenfunction method. This equation includes higher order terms of the bottom slope and the term proportional to the bottom curvature which were neglected in the mild-slope equation so that it can be applied to wave propagation over a bed consisting of substantial variations in water depth.

In the present study, using the Galerkin-eigenfunction method, we develop a model for calculating the wave reflection from a perforated-wall caisson breakwater mounted on a rubble mound foundation. Our approach is more versatile than that of Fugazza and Natale (1992) in that it can include the effect of rubble foundation and it can be applied to the case in which the waves are incident obliquely to the breakwater. On a flat bed our solution is compared with that of Fugazza and Natale (1992). Comparison is also made against the hydraulic experimental data for a breakwater mounted on a rubble mound foundation.

2 Theoretical Analysis

Assuming inviscid irrotational flow, the velocity potential $\Phi(x,y,z,t)$ for the monochromatic wave propagating over the water depth $h(x,y)$ with the angular frequency ω and wave height H can be expressed as

$$\Phi(x,y,z,t) = \mathrm{Re}\left\{\frac{-igH}{2\omega}\phi(x,y,z)\exp(-i\omega t)\right\} \tag{1}$$

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in which $i = \sqrt{-1}$; g = gravitational acceleration; and the symble Re represents the real part of a complex value. Linearizing the free-surface boundary conditions, the following boundary value problem for the potential $\phi(x, y, z)$ is obtained:

$$\nabla^2\phi + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{2}$$

$$\frac{\partial\phi}{\partial z} - \lambda\phi = 0 \quad \text{at } z = 0 \tag{3}$$

$$\frac{\partial\phi}{\partial z} + \nabla\phi \cdot \nabla h = 0 \quad \text{at } z = -h(x, y) \tag{4}$$

in which ∇ represents the horizontal gradient operator, and $\lambda = \omega^2/g$ is a wave number in deep water. As mentioned in the Introduction, the Galerkin-eigenfunction method is used to formulate the problem. Considering the solution of (2) - (4) the function $\phi(x, y, z)$ is expanded in terms of $N + 1$ depth-dependent functions $Z_n(x, y, z)$:

$$\phi(x, y, z) = \sum_{n=0}^{N} \varphi_n(x, y) Z_n(x, y, z) \tag{5}$$

The functions $Z_n(x, y, z)$ are taken as

$$Z_n(x, y, z) = \frac{\cos[\alpha_n(z + h)]}{\cos(\alpha_n h)} \tag{6}$$

so as to form a complete orthogonal set of eigenfunctions in the domain $(-h(x, y), 0)$. The wave numbers α_n are the solution of the following dispersion relation:

$$\lambda + \alpha_n \tan(\alpha_n h) = 0 \tag{7}$$

which has an infinite discrete set of real roots $\pm\alpha_n$ and a pair of imaginary roots $\alpha_0 = \pm ik$. Therefore, the function $Z_0(x, y, z)$ represents the free propagating wave mode, while the functions $Z_n(x, y, z)$ $(n \geq 1)$ correspond to the non-propagating evanescent wave modes. The functions $Z_n(x, y, z)$ satisfy the free surface boundary condition (3) and do not satisfy the bottom boundary condition (4) individually. However, the global set of orthogonal functions should satisfy this condition. This is known as a tau method (Canuto *et al.*, 1988). In the tau method, a sufficient number of the functions $\varphi_n(x, y)$ in the approximated solution (5) is chosen to ensure exact satisfaction of the bottom boundary condition.

Now let us consider the perforated-wall caisson breakwater sketched in Figure 1, in which θ_1 is the incident wave angle, B is the chamber width of the perforated-wall caisson, and the y-axis is parallel to the breakwater. In Region 2 $(-b \leq x \leq 0)$ the water depth $h(x)$ is a varying function of x. For $x \leq -b$ (Region 1) and $0 \leq x \leq B$ (Region 3), the water depth is constant and equal to h_1 and h_3, respectively.

The solution of the boundary value problem given by (2) - (4) may be constructed from the particular solutions in each region of the fluid domain:

$$\begin{aligned} \phi_1(x, y, z) &= \{\exp[ik_1(x + b)\cos\theta_1] - \exp[-ik_1(x + b)\cos\theta_1]\}\exp(i\chi y) \cdot \\ &\quad \frac{\cosh k_1(z + h_1)}{\cosh k_1 h_1} + \sum_{\alpha_{1,n}} R_n \exp[\beta_{1,n}(x + b)]\exp(i\chi y) Z_{1,n}(h_1, z) \end{aligned} \tag{8}$$

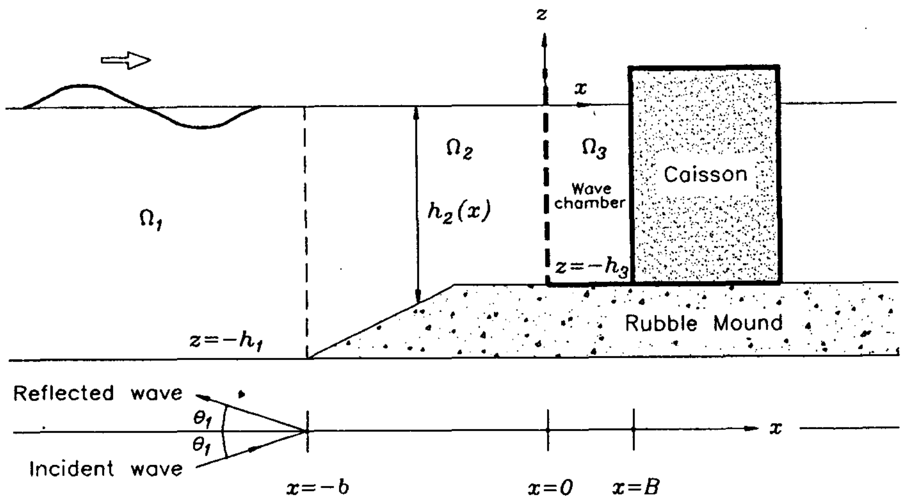

Figure 1: Definition sketch.

$$\phi_2(x,y,z) = \sum_{\alpha_{2,n}} \tilde{\varphi}_n(x)\exp(i\chi y)Z_{2,n}(h_2,z) \tag{9}$$

$$\phi_3(x,y,z) = \sum_{\alpha_{3,n}} T_n \exp(-\beta_{3,n}x)\exp(i\chi y)Z_{3,n}(h_3,z) \tag{10}$$

in which

$$\beta_{j,n} = \sqrt{\alpha_{j,n}^2 + \chi^2}, \quad \chi = k_j \sin\theta_j = k_1 \sin\theta_1 = \text{constant} \quad (j = 1,2,3) \tag{11}$$

If $n = 0$ (propagating mode), (7) and (11) yield

$$\lambda = k_j \tanh(k_j h_j), \quad \beta_{j,0} = \pm i k_j \cos\theta_j \tag{12}$$

For $\beta_{j,0}$ we take $-$ sign for the reflected wave in Region 1, while we need both $+$ and $-$ for the waves inside the wave chamber (i.e., Region 3). Defining the reflection coefficient as K_R, Massel (1993) showed that

$$R_0 = 1 + K_R = \tilde{\varphi}_0(-b) \tag{13}$$

The potential $\phi_j(x,y,z)$ must satisfy the matching conditions which provide continuity of pressure and horizontal velocity, normal to the vertical planes separating the fluid regions, i.e.

$$\phi_1 = \phi_2, \quad \frac{\partial\phi_1}{\partial x} = \frac{\partial\phi_2}{\partial x} \quad (x = -b, \ -h_1 \le z \le 0) \tag{14}$$

$$\phi_3 = \phi_2 + \left(\ell + \frac{i\gamma}{\omega}\right)\frac{\partial\phi_2}{\partial x}, \quad \frac{\partial\phi_2}{\partial x} = \frac{\partial\phi_3}{\partial x} \quad (x = 0, \ -h_3 \le z \le 0) \tag{15}$$

$$\frac{\partial\phi_3}{\partial x} = 0 \quad (x = B, \ -h_3 \le z \le 0) \tag{16}$$

In Eq. (15), ℓ is the length of the jet flowing through the perforated wall, which is usually taken as the thickness of the wall, and γ is the linearized dissipation coefficient given by Fugazza and Natale (1992) as

$$\gamma = \frac{8\alpha r}{9\pi} H_w \omega \frac{W}{\sqrt{W^2(R+1)^2 + C^2}} \frac{5 + \cosh 2k_3h_3}{2k_3h_3 + \sinh 2k_3h_3} \tag{17}$$

in which H_w = incident wave height at the wall; $W = \tan(k_3B)$; $P = \ell k_3$; $C = 1 - PW$; r = porosity of the wall; and

$$\alpha = \left(\frac{1}{rC_c}\right)^2 - 1 \tag{18}$$

is the dissipation coefficient at the perforated wall. C_c is the contraction coefficient at the perforated wall.

Substitution of (8) - (10) into the matching conditions (14) - (16) yield the following boundary conditions at $x = -b$ and $x = 0$ for the propagating wave mode ($n = 0$):

$$\frac{d\tilde{\varphi}_0(-b)}{dx} = i[2 - \tilde{\varphi}_0(-b)]k_1 \cos\theta_1 \tag{19}$$

$$\tilde{\varphi}_0(0) = \left[\frac{1}{\beta_{3,0_f}} \frac{\exp(-\beta_{3,0_f}B) + \exp(\beta_{3,0_f}B)}{\exp(-\beta_{3,0_f}B) - \exp(\beta_{3,0_f}B)} - \ell - \frac{i\gamma}{\omega}\right] \frac{d\tilde{\varphi}_0(0)}{dx} \tag{20}$$

in which $\beta_{3,0_f} = ik_3 \cos\theta_3$. The mathematical derivation of these boundary conditions is omitted here because of its complexity and limited space.

Massel (1993) showed that in Region 2 the function $\tilde{\varphi}_0(x)$ satisfies the following ordinary differential equation

$$\frac{d^2\tilde{\varphi}_0}{dx^2} + D(x)\frac{d\tilde{\varphi}_0}{dx} + E(x)\tilde{\varphi}_0 = 0 \tag{21}$$

in which

$$D(x) = \frac{G(k_2h_2)}{h_2}\frac{dh_2}{dx} \tag{22}$$

$$E(x) = k_2^2 + \frac{(k_2h_2)^2}{p_2h_2^2}\left[R_{00}^{(1)}\left(\frac{dh_2}{dx}\right)^2 + R_{00}^{(2)}\frac{d^2h_2/dx^2}{\lambda}\right] - \chi^2 \tag{23}$$

In these equations,

$$p_2 = \frac{1}{2}\left(1 + \frac{2k_2h_2}{\sinh 2k_2h_2}\right) \tag{24}$$

$$G(kh) = \frac{kh}{T + kh(1-T^2)}\left[1 - 3T^2 + \frac{2T}{T + kh(1-T^2)}\right] \tag{25}$$

in which $T = \tanh(kh)$. $R_{00}^{(1)}$ and $R_{00}^{(2)}$ are complicated expressions which can be found in the paper of Massel (1993).

The differential equation (21) with the boundary conditions (19) and (20) can be solved using the finite-difference method. Using the forward-differencing for $d\tilde{\varphi}_0(-b)/dx$, backward-differencing for $d\tilde{\varphi}_0(0)/dx$, and central-differencing for the derivatives in (21), the problem (19) - (21) was approximated by a system of linear equations, **AY** = **B**, where **A** is a tridiagonal band type matrix, **Y** is a column vector, and **B** is also a column vector. The subroutines given in the book of Press *et al.* (1992) were used to solve this matrix equation. In particular we are interested in $\tilde{\varphi}_0(-b)$, from which the reflection coefficient K_R is calculated using (13).

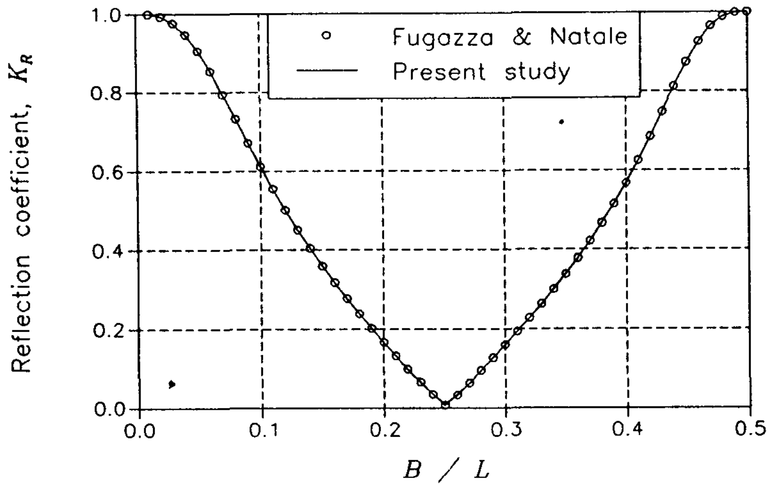

Figure 2: Variation of reflection coefficients w.r.t. B/L.

3 Comparison with Other Theory and Experimental Results

For a perforated-wall caisson breakwater situating on a flat bed, Fugazza and Natale (1992) showed that the resonance inside the wave chamber is important for wave reflection so that the reflection becomes minimum when $B/L = 0.25$ in which L is the wave length. Figure 2 shows the variation of reflection coefficient with respect to B/L calculated by the theory of Fugazza and Natale (1992) and the present theory. The porosity of the wall $r = 0.25$ was used. The present theory agrees exactly with that of Fugazza and Natale (1992).

In order to see the effect of the porosity of the wall upon the reflection coefficient, the contour plot of the reflection coefficient depending on the change of B/L and the porosity r is presented in Figure 3. The reflection coefficient approaches to 1.0 (perfect reflection) as r goes to either zero (solid wall) or unity (no perforated-wall), as it should do so, and it becomes minimum when $B/L = 0.25$ and $r = 0.25$. This result corresponds to other experimental results (personal communication with Dr. Shigeo Takahashi, Port and Harbour Research Institute, Japan).

Finally, to examine the performance of the present theory for a perforated-wall caisson breakwater mounted on a rubble foundation, the theory was compared against the experimental data reported by Park *et al.* (1993), who carried out hydraulic experiments for the reflection characteristics of perforated breakwaters with different types of perforated wall, i.e., vertical slits, horizontal slits, and circular holes. The porosity of the wall was 0.33 for all the types of the walls. Figure 4 shows the comparison between the theory and the experimental data. In the figure, L_c denotes the wave length inside the wave chamber. The theory slightly overpredicts the experimental data especially for a large wave height condition ($H = 0.1$ m), and the value of B/L_c for the minimum reflection coefficient in the calculation is somewhat greater than that in the experiment.

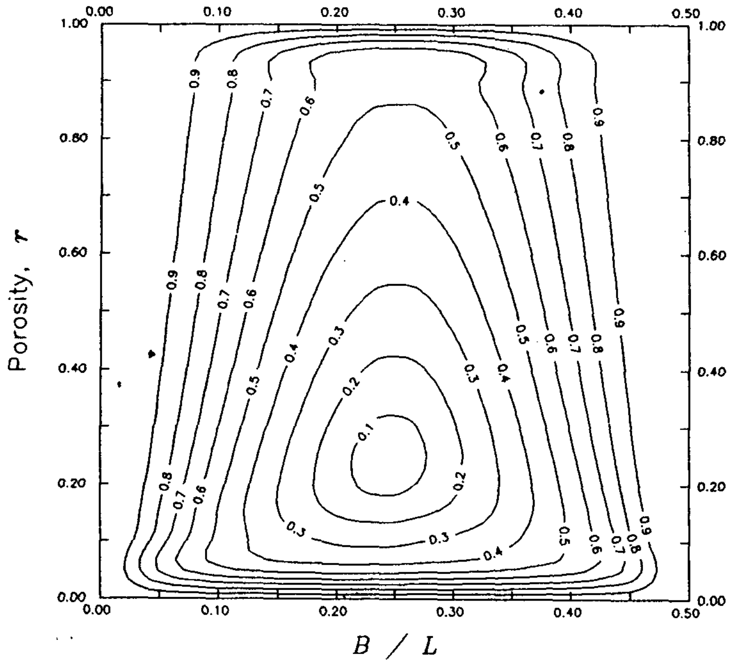

Figure 3: Contour plot of reflection coefficients w.r.t. B/L and r.

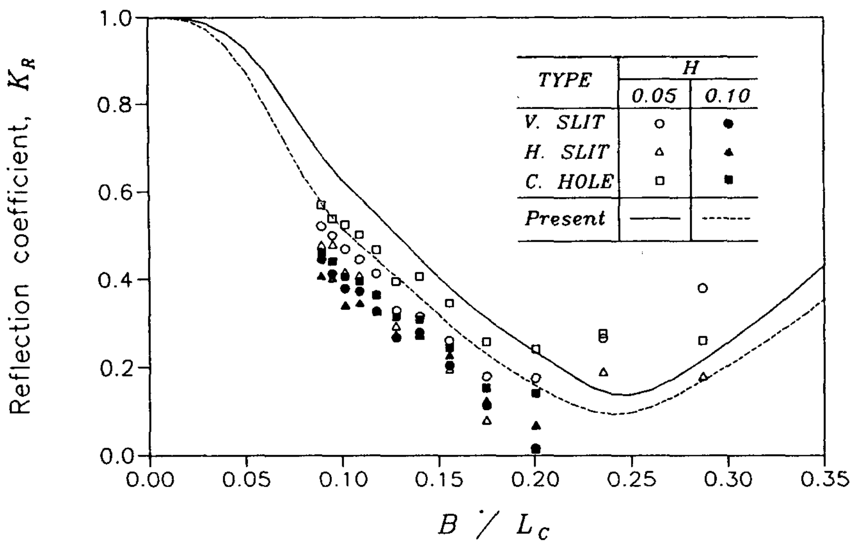

Figure 4: Comparison of reflection coefficients between theory and experimental data (ℓ = thickness of porous wall).

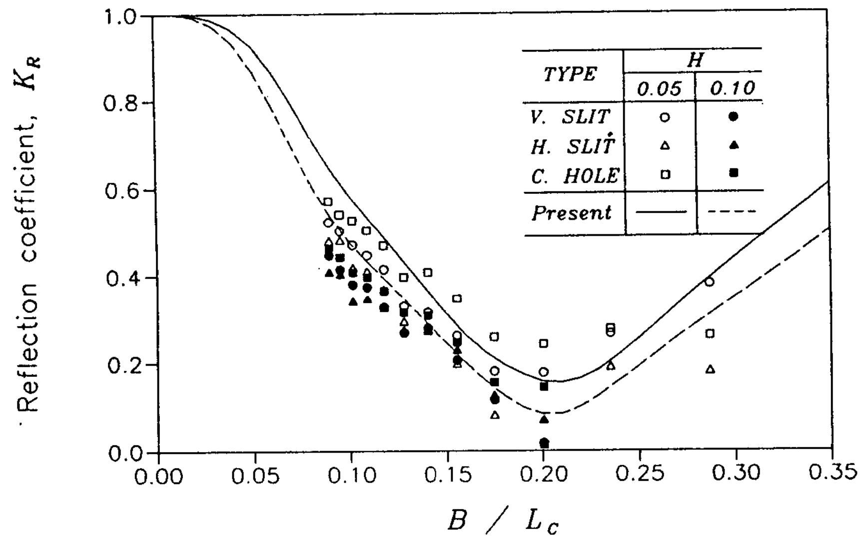

Figure 5: Comparison of reflection coefficients between theory and experimental data (ℓ = five times thickness of porous wall).

In the previous calculation to produce Figure 4, the length of the jet ℓ was taken the same as the thickness of the porous wall. Figure 5 is the results similar to Figure 4 when the length of the jet ℓ was taken as five times the thickness of the porous wall. The agreement between the theory and the experimental data is better than that in Figure 4, and the value of B/L_c for the minimum reflection coefficient is about 0.2 in both theory and experiment.

4 Conclusion

Using the Galerkin-eigenfunction method, a model was developed that can calculate the reflection coefficient from a perforated-wall caisson breakwater mounted on a rubble foundation. For a breakwater situating on a flat bed, the results of the proposed model were shown to exactly agree with the model developed by Fugazza and Natale (1992). For a breakwater mounted on a rubble mound foundation, the present model was tested against the hydraulic experimental data. The results show that the developed model is in reasonable agreement with observations. The present model is more versatile than that of Fugazza and Natale (1992) in that it can include the effect of rubble foundation and it can be applied to the case in which the waves are incident obliquely to the breakwater, even though the latter feature was not examined in this paper.

It was shown that a greater value of the length of the jet flowing through the porous wall which is usually taken the same as the thickness of the wall gives better agreement with the experimental data. It seems that the length of the jet should be related to the wave characteristics such as wave height and length. A further study for this may be needed.

REFERENCES

Canuto, C., Hussaini, M. Y., Quarteroni, A. and Zang, T. A. (1988). Spectral methods in fluid dynamics. Springer-Verlag. 557 pp.

Fugazza, M. and Natale, L. (1992). Hydraulic design of perforated breakwaters. J. Wtrway., Port, Coast. and Oc. Engrg., 118(1): 1-14.

Massel, S. R. (1993). Extended refraction-diffraction equation for surface waves. Coast. Engrg., 19: 97-126.

Park, W. S., Chun, I. S. and Lee, D. S. (1993). Hydraulic experiments for the reflection characteristics of perforated breakwaters. J. Korean Soc. of Coast. and Oc. Engrs., 5(3): 198-203.

Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. (1992). Numerical recipes in FORTRAN: the art of scientific computing. Cambridge University Press. 963 pp.

Tanimoto, K., Haranaka, S., Takahashi, S., Komatsu, K., Todoroki, M. and Osato, M. (1976). An experimental investigation of wave reflection, overtopping and wave forces for several types of breakwaters and sea walls. Tech. Note of Port and Harbour Res. Inst., Ministry of Transport, Japan, No. 246 (in Japanese).