'94 대한전기학회 추계학술대회 논문집(1994. 11. 18)

실 매개변수 불확실성을 가진 선형시스템의 외란 감소

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Disturbance Attenuation for Linear Systems with Real Parametric Uncertainties

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Abstract- This paper deals with the disturbance attenuation problem for linear systems with real parametric uncertainties. When there are time invariant parameter uncertainties whose sizes are bounded, a less conservative output feedback controller is constructed such that the closed loop system is asymptotically stable and achieves the prescribed disturbance attenuation level for all allowable parameter uncertainties. In order to demonstrate efficacy of the design method a numerical example is presented.

I. Introduction

When there are parameter uncertainties, the quadratic stabilization theory seems to be an effective means for designing a robust controller.[1][2] The problem of quadratic stabilization is to find a feedback controller such that the closed loop system is stable with a fixed and uncertainty-independent Lyapunov function. Xie and Souza consider the problem to design a controller such that the closed loop system is quadratically stable and achieves a prescribed level of disturbance attenuation.[3][4] But, their results may be conservative since they require a fixed Lyapunov function for all parameter uncertainties.

Very recently, the analysis results from absolute stability theory have been used to design a robust controller for systems with real parametric uncertainties.[5]-[7] The resulting state space stability criteria are in the form of Riccati equations and parameter dependent Lyapunov functions. Thus, the use of parameter dependent Lyapunov functions reduces conservatism for the real parameter robustness problem. Using a parameter dependent Lyapunov function, How et al. present a synthesis method for robust controllers that minimize an overbound of an H_2 performance objective.[7] However, in order to use the method given in [7] it is necessary to solve some coupled Riccati and Lyapunov type equations.

In this paper, we are going to attack the nearly same problem that Xie et al. considered in [4]. Xie et al. considered a linear system whose real parametric uncertainties (possibly time varying) are bounded. However, we consider a linear system with time invariant real parametric uncertainties whose sizes are bounded. Up to present time, an effective method to solve the disturbance attenuation problem for linear system with time invariant parametric uncertainties may be to use the method presented by Xie et al even if the wider class of parameter uncertainties is considered by them. When parameter uncertainties are time invariant, we will present a less conservative design method than that suggested by Xie et al.

The notation is standard. R denotes the set of real numbers, R^n denotes the n dimensional Euclidean space identified with $n*1$ vectors of real numbers, R^{n*m} denotes the set of all real matrices, and $Re(\cdot)$ denotes the real part. We denote the identity matrix by I and the zero matrix by 0. When it is not clear to know the dimension of the matrix from the context, we will use 0_{n*m} and I_m to denote the $n*m$ zero matrix and the $m*m$ identity matrix respectively. We will use A^T to denote the transpose of the matrix A. For symmetric matrices Q_1 and Q_2, $Q_1>Q_2$ ($Q_1\geq Q_2$) if Q_1-Q_2 is positive definite(positive semi-definite). Finally, matrix A is called stable if all its eigenvalues $(\lambda_i(A))$ have strictly negative real parts, and $\rho(A)$ denotes the spectral radius of A and $j=\sqrt{-1}$. $\|z\|_2$ denotes the usual L_2 norm of z, i.e. $\|z\|_2^2=\int_0^\infty z^Tz\,dt$. We say that $z\in L_2[0,\infty]$ if $\|z\|_2<\infty$.

II. Preliminaries

Consider a finite dimensional linear system described by

$$\dot{x} = Ax + Bu \qquad\qquad (1)$$
$$y = Cx + Du$$

where $A\in R^{n*n}$, $B\in R^{n*m}$, $C\in R^{p*n}$, $D\in R^{p*m}$ and $p=m$.

Let $G(s)=C(sI-A)^{-1}B+D$.

Definition :

1. The system (1) is said to be strictly positive real if its transfer function $G(s)$ is analytic in $Re(s)\geq 0$ and satisfies $G(j\omega)+G^T(-j\omega)>0$ for $\omega\in[0,\infty)$.
2. The system (1) is extended strictly positive real(ESPR) if it is strictly positive real and $D+D^T>0$.

With the aid of the following lemma, one can easily check whether the system (1) is ESPR or not.

Lemma 1 : (Sun et al. [8]) Define the following algebraic Riccati inequality(ARI)

$$A^TX+XA+(C-B^TX)^T(D+D^T)^{-1}(C-B^TX)<0 \qquad (2)$$

Then the following statements are equivalent.

i) The system (1) is ESPR and A is stable.

ii) The matrix $D+D^T$ is positive definite and the ARI (2) has a solution $X>0$.

Now, we are ready to state the ESPR control problem which is to synthesize a dynamic output feedback controller such that the closed loop system is ESPR. Consider the plant given by

$$\dot{x} = Ax + B_1w + B_2u$$
$$z_1 = C_1x + D_{11}w + D_{12}u \qquad (3)$$
$$y = C_2x + D_{21}w$$

where $A\in R^{n*n}$, $B_1\in R^{n*m1}$, $B_2\in R^{n*m2}$, $C_1\in R^{p1*n}$, $D_{11}\in R^{p1*m1}$, $D_{12}\in R^{p1*m2}$, $C_2\in R^{p2*n}$, $D_{21}\in R^{p2*m1}$ and $m_2\leq p_1, p_2\leq m_1$, $p_1=m_1$.

We briefly introduce a controller design method for the ESPR control problem.

Define

$$\begin{aligned} R(X) = & [A-B_1C_1-(B_2-B_1D_{12})(D_{12}^TD_{12})^{-1}D_{12}^TC_1]^TX \\ & +X[A-B_1C_1-(B_2-B_1D_{12})(D_{12}^TD_{12})^{-1}D_{12}^TC_1] \\ & +X[B_1B_1^T-(B_2-B_1D_{12})(D_{12}^TD_{12})^{-1}(B_2-B_1D_{12})^T]X \\ & +C_1^T[I-D_{12}(D_{12}^TD_{12})^{-1}D_{12}^T]C_1 \end{aligned} \qquad (4)$$

$$\begin{aligned} S(Y) = & [A-B_1C_1-B_1D_{21}^T(D_{21}^TD_{21})^{-1}(C_2-D_{21}C_1)]Y \\ & +Y[A-B_1C_1-B_1D_{21}^T(D_{21}^TD_{21})^{-1}(C_2-D_{21}C_1)]^T \\ & +Y[C_1^TC_1-(C_2-D_{21}C_1)^T(D_{21}D_{21}^T)^{-1}(C_2-D_{21}C_1)^T]Y \\ & +B_1^T[I-D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}]B_1^T \end{aligned} \qquad (5)$$

$$F(X) = -(D_{12}^T D_{12})^{-1}(B_2^T X + D_{12}^T(C_1 - B_1^T X))$$
$$L(Y) = -(YC_2^T + (B_1 - YC_1^T)D_{21}^T)(D_{21}D_{21}^T)^{-1} \tag{6}$$

Theorem 2 : (Sun et al. [8]) Assume that i) (A, B_2) and (C_2, A) are stabilizable and detectable respectively and ii) $D_{12}^T D_{12}$ and $D_{21}D_{21}^T$ are nonsingular. Then there exists a strictly proper controller such that the closed loop system is ESPR if and only if

i) The ARI $R(X)<0$ has a positive solution X_I,

ii) The ARI $S(Y)<0$ has a positive solution Y_I,

iii) The spectral radius $\rho(Y_I X_I)<1$.

Moreover, when these conditions are satisfied the controller $\dot{x}_k = A_k x_k + B_k y$, $u = C_k x_k$ achieves the ESPR control problem, where

$$\begin{aligned} A_k &= A + B_2 F_I + (I - Y_I X_I)^{-1} L_I C_2 + \Delta_I, \\ B_k &= -(I - Y_I X_I)^{-1} L_I, \quad C_k = F_I, \\ F_I &= F(X_I), \quad L_I = L(Y_I), \\ \Delta_I &= -(B_1 + (I - Y_I X_I)^{-1} L_I D_{21})(C_1 - B_1^T X_I + D_{12} F_I) \\ &\quad + (I - Y_I X_I)^{-1} Y_I R(X_I). \end{aligned} \tag{7}$$

Remark 1 : When $D_{12}^T D_{12}$ or $D_{21}^T D_{21}$ is singular, one can also use Theorem 2 by invoking perturbation arguments.

III. Problem Statement

We consider the class of uncertain linear systems described by state space models of the form

$$\begin{aligned} \dot{x} &= (A + \Delta A)\, x + B_1 w_1 + (B_2 + \Delta B)\, u \\ z_1 &= C_1 x + D_{12} u \\ y &= (C_2 + \Delta C)\, x + D_{21} w_1 + (D_{22} + \Delta D)\, u \end{aligned} \tag{8}$$

where $x \in R^n$ is the state, $u \in R^{m2}$ is the control input, $w_1 \in R^{m1}$ is the disturbance input, $y \in R^{p2}$ is the measured output, $z_1 \in R^{p1}$ is the controlled output, A, B_1, B_2, C_1, C_2, D_{12}, D_{21} and D_{22} are real constant matrices of appropriate dimensions that describe the nominal system and ΔA, ΔB, ΔC and ΔD are real valued matrix functions representing parametric uncertainties. The parametric uncertainties are of the form

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} B_0 \\ D_{20} \end{bmatrix} F [\, C_0 \;\; D_{02} \,] \tag{9}$$

where $B_0 \in R^{n*m0}$, $D_{20} \in R^{p2*m0}$, $C_0 \in R^{m0*n}$, $D_{02} \in R^{m0*m2}$ are known constant matrices denoting the structure of the parametric uncertainties. Without loss of generality, we assume $F \in R^{m0*m0}$ is an unknown diagonal matrix satisfying

$$0 \le F \le M \tag{10}$$

where M is an invertible diagonal matrix.

Remark 2 : Suppose that F satisfies the following general bound condition $M_1 \le F \le M_2$. Then,

$$\begin{bmatrix} A + \Delta A & B_2 + \Delta B \\ C_2 + \Delta C & D_{22} + \Delta D \end{bmatrix} = \begin{bmatrix} A + B_0 M_1 C_0 + \Delta\tilde{A} & B_2 + B_0 M_1 D_{02} + \Delta\tilde{B} \\ C_2 + D_{20} M_1 C_0 + \Delta\tilde{C} & D_{22} + D_{20} M_1 D_{02} + \Delta\tilde{D} \end{bmatrix}$$

where

$$\begin{bmatrix} \Delta\tilde{A} & \Delta\tilde{B} \\ \Delta\tilde{C} & \Delta\tilde{D} \end{bmatrix} = \begin{bmatrix} B_0 \\ D_{20} \end{bmatrix} \tilde{F} [\, C_0 \;\; D_{02} \,],$$
$$0 \le \tilde{F} \le M_2 - M_1$$

Hence one can recover the same kind of condition (10) if A, B_2, C_2, D_{22} and M are replaced with $A + B_0 M_1 C_0$, $B_2 + B_0 M_1 D_{02}$, $C_2 + D_{20} M_1 C_0$, $D_{22} + D_{20} M_1 D_{02}$ and $M_2 - M_1$ respectively.

Also we have the following assumption.

Assumption :

1) (A, B_2) is stabilizable.
2) (C_2, A) is detectable.
3) $D_{12}^T D_{12}$ and $D_{21} D_{21}^T$ are invertible.

The design objective is to synthesize a linear dynamic output feedback controller for the system (8) such that the closed loop system is asymptotically stable and $\| z_1 \|_2 < \gamma \| w_1 \|_2$ for any nonzero $w_1 \in L_2[0, \infty]$ where γ is a prescribed level of disturbance attenuation.

IV. Stability with H_∞ Norm Bound

In connection with the system (8), we introduce an equivalent system given by

$$\begin{aligned} \dot{x} &= A\, x + B_0 w_0 + B_1 w_1 + B_2 u \\ z_0 &= C_0 x + D_{02} u \\ z_1 &= C_1 x + D_{12} u \\ y &= C_2 x + D_{20} w_0 + D_{21} w_1 + D_{22} u \end{aligned} \tag{11}$$

where $w_0 = F z_0$.

Suppose that an output feedback controller described by

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k y \\ u &= C_k x_k \end{aligned} \tag{12}$$

is applied to the system (11). Then the closed loop system becomes

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_{0c} w_0 + B_{1c} w_1 \\ z_0 &= C_{0c} x_c \\ z_1 &= C_{1c} x_c \end{aligned} \tag{13}$$

where $x_c = \begin{bmatrix} x \\ x_k \end{bmatrix} \in R^{n_c}$, $A_c = \begin{bmatrix} A & B_2 C_k \\ B_k C_2 & A_k + B_k D_{22} C_k \end{bmatrix}$, $B_{0c} = \begin{bmatrix} B_0 \\ B_k D_{20} \end{bmatrix}$, $B_{1c} = \begin{bmatrix} B_1 \\ B_k D_{21} \end{bmatrix}$, $C_{0c} = [\, C_0 \;\; D_{02} C_k \,]$ and $C_{1c} = [\, C_1 \;\; D_{12} C_k \,]$.

For convenience, we define

$$B_c = [\, B_{0c} \;\; B_{1c} \,], \quad C_c = \begin{bmatrix} C_0 \\ 0_{m1*nc} \end{bmatrix}, \quad \tilde{N} = \begin{bmatrix} N & 0_{m0*m1} \\ 0_{m1*m0} & 0_{m1*m1} \end{bmatrix},$$
$$\tilde{D} = \begin{bmatrix} D & 0_{m0*m1} \\ 0_{m1*m0} & 0_{m1*m1} \end{bmatrix} \text{ and } R_c = \begin{bmatrix} DM^{-1} & 0 \\ 0 & 1/2\gamma^2 I_{m1} \end{bmatrix} - \tilde{N} C_c B_c \tag{14}$$

for some $m_0 * m_0$ diagonal matrices $N \ge 0$ and $D > 0$. In order to proceed further, it is necessary to state a technical lemma which is useful in the proof of Lemma 4.

Lemma 3 : Given any real matrix Y and any real symmetric $X>0$, $Z>0$ such that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$.

Then we have $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}^{-1} - \begin{bmatrix} X^{-1} & 0 \\ 0 & 0 \end{bmatrix} \ge 0$.

(proof) Using the block matrix inversion formula given in [9],

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}^{-1} = \begin{bmatrix} X^{-1} + X^{-1} Y \Delta^{-1} Y^T X^{-1} & -X^{-1} Y \Delta^{-1} \\ -\Delta^{-1} Y^T X^{-1} & \Delta^{-1} \end{bmatrix} \tag{15}$$

where $\Delta = Z - Y^T X^{-1} Y$.

Since $\Delta > 0$,

$$\begin{aligned} \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}^{-1} - \begin{bmatrix} X^{-1} & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} X^{-1} Y \Delta^{-1} Y^T X^{-1} & -X^{-1} Y \Delta^{-1} \\ -\Delta^{-1} Y^T X^{-1} & \Delta^{-1} \end{bmatrix} \\ &= \begin{bmatrix} -X^{-1} Y \\ I \end{bmatrix} \Delta^{-1} [\, -Y^T X^{-1} \;\; I \,] \ge 0 \end{aligned} \tag{16}$$

This completes the proof. **Q.E.D.**

Lemma 4 : Suppose that there exist a symmetric $P>0$, $N \ge 0$ and $D>0$ such that

i) $R_c + R_c^T > 0$ (17)

ii) $A_c^T P + P A_c + C_{1c}^T C_{1c} + (B_c^T P + \tilde{N} C_c A_c + \tilde{D} C_c)^T (R_c + R_c^T)^{-1} (B_c^T P + \tilde{N} C_c A_c + \tilde{D} C_c) < 0$ (18)

Then, for all admissible F satifying (10),

i) the closed loop system (13) is asymptotically stable and

ii) the controlled output z_1 satisfies $\| z_1 \|_2 < \gamma \| w_1 \|_2$ for any nonzero $w_1 \in L_2[0, \infty]$.

(proof) In order to prove asymptotic stability, we consider the candidate Lyapunov function

$$V = x_c^T P x_c + z_0^T F N z_0 \tag{19}$$

which is obviously positive definite. Note that our Lyapunov function candidate depends on unknown parametric uncertainties. Along the trajectories with $w_1 \equiv 0$, we have

$$\begin{aligned} \frac{dV}{dt} &= x_c^T (A_c^T P + P A_c) x_c + 2 w_0^T B_{0c}^T P x_c N z_0 + 2 w_0^T N \frac{dz_0}{dt} \\ &\le x_c^T (A_c^T P + P A_c) x_c + 2 w_0^T B_{0c}^T P x_c \\ &\quad + 2 w_0^T N C_{0c} (A_c x_c + B_{0c} w_0) + 2 w_0^T D (z_0 - M^{-1} w_0) \\ &= x_c^T (A_c^T P + P A_c) x_c + 2 w_0^T (B_{0c}^T P + N C_{0c} A_c + D C_{0c}) x_c \\ &\quad - 2 w_0^T (D M^{-1} - N C_{0c} B_{0c}) w_0 \end{aligned} \tag{20}$$

Define $R_0 = DM^{-1} - N C_{0c} B_{0c}$.

Since $R_c + R_c^T = \begin{bmatrix} R_0 + R_0^T & -N C_{0c} B_{1c} \\ -B_{1c}^T C_{0c}^T N & \gamma^2 I_{m1} \end{bmatrix} > 0$, $R_0 + R_0^T > 0$. By completion of square in (20) we have

$$\frac{dV}{dt} \le x_c^T[A_c^TP+PA_c+(B_{0c}^TP+NC_{0c}A_c+DC_{0c})^T(R_0+R_0^T)^{-1}(B_{0c}^TP+NC_{0c}A_c+DC_{0c})]x_c$$
$$-[w_0-(R_0+R_0^T)^{-1}(B_{0c}^TP+NC_{0c}A_c+DC_{0c})x_c]^T(R_0+R_0^T)[w_0-(R_0+R_0^T)^{-1}(B_{0c}^TP+NC_{0c}A_c+DC_{0c})x_c]$$
$$\le x_c^T[A_c^TP+PA_c+(B_{0c}^TP+NC_{0c}A_c+DC_{0c})^T(R_0+R_0^T)^{-1}(B_{0c}^TP+NC_{0c}A_c+DC_{0c})]x_c \qquad (21)$$

Using Lemma 3 and

$$B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c=\begin{bmatrix} B_{0c}^TP+NC_{0c}A_c+DC_{0c} \\ B_{1c}^TP \end{bmatrix},$$
$$(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)^T(R_c+R_c^T)^{-1}(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)-(B_{0c}^TP+NC_{0c}A_c+DC_{0c})^T(R_0+R_0^T)^{-1}(B_{0c}^TP+NC_{0c}A_c+DC_{0c})$$
$$=(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)^T\left((R_c+R_c^T)^{-1}-\begin{bmatrix}(R_0+R_0^T)^{-1} & 0 \\ 0 & 0\end{bmatrix}\right)(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c) \ge 0 \qquad (22)$$

Accordingly, we can conclude that the unforced system is asymptotically stable since $\frac{dV}{dt}<0$ from (18) and (22).

Finally to prove disturbance attenuation, we assume that $x_c(0)=0$ and we introduce

$$J=\int_0^\infty (z_1^Tz_1-\gamma^2w_1^Tw_1)dt. \qquad (23)$$

Similarly as in the proof of asymptotic stability part,

$$J\le \int_0^\infty\Big[z_1^Tz_1-\gamma^2w_1^Tw_1+\frac{d}{dt}(x_c^TPx_c+z_0^TFNz_0)+2w_0^TD(z_0-M^{-1}w_0)\Big]dt-x_c(\infty)^T(P+C_{0c}^TFNC_{0c})x_c(\infty)$$
$$\le \int_0^\infty\Big[z_1^Tz_1-\gamma^2w_1^Tw_1+\frac{d}{dt}(x_c^TPx_c)+2w_0^TN\frac{dz_0}{dt}+2w_0^TD(z_0-M^{-1}w_0)\Big]dt \qquad (24)$$
$$=\int_0^\infty H(x_c,w_0,w_1)dt$$

where

$$H(x_c,w_0,w_1)=x_c^T(A_c^TP+PA_c+C_{1c}^TC_{1c})x_c+2w^T(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)x_c-2w^T(R_c+R_c^T)w \qquad (25)$$

and $w=[w_0^T\ w_1^T]^T$.

After completion of square, we obtain

$$H(x_c,w_0,w_1)\le x_c^T[A_c^TP+PA_c+C_{1c}^TC_{1c}+(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)^T(R_c+R_c^T)^{-1}(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)]x_c \qquad (26)$$

Thus, $J<0$ and this completes the proof. Q.E.D.

Remark 3: Suppose that $F^TF\le\rho^2I$. This implies that $-\rho I\le F\le \rho I$. If we specialize Lemma 4 to the case where $N=0$ and $D=\frac{1}{\varepsilon\rho}I$ for a positive constant ε, $R_c+R_c^T$ is always positive definite and (18) becomes

$$A_c^TP+PA_c+\gamma^{-2}PB_{1c}B_{1c}^TP+\varepsilon\rho^2PB_{0c}B_{0c}^TP+1/\varepsilon\, C_{0c}^TC_{0c}+C_{1c}^TC_{1c}<0 \qquad (27)$$

Note that (27) is just the same condition derived by Xie et al.(see Lemma 3.1 in [4]). Even if there don't exist $P>0$ and $\varepsilon>0$ such that (27) holds, there may exist $P>0$, $N\ge0$ and $D>0$ such that *our sufficient conditions are satisfied*. Thus, we can expect that our conditions in Lemma 4 are less conservative than that given in [4].

V. Controller Synthesis

In this section we present a controller synthesis method by interconnecting our control problem with the ESPR control problem whose solution is presented in section II.

5.1 The case where $D_{02}=0$ and $D_{22}=0$

The fact that $D_{02}=0$ and $D_{22}=0$ implies that the original plant (8) is strictly proper and $\Delta B=0$. Define an auxiliary system from the equivalent system (11) as follows.

$$\dot{x} = \hat{A}\,x + \hat{B}_1\,\hat{w} + \hat{B}_2\,u$$
$$z = \hat{C}_1\,x + \hat{D}_{11}\,\hat{w} + \hat{D}_{12}\,u \qquad (28)$$
$$y = \hat{C}_2\,x + \hat{D}_{21}\,\hat{w}$$

where $B=[B_0\ B_1]$, $C=\begin{bmatrix}C_0 \\ 0_{m1\times n}\end{bmatrix}$, $R=\begin{bmatrix}DM^{-1} & 0 \\ 0 & 1/2\gamma^2I_{m1}\end{bmatrix}-\tilde{N}CB$,
$\hat{A}=A$, $\hat{B}_1=[B(R+R^T)^{-1/2}\ 0_{n\times p1}]$, $\hat{B}_2=B_2$,
$\hat{C}_1=-\begin{bmatrix}(R+R^T)^{-1/2}(\tilde{N}CA+\tilde{D}C) \\ C_1\end{bmatrix}$, $\hat{D}_{11}=1/2I$,
$\hat{D}_{12}=-\begin{bmatrix}(R+R^T)^{-1/2}\tilde{N}CB_2 \\ D_{12}\end{bmatrix}$, $\hat{C}_2=C_2$,
$\hat{D}_{21}=[\,[D_{20}\ D_{21}](R+R^T)^{-1/2}\ 0_{p2\times p1}]$

Note that the system (28) satifies the conditions given in Lemma 2.

Theorem 5: Suppose that there exist $N\ge0$ and $D>0$ such that i) $R+R^T>0$ and ii) the ESPR control problem is solvable for the auxiliary system (28). If the controller from the above ESPR control problem is applied to the equivalent system (11), then the resulting closed loop system is asymptotically stable and furthermore $\|z_1\|_2<\gamma\|w_1\|_2$ for any nonzero $w_1\in L_2[0,\infty)$.

(proof) Let the controller designed from the ESPR control problem is given by

$$\dot{x}_k = A_k\,x_k + B_k\,y \qquad (29)$$
$$u = C_k\,x_k$$

When the controller (29) is applied to the auxiliary system (28), the closed loop system is ESPR and represented as.

$$\dot{x}_c = \hat{A}_c\,x_c + \hat{B}_c\,\hat{w} \qquad (30)$$
$$z = \hat{C}_c\,x_c + \hat{D}_c\,\hat{w}$$

where $\hat{A}_c=\begin{bmatrix}\hat{A} & \hat{B}_2C_k \\ B_k\hat{C}_2 & A_k\end{bmatrix}$, $\hat{B}_c=\begin{bmatrix}\hat{B}_1 \\ B_k\hat{D}_{21}\end{bmatrix}$,
$\hat{C}_c=[\ \hat{C}_1\ \ \hat{D}_{12}C_k]$, $\hat{D}_c=\hat{D}_{11}$.

Since the closed loop system (30) is ESPR we have

$$\hat{A}_c^TP+P\hat{A}_c+(\hat{C}_c-\hat{B}_c^TP)^T(\hat{C}_c-\hat{B}_c^TP)<0 \qquad (31)$$

for some $P>0$.

Suppose that the controller (29) is applied to the equivalent system (11). Since D_{02} and D_{22} are assumed to be zero, the closed loop system is given by (13) with $A_c=\begin{bmatrix}A & B_2C_k \\ B_kC_2 & A_k\end{bmatrix}$ and $C_{0c}=[C_0\ 0]$.

Hence, in order to prove the theorem it suffices to prove that the conditions given in Lemma 4 are satisfied. Since

$$R_c=\begin{bmatrix}DM^{-1} & 0 \\ 0 & 1/2\gamma^2I\end{bmatrix}-\tilde{N}C_cB_c$$
$$=\begin{bmatrix}DM^{-1} & 0 \\ 0 & 1/2\gamma^2I\end{bmatrix}-\begin{bmatrix}N & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}C_0 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}B_0 & B_1 \\ B_kD_{20} & B_kD_{21}\end{bmatrix}=R \qquad (32)$$

$R+R^T>0$ implies $R_c+R_c^T>0$.

Using simple block matrix manipulations and definitions of $\hat{A}_c$, $\hat{B}_c$ and $\hat{C}_c$ it is easy to see that

$$A_c^TP+PA_c+C_{1c}^TC_{1c}+(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)^T(R_c+R_c^T)^{-1}(B_c^TP+\tilde{N}C_cA_c+\tilde{D}C_c)$$
$$=\hat{A}_c^TP+P\hat{A}_c+(\hat{C}_c-\hat{B}_c^TP)^T(\hat{C}_c-\hat{B}_c^TP)<0 \qquad (33)$$

This completes the proof. Q.E.D.

5.2 The case where D_{02} and D_{22} are not necessarily zero

One can not synthesize a controller by directly following the proof of Theorem 5 since $R_c\ne R$ in this case. Thus, in order to check $R_c+R_c^T>0$, B_k must be known. Accordingly we are going to use an augmentation method so that an output feedback controller can be constructed by directly following the method given in section 5.1. The augmentation scheme is shown in Fig. 1.

In Fig. 1, an m_2*m_2 strictly proper stable transfer function $Q(s)$ is connected to the input channel. For simplicity, let $Q(s)=\frac{\alpha}{s+\alpha}I$ where α is a positive constant. Then the augmented system $G_a(s)$ in Fig. 1 can be represented as

$$\dot{x}_a = A_a\,x_a + B_{0a}\,w_0 + B_{1a}\,w_1 + B_{2a}\,\bar{u}$$
$$z_0 = C_{0a}\,x_a + D_{02a}\,\bar{u} \qquad (34)$$
$$z_1 = C_{1a}\,x_a + D_{12a}\,\bar{u}$$
$$y = C_{2a}\,x_a + D_{20a}\,w_0 + D_{21a}\,w_1 + D_{22a}\,\bar{u}$$

where

$$x_a \in R^{n+m2},\ A_a=\begin{bmatrix} A & B_2 \\ 0 & -\alpha I \end{bmatrix},\ B_{0a}=\begin{bmatrix} B_0 \\ 0 \end{bmatrix},\ B_{1a}=\begin{bmatrix} B_1 \\ 0 \end{bmatrix},\ B_{2a}=\begin{bmatrix} 0 \\ \alpha I \end{bmatrix},$$

$$C_{0a}=[\,C_0\ \ D_{02}],\ D_{02a}=0,\ C_{1a}=[\,C_1\ \ D_{12}],\ D_{12a}=0,$$

$$C_{2a}=[\,C_2\ \ D_{22}],\ D_{20a}=D_{20},\ D_{21a}=D_{21},\ D_{22a}=0$$

Now, the system (34) is in the form of the system considered in section 5.1 so that we can construct $n+m_2$ dimensional controller $\bar{u}=K(s)y$. Finally we can synthesize an $n+2m_2$ dimensional output feedback controller $u=Q(s)K(s)y$. One may use one of the controller reduction methods in order to reduce the dimension of the controller.

VI Illustrated Example

In order to demonstrate our design method we present a simple numerical example. In (8) we assume that our system is given by

$$A=\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix},\ B_1=\begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix},\ B_2=\begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_1=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ D_{12}=\begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_2=[5\ \ 5],\ D_{21}=[0\ \ 1], D_{22}=0,\ \Delta A=\begin{bmatrix} f & f \\ 0 & 0 \end{bmatrix},\ \Delta B=\begin{bmatrix} f \\ 0 \end{bmatrix},$$

$$\Delta C=[0\ \ 0],\ \Delta D=0$$

where f is an unknown but constant uncertain parameter. We choose the prescribed level of disturbance attenuation $\gamma=4$. Hence the structure matrix of the parameter uncertainty B_0, C_0, D_{20} and D_{02} can be obtained by

$$B_0=\begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C_0=[1\ \ 1],\ D_{20}=0,\ D_{02}=1.$$

Let $-M_0 \le f \le M_0$. Note that A, B_2 and M must be replaced with $A-B_0M_0C_0$, $B_2-B_0M_0D_{02}$ and $2M_0$ respectively in constructing (28). We choose $\alpha=10000$ in obtaining the augmented system (34). By using a search technique, we are going to find the maximum M_0 such that our design method can give an admissible controller equation. When $M_0=0.5379$ we can construct an output feedback controller by choosing $N=1.579$ and $D=8.316$. The resulting controller $Q(s)K(s)$ in Fig. 1 is given by

$$Q(s)K(s)=\frac{-10000(8.732s^2+87320s+258)}{(s+10000)(s^3+9847s^2+117660s+349)}. \quad (35)$$

In order to reduce the controller dimension we use the balance and truncate method which yields the second order controller given by

$$K_r(s)=-\frac{8.878s+0.0265}{s^2+11.96s+0.03583}. \quad (36)$$

When $Q(s)K(s)$ and $K_r(s)$ are applied to the system (8), the resulting H_∞ norm of the closed loop system are depicted in Fig. 2.

In order to illustrate that our design method is less conservative, we also synthesize a controller by using the Xie's method presented in [4]. When $M_0>0.5172$ we can't design an admissible controller by the Xie's method. The reason is that the Xie's method is developed based on the quadratic stabilization theory which is somewhat conservative while our method is developed using an uncertainty dependent Lyapunov-like function.

VII Conclusions

In this paper, we have considered a disturbance attenuation problem for linear system with parametric uncertainties. Based on the uncertain parameter dependent Lyapunov function, we have derived a sufficient condition which guarantees that the closed loop system is asymptotically stable and achieves the prescribed level of disturbance attenuation for all allowable parametric uncertainties. Next, we also have constructed an output feedback controller achieving our design goal. It has been shown that our design method is less conservative than that suggested in [4].

References

[1] B. R. Barmish, "Necessary and sufficient conditions for quadratic stabilizability of an uncertain system", J. Optimiz. Theory Appl., vol. 46, no. 4, pp.399-408, 1985

[2] G. Leitmann, "Guaranteed asymptotic stability for some linear systems with bounded uncertainties", J. Dynamics. Syst. Meas., Control, vol. 101, pp.212-216, 1979.

[3] L. Xie and C. E. de Souza, "Robust H_∞ control for linear systems with norm-bounded time varying uncertainties", IEEE Trans. Automatic Control, vol. 37, no. 8, pp.1188-1191, 1992.

[4] L. Xie, M. Fu and C. E. de Souza, "H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback", IEEE Trans. Automatic Control, vol. 37, no. 8, 1992.

[5] E. G. Collins, W. M. Haddad and L. D. Davis, "Riccati equation approaches for robust stability and performance analysis using the small gain, positivity and Popov theorems", Proceedings of the American Control Conference, pp.1079-1083, San Francisco, California, June 1993.

[6] J. P. How and S. R. Hall, "Connections between the Popov stability criterion and bounds for real parameter uncertainty", Proceedings of the American Control Conference, pp.1084-1089, San Francisco, California, June 1993.

[7] J. P. How, W. M. Haddad and S. R. Hall, "Robust control synthesis examples with real parameter uncertainty using the Popov criterion", Proceedings of the American Control Conference, pp.1090-1095, San Francisco, California, June 1993.

[8] W. Sun, P. P. Khargonekar and D. Shim, "Controller synthesis to render a closed loop transfer function positive real", Proceedings of the American Control Conference, pp.1074-1078, San Francisco, California, June 1993.

[9] T. Kailath, "Linear Systems", Prentice-Hall, Inc., Englewood Cliffs, N.J., 1980.

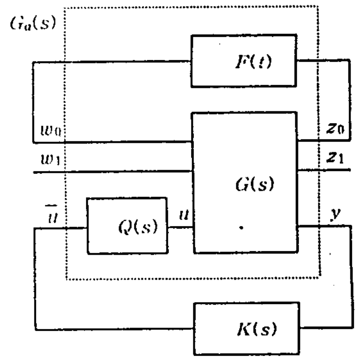

Fig. 1 Augmented System

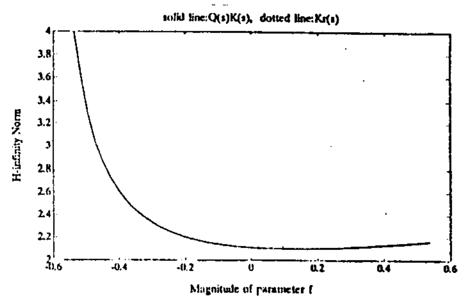

Fig. 2 H_∞ norm of the closed loop system