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Mutual Inductance of the Non-Coaxial Coils in Coil Gun

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Abstract - The purpose of this paper is to describe the mutual inductance between two non-coaxial circular coils in coil gun. As being in many electromagnetic applications, one of them is fixed and the other one is moving and, being not supported, its axis may not coincide with the axis of the fixed coil. This paper presents a method for the calculation of the mutual inductance in case of non-coaxial coupled coils, the characteristics of this inductance and experimental results. For the computation, the complete elliptic integrals formula and mesh matrix technique were introduced. This method enables accurate results from relatively simple procedure and calculation program.

1. LUMPED CIRCUIT MODEL AND SYSTEM EQUATIONS

The poly-phase Coil Gun is consist of drive coils which are linear array of circular coils and a projectile which has a finite length of continuos conductive tube. In capacitor-driven coil gun, the energy to the coil gun is supplied by pre-charged external capacitor bank.[1] In conductive part of projectile, called sleeve, induced current is flowing due to the traveling magnetic flux wave of the drive coils. Since the axial distribution of the induced current in the sleeve is not uniform, the sleeve is virtually divided into several zones electrically separated from one another for the electrical analysis. Each zone of the sleeve is called sleeve coil and number of the virtual sleeve coil is depends on the requirement of accuracy. By introducing the lumped parameter circuit model of the capacitor-driven coil gun, one can establish the system equations in matrix form ; [2]

$$[C]\frac{d}{dt}[V_C]=-[I_d]$$

$$\{[L]+[M]\}\frac{d}{dt}[I]=[V]-[R][I]-v_P[G][I]$$

$$m_p\frac{dv_p}{dt}=\sum_{p=1}^{N_p}\sum_{d=1}^{N_d}[I_p][I_d]\frac{d[M]}{dz}$$

$$v_p=\frac{dz}{dt}$$

$$F=\frac{1}{2}[I]^T[G][I]$$

$$[G]=\frac{d[M]}{dz}$$

where, m_p refers to the mass of projectile, v_p to the z-axial velocity of projectile, z is the distance traveled by the projectile, [R] to the matrix of resistance in the drive and sleeve coil circuits, $[C]$ to the matrix of the drive circuit capacitors, $[L]$ to the matrix of self inductances in the drive and sleeve coil, [M] to the matrix of mutual inductances and $[I]$ to the matrix of the drive and sleeve coil currents.

As shown in these equations, one can find that mutual inductance is the principal factor for system analysis and the accurate calculation of the mutual inductance is very important. Various calculations have been presented in case of coaxial coils couple.[5] However, since the projectile in coil gun is not supported by any other physical means except air, the projectile can be off-centered from the z-axis of the barrel due to main possible reasons.[1] In this case, radial distance between the drive and sleeve coil couple is not symmetric and the mutual inductance between the couple is different from that in case of coaxial.

2. CALCULATION WITH THE COMPLETE ELLIPTIC INTEGRALS AND MESH-MATRIX METHOD

One couple of the non-coaxial drive and sleeve coils is modeled in Fig. 1. The drive and sleeve coils have the dimensions shown in the picture; the thickness of the sleeve is assumed to be relatively thin. The drive coils of coil guns usually have multi-turn windings and, in this analysis, the winding turns in the drive coil is denoted N_d. Therefore, it is reasonable to assume that the current distribution in the drive coil is uniform. The winding turns in the sleeve coil is denoted N_p. Since the sleeves in coil guns are made of continuous and rigid aluminum tube, turns in sleeve coil is assumed one. The coils used in coil guns are not filamenatary. Instead, they have sectional areas. In this case, the drive and sleeve coils are considered as a mesh matrix of filamentary coils as shown in Fig.1. [1],[3],[4] In Fig.1, sectional area of the drive coil is divided into (2M+1) by (2N+1) meshes and that of the sleeve coil is divided into (2m+1) by (2n+1). If the cross-section of the coils is not rectangular, e.g. circular, the current densities in non-existing elements can be regarded as zero when the calculation is performed.

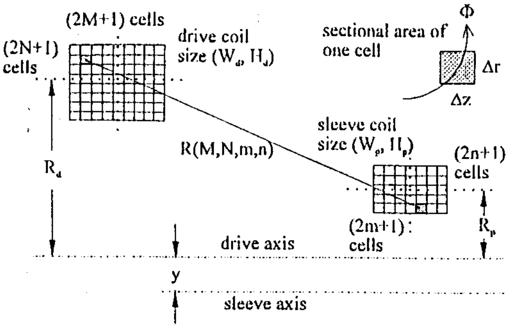

Fig.1. Configuration of mesh matrix

Each cell in the drive coil contains one filament, and the current densities in each filament are assumed to be uniform. Hence, the magnetic flux interlinked with any sleeve cell is the cumulation of the magnetic fluxes due to each filamentary cell in the drive coil. Fig.2 explains the calculation of the magnetic flux and mutual inductance in each filamentary cell.

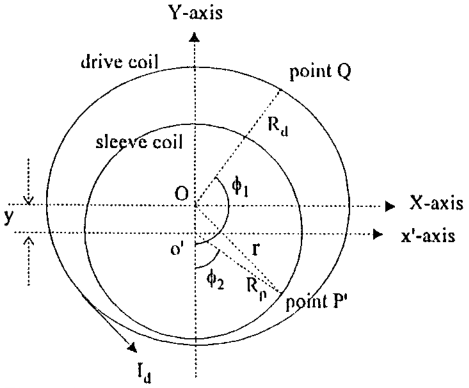

Fig. 2. Front View of the Two filamentary Coils

The net mutual inductance between two coils shown in Fig.1 can be defined as

$$M = \frac{\Psi_{dp}}{I_d} = \frac{N_p\,\phi_d}{I_d} \tag{1}$$

where, Ψ_{dp} is the magnetic flux interlinkage produced by the drive current I_d and interlinked with the sleeve, ϕ_d is magnetic flux linkage produced by the drive current.

When the turns-density in the sleeve filamentary cell is denoted N_{pe} and ϕ_{dpe} is magnetic flux linkage produced by one drive filament current, Eq.1 can be rewritten as

$$M = \sum_{k=-m}^{m}\sum_{l=-n}^{n}\sum_{i=-M}^{M}\sum_{j=-N}^{N} \frac{N_{pe}\,\phi_{dpe(i,j,k,l)}}{I_d} \tag{2}$$

$$N_{pe} = \frac{N_p}{(2m+1)(2n+1)} \tag{3}$$

The magnetic flux interlinking a cell of the sleeve is

$$\Phi_{dpe} = \int_{s_{2e}} \vec{B}\cdot ds_{2e} \tag{4}$$

where, B is the flux density at the sleeve cell, s_{2e} is the sectional area, ℓ_{2e} is the length at the sleeve coil.

The magnetic flux density has three vector components in the cylindrical coordinates. Therefore, the interlinking flux at the differential area of the sleeve coil can be obtained from the relationship as,

$$d\phi_{dpe} = \left\| B_r \Delta z d\ell_{se} + B_\phi \Delta\phi\Delta z + B_z \Delta r d\ell_{se} \right\| \tag{5}$$

The magnetic flux density at an arbitrary point on the sleeve cell in Fig.2 can be obtained from relationship between the vector magnetic potential A, which is

$$\vec{B} = \nabla\times\vec{A} \tag{6}$$

The vector magnetic potential at point P on the (k,l) sleeve filament due to the entire length of the (i,j) drive filament has a tangential component only and which can be expressed as in Eq.(7) by introducing the complete elliptic integrals. [6] [7]

$$A_\phi = \frac{\mu_0 I_{de}}{2\pi r_\ell}\left[(R_{dj}+r_\ell)^2 + z_{i,k}^2\right]^{1/2} \left[(1-\frac{1}{2}k^2)K(k) - E(k)\right] \tag{7}$$

where E and K are the complete elliptic integrals of first and second kind, respectively.

$$E = \int_0^{\pi/2} (1-\kappa^2\sin^2\vartheta)^{1/2}\, d\vartheta \tag{8}$$

$$K = \int_0^{\pi/2} \frac{1}{(1-\kappa^2\sin^2\vartheta)^{1/2}}\, d\vartheta \tag{9}$$

The modules is

$$\kappa_{i,j,k,l}^{\;2} = \frac{4R_{dj}\,r_\ell}{(R_{dj}+r_\ell)^2 + z_{i,k}^{\;2}} \tag{10}$$

where,

$$r_\ell^2 = (R_{p\ell}\cos\phi_2 + y)^2 + (R_{p\ell}\sin\phi_2)^2 \tag{11}$$

$$R_{dj} = R_d + \frac{H_d}{2N+1}j \tag{12}$$

$$R_{p\ell} = R_p + \frac{H_p}{2n+1}\ell \tag{13}$$

$$z_{i,k} = z - \frac{W_d}{2M+1}i + \frac{W_p}{2m+1}k \tag{14}$$

$$I_{de} = \frac{N_d I_d}{(2M+1)(2N+1)} \tag{15}$$

Introducing the relation ship in Eq.6, one can find the flux density in cylindrical coordinates.

$$B_z = \frac{\mu_o I_{de}}{2\pi[(R_{dj}+r_t)^2+z_{i,k}^2]^{1/2}}\left[\frac{R_{dj}^2-r_t^2-z_{i,k}^2}{(R_{dj}-r_t)^2+z_{i,k}^2}\cdot E+K\right] \tag{16}$$

$$B_r = \frac{\mu_o I_{de} z_{i,k}}{2\pi r_t[(R_{dj}+r_t)^2+z_{i,k}^2]^{1/2}}\left[\frac{R_{dj}^2+r_t^2+z_{i,k}^2}{(R_{dj}-r_t)^2+z_{i,k}^2}\cdot E-K\right] \tag{17}$$

$$B_\phi = 0 \tag{18}$$

Magnetic flux interlinking to one entire sleeve coil cell due to one drive coil cell is

$$\phi_r^{(1)} = \sum_{\phi_2=0}^{N_\phi} R_{pl} B_r \Delta z \Delta\varphi_2 \tag{19}$$

$$\phi_z^{(1)} = \sum_{\phi_2=0}^{N_\phi} R_{pl} B_z \Delta r \Delta\varphi_2 \tag{20}$$

where,

$$\Delta\varphi_2 N_\varphi = 2\pi \tag{21}$$

Considering the entire volume of the drive coil, the magnetic flux interlinking to one entire sleeve coil cell is

$$\phi_r^{(2)} = \sum_{i=-M}^{M}\sum_{j=-N}^{N}\sum_{\phi_2=0}^{N_\phi} R_{pl} B_r \Delta z \Delta\varphi_2 \tag{22}$$

$$\phi_z^{(2)} = \sum_{i=-M}^{M}\sum_{j=-N}^{N}\sum_{\phi_2=0}^{N_\phi} R_{pl} B_z \Delta r \Delta\varphi_2 \tag{23}$$

Inserting Eqs.(3)(15)(16)(17)(22)(23) to Eq.(2), one can get the equation for the net mutual inductance such that

$$M = \frac{N_d N_p \Delta\varphi_2}{(2M+1)(2N+1)(2m+1)(2n+1)} \sum_{i=-M}^{M}\sum_{j=-N}^{N}\sum_{k=-m}^{m}\sum_{l=-n}^{n}\sum_{\varphi=0}^{N_\phi}[(B_r R_{pl}\Delta z)^2+(B_z R_{pl}\Delta r)^2] \tag{24}$$

For computation of B_r and B_z in Eq.24, drive current I_d is considered being unit value, i.e. I_d = 1(A). Eq. (24) is the final form for the mutual inductance between two non coaxial circular coils.

3. CALCULATION RESULTS

The calculation was done with the following coil dimensions : 1) drive coil : R_d = 0.0397m, W_d = 0.0302m, H_d = 0.022 m, N_d = 10. 2) sleeve coil : R_p = 0.0246 m, W_p = 0.01m, H_p = 0.00165 m, N_p = 1,

The mutual inductance between the drive coil and the sleeve coil is calculated with different values of the off-center distance. In Fig.3, the off-centered distances are y = 0.0 mm, 1.5 mm, 3.0 mm, 4.5 mm, the last one being the uppermost curve.

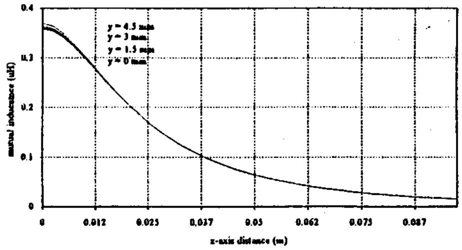

Fig.3. Mutual inductance with various off-centered distance

In Fig.3, it is shown that the mutual inductance increases as the off-centered distance increases. This increment can be investigated more in detail in Fig.4, where the mutual inductance, at axial distance z = 0.0 m, is calculated as a function of y and the value of y is varied from 0.0 to 7.2 mm.

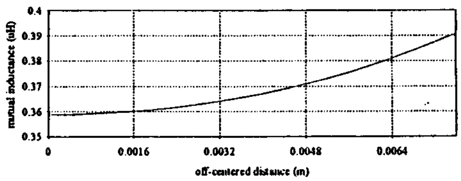

Fig.4. Mutual inductance.vs. y, with z = 0.0 m

From Figs.3 and 4, it can be concluded that the mutual inductance increases as the off-centered distance gets larger. However, if the air gap is relatively small, the mutual inductance does not change much.

4. EXPERIMENTAL MEASUREMENT

The computer calculation results are verified by actual experiments in the laboratory. Two experiments were performed with the experiment setup shown in Fig. 5. The sleeve coil was cut in longitudinal direction to measure the induced voltage. If an AC current is applied to the drive coil, the change in the drive coil flux induces a voltage in the sleeve coil. Neglecting the resistance of the sleeve coil, the voltage induced in the sleeve coil is

$$e_2 = R_2 i_2 + \frac{d\psi_2}{dt} + \frac{d\psi_{12}}{dt}$$
$$= R_2 i_2 + L_2\frac{di_2}{dt} + i_2\frac{dL_2}{dt} + M_{12}\frac{di_1}{dt} + i_1\frac{dM_{12}}{dt}$$

The internal impedance of the oscilloscope was set to be 1 (MΩ) while R_2 is as small as less than 0.2(Ω) and L_2 is as small as 0.3 (mH). Therefore, the current I_2 during the measurement is negligibly small. Hence, terms (1),(2),(3) are to be neglected. At the fixed distance, permeability of the air is not varying. Hence, value of the term (5) is zero. Thus, the final form of the mutual inductance for sinusoidal input current is;

$$M_{12} = \frac{2\pi f e_2}{i_1}$$

(1) Experiment 1

The experiment was done with the set up as shown in Fig.5.

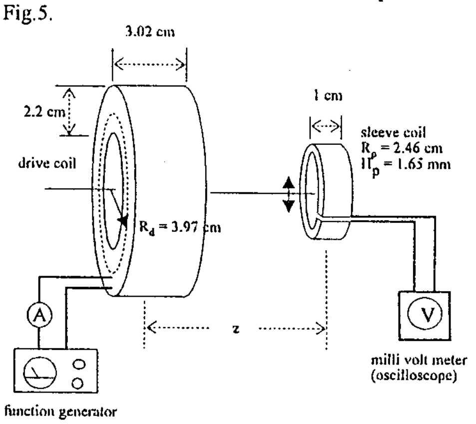

Fig. 5. Experiment setup

The drive coil and the sleeve coil were placed on the same longitudinal axis and the sleeve coil was shifted from z = 0 (m) to the right. To measure the induced voltage in the sleeve coil, the sleeve coil was cut with a very small air-gap as shown in Fig.5. The off-centered distance between two coils is about 2 mm.

Table 1. Summary of experiment 1 results

no.	distance z(m)	mutual inductance (μH)
1	0.000	0.36
2	0.005	0.34
3	0.010	0.30
4	0.015	0.27
5	0.020	0.22
6	0.025	0.18
7	0.030	0.16
8	0.035	0.11
9	0.040	0.08
10	0.045	0.07
11	0.050	0.05

(2) Experiment 2

In experiment 1, the sleeve coil was moved up and down in the vertical direction. However, no significant voltage change was detected. It was concluded that the change was too small to be measured. Hence, the drive coil was changed to a bigger radius (R_d = 7.7 cm, coil thickness = 1.7 cm) instead of the 3.97 cm radius coil. The sleeve coil was moved up and down in radial direction while the drive coil is fixed and the longitudinal distance between the drive coil and the sleeve coil is zero. The results of this experiment are listed in Table (2).

Table 2. Summary of experiment 2 results

No.	y (m)	mutual inductance (uH)
1	0.0000	1.81
2	0.0125	1.87
3	0.0254	1.98
4	0.0375	2.15

From Table (2), it can be seen that the mutual inductance increases as the off-center distance increases, and the pattern of increment is similar to Fig.3. Thus, the calculation results shown in Fig.3 appear to be reasonable.

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