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Issues Involved In The Study Of The Voltage Stability of A Power System Network Modeled By DAE

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Abstract

In this paper an attempt is made to understand the voltage stability when the power system networks are represented by the differential-algebraic equations (DAEs) form. The problem is analyzed by interpreting the shape of constraint manifold, based on the singular perturbation model. The global picture of constraint manifold is given to show how the local shape of constraint manifold can be used to guess for the system behavior. The gradient analysis is used systematically to obtain a local shape of the constraint manifold.

I. Introduction

The voltage stability problem has become one of the most significant challenges in the operation of power system networks. The voltage stability problem involves both static and dynamic aspects. Even though there is a fair understanding of the steady-state voltage stability, there still exists confusion regarding the interaction of load, generator and other component dynamics in the study of voltage collapse [1, 2, 3].

The dynamic formulation requires the consideration of appropriate models for the system components. A parameter dependent differential-algebraic equations (DAEs), eq. 1 and 2, can be used to study the interaction between static and dynamic aspects of voltage collapse.

$$\dot{x} = f(x, y, p) \quad (1)$$
$$0 = g(x, y, p). \quad (2)$$

This model usually represents the dynamic equations of the generators and control devices, the stator and network algebraic equations. The underlying dynamics in the algebraic parts are assumed infinitely fast. Many researchers think that the DAE models of the power system are equivalent to the quasi-stationary model of the singular perturbation model. Thus within the quasi-stationary range, the DAE model can be used without any restrictions. However, the stability analysis, beyond the quasi-stationary limit, may not be valid with the DAE model. In that case, we may have to consider the underlying dynamics of algebraic parts.

Recently, starting with the work of Venkatasubramanian et al. [4], the case in which the quasi-stationary condition is violated have been reported [5, 6, 7].

This paper is an attempt to carry out a study to understand the phenomenon involved in the modelling deficiency of DAE formulation. The same problem is commented on the ref. [8]. The best way to figure out this aspect is to know the underlying dynamics of the algebraic equations [4].

In this paper, the instance at which the algebraic Jacobian becomes singular is focussed in a view to understand the possible system behavior around this operating point. This is possible by observing the geometrical shape of algebraic equations in state and algebraic space. The gradient-analysis [9] is utilized to systematically analyze the shape of the constraint manifold. Using this technique, the critical regions can be identified at least, around which the underlying dynamics must be considered.

This paper is organized as follows. In section 2, a simple nonlinear oscillator is considered to motivate the difficulty involved in the analysis of a system which is modelled in DAE form. Section 3 describes various issues involved in the power system networks modelled in DAE form. When the DAE model is not valid, it is claimed that the understanding of the local shape of algebraic constraint may provide the way to guess the system behavior. Section 4 clarifies the above issues with a simple power system example. The global pictures of this constraint manifold are given to show how the local shape of constraint manifold can be used to guess for the system behavior without knowing the underlying dynamics.

II. Motivation Example

In this section we consider the basic circuit structure of electronic oscillators. This example has been thoroughly analyzed by many researches. We use this example to show the difficulty involved in the analysis of DAE model.

In that example, the oscillator circuit is represented as follows,

$$\dot{x}_1 = x_2 \quad (3)$$
$$\epsilon\dot{x}_2 = -x_1 + x_2 - x_2^3/3, \quad (4)$$

where $0 < \epsilon = CL \ll 1$. Here x_2 is input voltage (v), and the nonlinear resistor has $i - v$ characteristic of $h(v) = -v + v^3/3$.

Since $\epsilon \ll 1$, the dynamics of x_2 is very fast (fast dynamics) rather than that of x_1 (slow dynamics). Figure 1 shows the vector flows of eq. 3 and 4. On the equilibrium space of eq. 4, which consists of all the solutions satisfying $x_1 = x_2 - x_2^3/3$, the left hand side $\epsilon\dot{x}_2 = 0$ becomes zero. Therefore the field is vertical with respect to the x_2 axis. This equilibrium space is also called slow manifold because only slow dynamics is activated on this space. Outside this space the vector field is almost horizontal to x_2 axis because the speed of x_2 dynamics is very fast compared to x_1 dynamics. On the branch AB, the system slides down along the solution space. At point B, the system jumps onto the branch CD. Once on CD, it moves along the branch DC. The flow is restriced to the branches AB and DC. BC is unstable branch. At point B and C, jumping behavior occurs (i.e., the system exhibits limit cycle behavior).

As mentioned in the above paragraph, because the speed of the fast dynamics is usually very high, we concentrate the dynamic behavior due to x_1 only.

$$\dot{x}_1 = x_2 \quad (5)$$

unstable, there is no meaning to extend the analysis to determine the stability of the slow dynamics. This fact can be conveyed to the linear analysis of DAE model. Let us linearize the eq. 1 and 2 at certain point (x_1^*, x_2^*) as follows

$$\begin{bmatrix} \Delta \dot{x}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \quad (8)$$

If we express Δx_2 in terms of Δx_1, then

$$\Delta \dot{x}_1 = [a - b * d^{-1} * c]\Delta x_1. \quad (9)$$

Calculating eigenvalue of reduced Jacobian matrix $[a - b * d^{-1} * c]$, the stability is determined. Here the point at which d becomes zero is the limit of quasi-stationary range. Beyond that limit, the fast dynamics may be unstable. Thus the reduced Jacobian matrix is valid only up to the limit.

III. Stability of the power system based on DAE model

For a system defined by DAE, the usual way to determine the system stability is solving the algebraic equations for y in terms of x. Possibly, many solutions exist and these solutions are to be substituted into the differential equations to find equilibria. Once we have the equilibrium point, the stability can be determined by using various tools. However, it must be recalled that the DAE model is an approximation of physical power systems because the algebraic equations represent the stator and transmission network. In nature, the dynamics corresponding to those components are very fast and assumed to be steady-state [4]. If it is true, the DAE model of power systems is valid only within the quasi-stationary range. This fact can be explained as follows.

Let eq. 10 and 11 are the singular perturbation equivalents of eq. 1 and 2.

$$\dot{x} = f(x, y, p) \quad (10)$$

$$\epsilon \dot{y} = g(x, y, p), \quad (11)$$

where $0 < \epsilon \ll 1$. If we rescale time t to $\tau = t/\epsilon$, i.e., focusing on the time frame of fast dynamics, then

$$\frac{dx}{d\tau} = \epsilon f(x, y, p) \quad (12)$$

$$\frac{dy}{d\tau} = g(x, y, p), \quad (13)$$

As $\epsilon \to 0$, the set of equilibria for this system is exactly on the constraint manifold and the slow dynamics are neglected because $dx/d\tau = 0$. The fast dynamics describes whether the constraint manifold is attracting or not. When we have an equilibrium point (x^*, y^*), the fast dynamic behavior near an equilibrium point is determined by the eigenvalue condition of the Jacobian $\frac{\partial g}{\partial y}$ at the equilibrium. If all the eigenvalues have negative real part, then it remains at the (x^*, y^*) in the time frame corresponding the fast dynamics. To know the entire system behavior, we also need to analyze the slow dynamics. It is performed as mentioned above. However, If any eigenvalues have a positive real parts, then the constraint manifold at that point is not attracting any more. In that case, there is no meaning to perform the analysis to determine the stability of slow dynamics at that equilibrium point.

If the entire dynamics is known, then the system behavior can be traced by considering the whole time frame (fast and slow). In that case, the system may come back to the original equilibrium point or it may be attracted to another equilibrium point

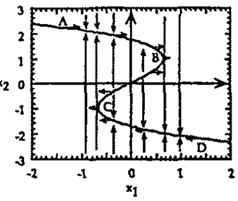

Figure 1: Vector flows of eqs. 3 and 4

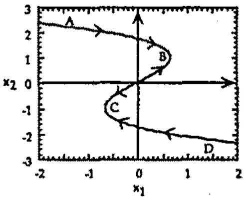

Figure 2: Vector flows of eqs. 5 and 6

$$0 = -x_1 + x_2 - x_2^3/3, \quad (6)$$

Eq 5 and 6 are called the differential-algebraic equations (DAEs). This algebraic constraint (eq. 6) defines the solution space which is same as the slow manifold defined in eq. 3 and 4. The solution space defined by the algebraic constraint is called constraint manifold. Figure 2 shows the constraint manifold. The arrows on the constraint manifold give the direction of the system movement. At point B, the tangent to the constraint manifold becomes parallel to the x_2 axis. In that case, it is difficult to predict what happens at that point because $\dot{x}_1$ can not be moved to the direction as indicated on the constraint manifold. There does not exist the algebraic solution in that direction at the point B. The same situation occurs at the point C. We need to go back to the original differential equations to understand the system behavior

Comparing the DAEs with the full differential equations of this example, the vector flow of the DAE is same as that of the full differential equations on the branches of AB and CD. Thus if the DAE is considered, the stability information is valid only on the branches AB and CD. This is called the consistency in the theory of singular perturbations. The condition for consistency is the local asymptotic stability of an equilibrium point on the constraint manifold. This is guaranteed by

$$\left. \frac{\partial h(x_2)}{\partial x_2} \right|_{x_2 = x_2^*} < 0 \quad (7)$$

The region in which this condition is satisfied is called quasi-stationary region. $\partial h(x_2)/\partial x_2$ is nothong but one dimensional algebraic Jacobian matrix. The point at which the algebraic Jacobian becomes singular is called impasse point or Singularity-Induced bifurcation point [4]. At this point consistency condition (7) is violated.

From this example, it is concluded that DAE model is valid only if the fast dynamic part is stable. If the fast dynamics becomes or limit cycle, etc.). If the algebraic Jacobian is singular at an equilibrium, then it is impossible to trace the system behavior, without knowing the underlying dynamics of the algebraic variables. However, the local behavior around the equilibrium can be guessed by checking the existence of solution of algebraic equations in the direction in which the system will change by a small disturbance. This can be achieved by observing the shape of the constraint manifold. Next section explains how to obtain this constraint manifold.

IV. A single machine power system example

We consider a single machine supplying power to a load modeled with constant active and reactive power components used in Chow and Gebreselassie paper [6]. They analyzed a dynamic voltage instability with the variation of active power load demand.

Our analysis is focussed on the load level at which the algebraic Jacobian becomes singular. This level is the boundary of quasi stationary range.

Algebraic equations are manipulated to have the state variables decoupled

$$E_d' - V_L sin(\delta - \theta) - 0.6706 P_L cos(\delta - \theta)/V_L = 0 \quad (14)$$
$$E_q' - V_L cos(\delta - \theta) - 0.6706 P_L sin(\delta - \theta)/V_L = 0 \quad (15)$$

At the load $P_L = 1.3068$, the equilibrium meets the limit of quasi stationary range. At that point, the tangent space of the constraint manifolds becomes parallel with the algebraic space. At this point, the linearized form of algebraic constraints Eq.14 and 15 at an equilibrium point are

$$\begin{bmatrix} E_d' \\ E_q' \end{bmatrix} = \begin{bmatrix} -1.322 & 1.238 \\ 0.502 & -0.470 \end{bmatrix} \begin{bmatrix} \Delta V_L \\ \Delta\theta \end{bmatrix}$$

The algebraic Jacobian matrix has eigenvalues $\lambda_1 = 0.000$ and $\lambda_2 = -1.792$. The corresponding eigenvectors are: right eigenvectors $(-0.683, -0.730)^t$, $(-0.935, 0.355)^t$, left eigenvectors $(-0.355, -0.935)$, $(-0.730, 0.683)$, respectively. Here, t denotes transpose of the vector. Using the gradient technique, we can separate the Jacobian matrix into the matrices corresponding to each mode as follows,

$$\begin{bmatrix} \Delta V_L \\ \Delta\theta \end{bmatrix} = \frac{1}{\lambda_1} \begin{bmatrix} 0.242 & 0.639 \\ 0.259 & 0.683 \end{bmatrix} \begin{bmatrix} E_d' \\ E_q' \end{bmatrix} + \frac{1}{\lambda_2} \begin{bmatrix} 0.683 & -0.638 \\ -0.259 & 0.242 \end{bmatrix} \begin{bmatrix} E_d' \\ E_q' \end{bmatrix}$$

In the above matrices, $1/\lambda_1$ is very large compared to $1/\lambda_2$. So, we only consider the matrix corresponding to λ_1.

$$\begin{bmatrix} \Delta V_L \\ \Delta\theta \end{bmatrix} = \frac{1}{\lambda_1} \begin{bmatrix} 0.242 & 0.639 \\ 0.259 & 0.683 \end{bmatrix} \begin{bmatrix} E_d' \\ E_q' \end{bmatrix}$$

It is obvious that the algebraic space is very sensitive to the change of E_d' and E_q', which verifies that the equilibrium is close to the limit of quasi-stationary range. The second column is much bigger than the first column. Thus E_q' is more dominant in this mode than the E_d'.

Here the right eigenvector corresponding to the zero eigenvalue gives the initial direction of the unstable region. The right eigenvector corresponding to the negative real eigenvalues indicates the initial direction of separatrix. The region in the opposite direction of the right eigenvector of zero eigenvalue is stable, i.e., the constraint manifold is attracting. So, any disturbance which make the system crosses the separatrix may lead to radical change in system behavior.

V. Conclusions

In this paper, we tried to understand the voltage instability at the instance of the singularity of the algebraic Jacobian matrix, by using the singular perturbation model. By observing the geometrical shape of the algebraic equations, the possible system behavior is analyzed around this operating point. The gradient analysis is used to systematically analyze the shape of the constraint manifold.

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