A Study on the Nonlinear Characteristics of EEG Using Bispectral Analysis.

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INTRODUCTION

The random characteristic of the EEG background activity makes the concept of random signals and stochastic process applicable to this field of signal analysis. The information carried by the EEG signal is to be found in its statistical properties. Through possible clinical relations, the statistical characteristics of the EEG may be of great interest in diagnosis as well as in the study of basic EEG mechanism.

Heuristic justification for the belief that EEG signal is linear and Gaussian is based on multiple small currents generated by multiple neurons, multipaths, and an assumed central limit theorem effect. Indeed, when one examines the univariate distribution of recorded EEG signal, the resulting distribution is often statistically undistinguishable from a univariate normal distribution. However, if the sources are not independent but contain significant frequency coherence at certain frequency components, and quadratic phase coupling arises between frequency pairs, then the resultant time series may be nonlinear.

In this study, we examine experimentally both the non-Gaussianity of EEG signals and their nonlinearity at two different states and five different sites.

The method used to test for the linearity and Gaussianity of EEG is based on the sample estimate of the bispectrum [7]-[11]. This bispectrum analysis is genarally used i) to extent information in the signal of interest pertaining to deviations from Gaussianity, and ii) to detect the presence of nonlinear properties, such as quadratic phase coupling [4]-[6],[11].

In this following sections, we provide a summary of definitions used in this paper and the conventional method for estimating the bispectrum. And in the latter part, we apply the bispectrum to the EEG analysis.

I. LINEAR AND NONLINEAR SERIES

A linear process is a time series which can be experessed in the form

$$X(n)= \sum_{m=-\infty}^{\infty} h(m)w(n-m) \qquad (1)$$

where $\{w(n)\}$ is a purely random series. This model includes all AR, ARMA models. If the $\{w(n)\}$ is Gaussian, the original process $\{X(n)\}$ is also Gaussian. The converse is also true .

Standard test for whiteness of a time series uses the sample autovariance function (or the power spectrum) of the series. If the series, in fact, nonlinear, then the nonlinearities present when fitting a linear series model to the data will be missed by standard tests based solely on the mean and covariance function.

The quadratic phase coupling is the case of these nonlinear processes.

If three frequencies f_1, f_2, and f_3 are such that $f_3 = f_1 + f_2$, then the triple (f_1, f_2, f_3) is called a bifriquency. The discrete-time system $y(n)=x(n)+\varepsilon x(n)^2$ with input $x(n)$ and output $y(n)$ provides an example of a mechanism that could generate bifrequencies. The quadratic phase coupling is the special case of this phenomenon.

Consider ,

$$X(n) = \cos(\lambda_1 n + \phi_1)+\cos(\lambda_2 n + \phi_2)+\cos(\lambda_3 n + \phi_3)$$

where $\lambda_3 = \lambda_1 + \lambda_2$ and $\phi_3 = \phi_1 + \phi_2$, then the frequency is caused by a quadratic coupling between λ_1 and λ_2 .

II.CUMULANTS AND BISPECTRA

If {X(K)}, k = 0 ,±1 ,±2,... is a real stationary discrete-time signal and its moments up to order 3 exist, than the third-order cumulant can be written as

$$c_3^x(\tau 1,\tau 2)=m_3^x(\tau 1,\tau 2)-m_3^G(\tau 1,\tau 2) \quad (2)$$

where $m_3^x(\tau 1,\tau 2) = E\{X(k)X(k+\tau 1)X(k+\tau 2)\}$ is the third -order moment and $m_3^G(t1,t2)$ is the third-order moment of an equivalent Gaussian signal that has the same mean value and autocorrelation sequence as X(k). Clearly, if it is Gaussian $m_3^x(\tau 1,\tau 2)=m_3^G(\tau,\tau 2)$ and thus $c_3^x(\tau 1,\tau 2)=0$.

Bispectrum is the Fourier transform of the third-order cumulant sequence defined as

$$C_3^x(\omega_1,\omega_2) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} c_3^x(\tau_1,\tau_2) \exp\{-j(\omega_1\tau_1 + \omega_2\tau_2)\} \quad (3)$$

$$|\omega_1|\le\pi , \quad |\omega_2|\le\pi, \quad |\omega_1+\omega_2|\le\pi$$

The bicoherence is defined by

$$P_3^x(\omega_1,\omega_2) = \frac{C_3^x(\omega_1,\omega_2)}{[C_2^x(\omega_1)C_2^x(\omega_2)\ C_2^x(\omega_1+\omega_2)]^{1/2}} \quad (4)$$

where $C_2^x(\omega)$ is the power spectrum of X(n).

This bicoherence function is very useful in the detection and characterization of nonlinearities in time series and in discriminating linear process from nonlinear ones. In fact, a signal is said to be a linear non-Gaussian process of order n if the magnitude of the nth-order coherence, $|P_3^x(\omega_1,\omega_2)|$, is constant over all frequencies : otherwise, the signal is said to be a non-linear process.

The bispectrum of a linear signal, {X(k)} in Eq.1, can be written in the form

$$C_3^x(\omega_1,\omega_2)= H(\omega_1)H(\omega_2)H^*(\omega_1+\omega_2)C_3^w(\omega_1,\omega_2) \quad (5)$$

where $C_3^w(\omega_1,\omega_2)$ is the bispectrum of source purely random series {w(k)} and $H(\omega)$ is the frequency transfer function of a LTI system. If {w(k)} is a white process, then $C_3^w(\omega_1,\omega_2)=\gamma_3^w$ (skewness) , which is constant for all frequency pairs. In Gaussian case, $\gamma_3^w = 0$, such that $C_3^x(\omega_1,\omega_2) = 0$ independent of frequency pairs. It can be shown that the power spectrum of {X(k)} is of the form $C_2^x(\omega)=\sigma_w^2|H(\omega)|^2$, where σ_w^2 is the varience of w(k). Then the bicoherence is $P_3^x(\omega_1,\omega_2)=\gamma_3^w/\sigma_w$ in LTI, white driven series. That is constant for all frequency pairs.

Detailed explanations are in [4],[5], and the reader is also referred to them.

III. CALCULATION OF BISPECTRUM AND MEASURES

The bispectrum of the zero mean stationary process {x(t)} can be consistently estimated by using a sample {x(0),x(1),...,x(N-1)} as follows [12],[7],[8].

Given N samples of the zero-mean process {x(t)}, thet are devided into K records, each having M samples. Let

$$A_N(j)= \sum_{n=0}^{M-1} X(n) \exp\{-2\pi jn/M\} \quad (6)$$

$$Fx(j,k)=A_N(j)A_N(k)A_N^*[(j+k)] / M \quad (7)$$

The smoothed estimate of the bispectrum $C_3^x(\omega_1,\omega_2)$ is

$$B_3^x(g_m,g_n)= M^{-2} \sum_{j=(m-1)L}^{mL-1} \sum_{k=(n-1)L}^{nL-1} Fx(j,k) \quad (8)$$

where $g_j=(2j-1)L/(2M)$. Bicoherence function is

$$\gamma_{m,n} = \frac{B_3^x(g_m,g_n)}{[M/KL^2]^{1/2}[S(g_m)S(g_n)S(g_m+g_n)]^{1/2}} \quad (9)$$

where S(g) is the smoothed estimator of the power spectrum $C_2^x(\omega)$.

Hinich shows that the estimators $2|\gamma_{m,n}|^2$ (denoted by $\lambda_{m,n}$) are asymptotically distributed as independent, non-central chi-squared variates for all m and n within the principal domain.

The measure of deviation from Gaussianity (i.e., non-Gaussianity) we used is

$$S_g = 2 \sum_m \sum_n |\gamma_{m,n}|^2 \quad (10)$$

,which is asymptotically distributed $\chi^2(2P)$ under the hypothesis of Gaussianity. Here, P denotes the number of frequency pairs in principal domain.

Under the hypothesis that {X(t)} is linear, $\lambda_{m,n}$ is constant independent of m and n. This constant is consistently estimated by $\lambda_e = S_g / P$, and $\lambda_{m,n}$ is distributed $\chi^2(2,\lambda_e)$.

And to quantify the nonlinearity, we used the variance of the bicoherence (= $|\gamma_{m,n}|$) sequencies as a measure. If an EEG series are more linear, it implies that $|\gamma_{m,n}|$ sequencies are asymptotically more constant, which means lower variance. Especially, to quantify the degree of the quadratic phase coupling, we find the peak frequency pair and the magnitude of bicoherence $|\gamma_{m,n}|$.

IV. MEATHODS AND RESULTS

EEG's were collected from 5 different sites of the brain and sampled at 200Hz for 100 second per records at two different brain states. eye-open and eye-close.

The electrode sites are A1-Cz(Ts.1), Fp1-Cz (Ts.2), A1-O1(Ts.3), O1-P3 (Ts.4), O1-Cz (Ts.5). And at each sites, we collected the data during eye-open(EO) and eye-close(EC).

The bispectrum and the bicoherenc of EEG for Ts.5 EO,EC is seen in Fig.1 - Fig.4, and the contour is seen below the each figure. The result of each experiment is seen in Table.1.

The location of the bispectrum peaks accours at (0,0) . But this does not mean zero frequency because we calculated the signal subtracted by average value.It is because of the low frequency resolution of the conventional estimation. It means the lower frequency below 2 Hz.

We can find that the location of eye-close bispectrum peak is at higher frequency pair than that of eye-open at each case. This is consistent with the fact that α-state(eye closed awake) has lower frequency than β-state (eye open).

For the degree of phase coupling, eye-close state is higher than eye-open state except at Ts.2. the difference of which is very small. And the variance of $|\gamma_{m,n}|$ is higher at the eye-close state than eye-open state except at Ts.2. These two results show that, in general, eye-close state is more nonlinear than eye-open except at Ts.2.

During eye-close state, the distance from Gaussianity is higher than that of eye-open state except at Ts.2 .

Among 5 electrode sites, the distance from Gaussianity and the non-linearity(i.e.,phase coupling and varience) is highest at Ts.5 .

Thus, the bispectrum approach does provide a method to differentiate the relative nature of the distributions of the EEG's .

V. CONCLUSION

In this study we examined the possibility of using bispectrum as a measure of EEG statistic characters, such as the deviation from Gaussianity, quadratic phase coupling, and nonlinearity. This experiment shows that EEG has different statistical properties according to the electrode location, and the brain state. This means that the various modeling of the brain may be possible. Non-linear properties are appeared and this implies that non-linear approaches of EEG are required, such as Volterra system representation or higher order spectrum in point of random process and chaotic analysis in dynamical point of view.

The results indicate that the bispectral analysis of the EEG can reveal extra information not obtainable from the power spectrum and may provide insights regarding the formation of the EEG within different brain structures during various states. Additional researches are required in estimating the bispectrum and more exact statistical verification of this method using more data are also remained to confide the conclusion of these EEG properties.

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TABLE 1

RESULT OF BISPECTRAL ANALYSIS OF EEG.

	Sg	var($\|\gamma_{m,n}\|$)	($m,n,B_3^x(m,n)$)	($m,n,\|\gamma_{m,n}\|$)
Ts.1 EO	2.1321	0.5596	(0,0,114.98)	(0,0,9.337)
Ts.1 EC	2.7170	0.9227	(2,2,13.776)	(2,2,15.3101)
Ts.2 EO	3.1012	0.9795	(0,0,89.721)	(0,0,15.3555)
Ts.2 EC	2.5152	0.8777	(4,4,2.7714)	(4,4,15.0399)
Ts.3 EO	2.1728	0.5325	(0,0,3.6919)	(15,15,6.904)
Ts.3 EC	3.1439	1.1716	(5,5,108.71)	(2,2,16.8931)
Ts.4 EO	3.3671	0.9688	(0,0,10.743)	(1,1,8.0241)
Ts.4 EC	4.9994	2.6570	(0,0,13.246)	(4,4,28.9681)
Ts.5 EO	5.9117	2.1745	(0,0,82.691)	(24,24,13.0227)
Ts.5 EC	10.747	7.2097	(5,5,65.800)	(4,4,45.4358)

* ($m,n,B_3^x(m,n)$) means maxmum bispectrum peak frequency at (m x 2 Hz, n x 2 Hz) and its peak value $B_3^x(m,n)$.
* ($m,n,|\gamma_{m,n}|$) means maxmum bicoherence peak frequency at (m x 2 Hz, n x 2 Hz) and its peak value $|\gamma_{m,n}|$.

* The unit of Sg is 10^3 and that of B_3^x is 10^7

Fig.1.
Bispectrum(B_3^x) Ts.5 EO.

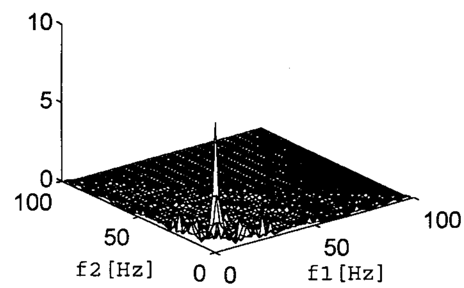

Fig.2.
Bispectrum(B_3^x) Ts.5 EC.

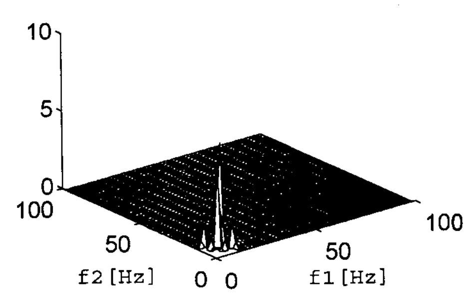

Fig.3.
Bicoherence($\gamma_{m,n}$) Ts.5 EO.

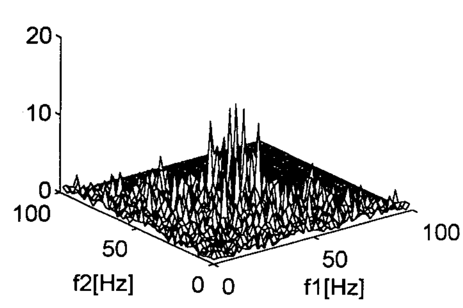

Fig.4.
Bicoherence($\gamma_{m,n}$) Ts.5 EC.