# Two-Parameter Optimization of CANDU Reactor Power Controller

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#### ABSTRACT

A nonlinear dynamic optimization has been performed for reactor power control system of CANDU 6 nuclear power plant considering xenon, fuel and moderator temperature feedback effects. Integral-of-Time-multiplied Absolute-Error (ITAE) criterion has been used as a performance index of the system behavior. Optimum controller gains are found by searching algorithm of Sequential Quadratic Programming (SQP). System models are referenced from most recent literatures. Signal flow network construction and optimization have been done by using commercial computer software package.

# 1. Introduction

Requirements of optimal control of process systems used in power plants in normal and/or abnormal conditions have been arisen for the sake of safe and economic operation. In addition, for the life extension of system components, it is desirable to prevent them from wears and failures as a result of inefficient control actions. Optimization of dynamic responses of such systems are also inevitable for real-time plant control.

In particular, for CANDU nuclear power plant, such needs should be more emphasized since its typical reactivity control mechanism, i.e., liquid zone control system is a complicated one composed of a number of components. Fourteen zone control valves are operated by the computer control signals to make the measured power agree with demanded power. The valves are manipulated in unison to control bulk power and tilts. [1,2] However, the response characteristic of zone control valves are slower than those of solid control rods since it is a hydraulic mechanism other than electric. Therefore, the controller must be designed to be as *optimal* as possible.

Techniques for the time-optimal control have been studied by many researchers. However, they have some limitations in practical applications. Most of them are unnecessarily theoretical to be applied to classical controllers of nuclear power plants. [3]

The purpose of this paper is to optimize the reactor power controller gains used in CANDU by using simplified process models published in literatures, and commercial software package for optimization procedure. Cost reductions in terms of Integral-of-Time-multiplied Absolute-Error (ITAE) obtained by using present optimized controller gains are presented as results.

## 2. Mathematical Models

### 2.1 Process

### Reactor Power

One point neutron kinetics model with 6 delayed neutron groups, and one point iodine and xenon rate equations are given by [4]

$$\frac{dn(t)}{dt} = \frac{\Delta k_T(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^6 \lambda_i C_i(t)$$
 (1)

$$\frac{dC_i(t)}{dt} = \frac{\beta_i \, \mathbf{n}(t)}{\Lambda} - \lambda_i C_i(t) \qquad i = 1, 2, 3, \ldots, 6 \tag{2}$$

$$\frac{dI(t)}{dt} = \gamma_I \Sigma_f \, n(t) - \lambda_I \, I(t) \tag{3}$$

$$\frac{dX(t)}{dt} = \gamma_x \, \Sigma_f \, n(t) + \lambda_I \, I(t) - \lambda_x \, X(t) - \sigma_X \, X(t) \, n(t) \tag{4}$$

In Eq.(1), feedback reactivity  $\Delta k_T(t)$  is a sum of those from control rods, xenon, fuel and moderator, i.e.,

$$\Delta k_T(t) = \Delta k_c(t) + \Delta k_{Xe}(t) + \Delta k_f(t) + \Delta K_m(t)$$
 (5)

## Fuel and Moderator Temperature

The fuel temperature  $T_f$  is assumed to have one representative temperature with heat input from fission  $Q_f$  and forced convective heat removal by heavy water coolant with temperature  $T_c$  as following:

$$M_f C_f \frac{dT_f}{dt} = Q_f - h_{fo} A_{fo} (T_f - T_c) \tag{6}$$

The moderator system is also assumed to have one temperature  $T_m$  with heat flow into and out of cooling heat exchanger and natural convection from calandria tubes  $Q_m$  as following:

$$M_m C_m \frac{dT_m}{dt} = \dot{m}_i C_i T_i - \dot{m}_o C_o T_o + \dot{Q}_m \tag{7}$$

In Eq.(7), we assumed that  $T_m$  is given by linear combination of inlet and outlet temperatures from heat exchangers  $T_i$  and  $T_o$ , constant specific heat capacities.

## Fuel and Moderator temperature Feedback

The approximate changes in reactivity with respect to fuel and moderator temperature changes in equilibrium core conditions are obtained from curve-fit of graphs in Ref. 2 as following [2,5]:

$$\frac{d(\Delta k_f)}{dT_f} = 10^{-3} \times \left[ 1.85194 \times 10^{-9} (3T_f^2) + 4.96047 \times 10^{-7} (2T_f) - 9.31416 \times 10^{-3} \right] (8)$$

$$\frac{d(\Delta k_m)}{dT_m} = 0.086 \times \left( -2.77451 \times 10^{-14} T_m^6 + 1.27442 \times 10^{-11} T_m^4 - 2.39436 \times 10^{-9} T_m^3 + 2.3802 \times 10^{-7} T_m^2 - 1.81723 \times 10^{-5} T_m + 7.83592 \times 10^{-5} \right) (9)$$

# 2.2 Control System

#### Zone Control Absorber

Reactor bulk power control in CANDU system is primarily done by light water liquid zone control absorber(ZCA) level. It is thus assumed that the only reactivity control mechanism is ZCA.

If we assume the light water net flow into zone control absorber is proportional to the area of the control valve opening and if initial flow is 50 % rated flow, then the rate of change of zone liquid level  $L_z$  can be approximately given by

$$\frac{dL_z(t)}{dt} = \frac{k_z}{k_{tz}} \left( \frac{s_z^2(t) - 0.5}{0.5} \right)$$
 (10)

where  $k_z$  is the total reactivity change rate due to zones (1.0 x 10<sup>4</sup> K/sec) and  $k_{lz}$  is the constant (7.1 x 10-3 K) relating zone reactivity with zone level.

# Moderator Cooling Heat Exchanger

The moderator D<sub>2</sub>O is continuously circulated by forced convection through two U tube and shell heat exchangers with 50% capacity. It is assumed that we have only one 100% heat exchanger.

The heat exchanger model is based on the direct lumping of Jonsson and Palsson's [6]. The inherent assumptions are: (1) no heat conduction in the direction of the flow in the metal between the fluids nor in the fluids themselves; (2) uniform temperature in each section; (3) constant heat capacity of the fluids. The equation for the

case of internal node = 2 is given by

$$\frac{d}{dt} T = \begin{bmatrix}
-(1 + \frac{\alpha}{2})/\tau_{h} & 0 & \frac{\alpha}{2\tau_{h}} & \frac{\alpha}{2\tau_{h}} \\
(1 - \frac{\alpha}{2})/\tau_{h} & -(1 + \frac{\alpha}{2})/\tau_{h} & \frac{\alpha}{2\tau_{h}} & 0 \\
\frac{\beta}{2\tau_{c}} & \frac{\beta}{2\tau_{c}} & -(1 + \frac{\beta}{2})/\tau_{c} & 0 \\
\frac{\beta}{2\tau_{c}} & 0 & (1 - \frac{\beta}{2})/\tau_{c} & -(1 + \frac{\beta}{2})/\tau_{c}
\end{bmatrix} T + \begin{bmatrix}
(1 - \frac{\alpha}{2})/\tau_{h} & 0 \\
0 & \frac{\alpha}{2\tau_{h}} \\
0 & (1 - \frac{\beta}{2})/\tau_{c} \\
\frac{\beta}{2\tau_{c}} & 0
\end{bmatrix} T_{inlet} (11)$$

where  $T = [T_{h,1} \ T_{h,2} \ T_{c,1} \ T_{c,2}]^T$ ,  $T_{inlet} = [T_{h,inlet} \ T_{c,inlet}]^T$  and  $\alpha = A_h U / m_h c_{h'}$  $\beta = A_c U / m_c c_{c'} \ \tau_h = M_h / m_h$  and  $\tau_c = M_c / m_c$  are model parameters.

## Actuator - Control Valve Stem

For a simple system where a loaded actuator is powered from a charged accumulator, if we ignore liquid compressibility and the pipe and valve pressure drop is assumed of the form  $\Delta P = C_Q Q^2$ , the force balance on the mass  $M_a$  is given by [7]

$$M_a \ddot{X} + f \dot{X} + C_Q A^3 \dot{X}^2 + kX = \frac{GA}{V_2 + AX} - F$$
 (12)

# Reactor Power Control Signal Signal

The demanded zone control valve position signal  $s_z^*$  can be calculated as following [8,9]:

$$s_z^*(t+T_S) = 0.133 E_p(t+T_S) + \sqrt{0.5}$$
 (13)

where

$$E_{P}(t+T_{S}) = K_{b} \left[ \log_{10} n(t+T_{S}) - \log_{10} n_{set}(t) \right] + K_{r} \left[ 1/\log_{10} n(t+T_{S}) - 1/\log_{10} n(t) \right]$$
(14)

where  $n_{set}(t)$  is the power setpoint. The original values of gains in plant computer are  $K_b = 1.0$  and  $K_r = 1.0$  and  $T_S = 0.5$  sec.

# Moderator Temperature Control Signal

Moderator temperature control is done by regulating flow through cooling heat exchanger. The flow is controlled by the following control valve position signal:

$$s_{m}*(t+T_{S}) = 1.0 - \left\{ K_{c} E_{m}(t+T_{S}) + s_{m}*(t) + K_{c} \frac{T_{s}}{T_{I}} E_{m}(t+T_{S}) + K_{c} \frac{T_{D}}{T_{S}} \left[ 0.3 E_{m}(t+T_{S}) + 0.1 E_{m}(t) - E_{m}(t-T_{S}) - 0.3 E_{m}(t-2T_{S}) \right] \right\}$$
(15)

where  $E_m$  ( $t+T_S$ ) =  $T_{set}$  -  $T_m$ ( $t+T_S$ ), gain  $K_c$  = 0.15 fractional lift /°C, integral time  $T_I$  = 100 sec, differential time  $T_D$  = 20 sec and sampling time  $T_S$  = 2 sec.

# 3. Optimization Method

Performance index for controller optimization used in this paper is Integral-of-Time- multiplied Absolute-Error (ITAE) criterion defined by following integral [10]:

$$\text{ITAE} = \int_0^{t_f} t |E_P(t)| dt \tag{16}$$

A system with above minimum ITAE has a characteristic that the overshoot in the transient response is small and oscillations are well damped. In addition, a large initial error can be weighted lightly, and errors occurring late in the transient response can be penalized heavily.

In order to obtain optimized gains  $K_b$  and  $K_r$  used in Eq. (14), we used Sequential Quadratic Programming Method [11] which can be easily run from a commercial mathematical package [12].

#### 4. Results and Conclusion

Table 1 shows the original gains used in CANDU computer program used for reactor control and resulting optimized gains. The optimized value of proportional gain  $K_b$  is nearly the same as the original value. However, the derivative controller gain  $K_r$  is much different. The optimized value is larger that the original value by a factor of about two.

Table 1. Optimized controller gains setting

Variable	Original	Optimized
$K_b$	1.0000	1.0234
K,	1.0000	1.8383

The comparison of ITAE costs between original and optimized cases is presented in Table 2. Increase in the gain for derivative action resulted in improved behavior. Table 2 also shows that the cost reduction of more than 10 % can be achieved if the system controller gains are optimized. When we compare the two cases with different power descent rate of 1 and 2 %, we can achieve higher cost reduction when power excursion speed is high.

Table 2. Comparison of ITAEs between original and optimized cases (Full power to 60% at a rate designated, time span = 50 secs)

Power Descent Rate (%)	Original	Optimized	cost reduction(%)
1.0	6.3415	5.6169	11.4
2.0	5.0121	4.3583	13.1

Present paper leads to the conclusion that the whole power plant operation can be optimized and corrective maintenance costs can be further decreased merely by optimizing controller gain settings.

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