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수중블럭이 사각형탱크의 자유수면 유동에 미치는 영향
Effects of a Submerged Block on the Sloshing of a Fluid in Rectangular Tanks

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요 약

사각형 유체저장탱크의 바닥에 놓여져 있는 사각형 블럭의 크기 및 위치가 자유수면의 유동진동수와 모드형상에 미치는 영향에 대하여 선형파이론을 적용하여 검토하였다. 유체의 영역을 3부분으로 나누고 각 영역에서의 속도포텐셜을 입사파와 블럭으로 인해 발생되는 반사파 및 전달파의 항으로써 표현하였다. 그리고 블럭에 의한 파의 반사율과 전달율은 유체영역의 경계에서 속도포텐셜과 유체입자의 속도가 연속인 조건을 사용하여 구하였다. 연구결과 블럭의 높이와 폭은 자유수면의 고유진동수에 크게 영향을 미치며 높이와 위치는 모드형상에 주로 영향을 준다. 블럭이 높고 폭이 넓을수록 고유진동수는 감소하며 블럭이 높고 탱크의 벽면으로 이동할수록 모드형상은 크게 변한다.

1 INTRODUCTION

A sloshing behavior is of important concern in the design of the liquid containers with internal bodies and the submerged components. Specially, the sloshing is more important problem to analyze the stability of freestanding submerged bodies.

Sloshing behavior of liquid in containers has been investigated by analytical and experimental method. Hunt and Priestley[1], Liu[2], Utsumi, et.al.[3], Haroun and Bashardoust[4], and Haroun and Chen[5] developed equations for the behavior of the surface water wave of rectangular tanks. Their studies, however, are limited to the containers without internal bodies. In general, since sloshing frequencies are decreased in case of containers with internal bodies[6], the results of those studies could be no more applicable to solve the sloshing problems of containers with internal bodies.

There are a few studies on the sloshing behavior of the liquid of containers with a submerged body. Evans and McIver[7] investigated the effects of a thin vertical baffle on the resonant frequencies of a fluid within a rectangular container using the linearized theory of water waves. They obtained an integral equation for the unknown velocity across the gap, which was not covered by the baffle in the fluid, by matching eigenfunction expansions across the gap. The thickness of the baffle was neglected in their study. Watson and Evans[8] studied the resonant frequencies of oscillation of a fluid in a rectangular container that contained either a partly immersed rectangular surface block or a totally immersed bottom-mounted rectangular block, which was symmetrically placed on the bottom of the tank. By considering symmetric and antisymmetric potentials separately, they constructed eigenfunction expansions appropriate to two distinct fluid regions that could be matched across their common vertical boundary. The matched eigenfunction expansions and Galerkin expansions were used to find out the resonant frequencies.

The purpose of present study is to find out the resonant frequencies and mode shapes of a fluid in rectangular containers with a totally submerged and unsymmetrically placed rectangular block, and to investigate the effects of the height, width, and position of the block on the resonant frequencies and mode shapes through some numerical computations. Fluid is divided into three regions and the velocity potentials, which satisfy Laplace equation and boundary conditions in each region, are introduced. By finding complex constants satisfying the continuity conditions of the mass flux and energy flux at the common vertical boundaries between fluid and block, the resonant frequencies and mode shapes are found.

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2 RESONANT FREQUENCIES

A rectangular rigid tank with an unsymmetrically placed rectangular block is considered. Figure 1 shows the geometry and the coordinate system of the tank.

With the assumption of irrotational motion and an incompressible and inviscid fluid, there exists a velocity potential

$$\Phi(x,z,t) = \phi(x,z)e^{i\omega t} \tag{1}$$

where the time-independent potential, $\phi(x,z)$, satisfies the Laplace equation in the fluid region.

$$\nabla^2\phi(x,z) = 0 \tag{2}$$

From figure 1, the velocity potential should satisfy following boundary conditions:

$$\phi_{,z} = 0. \qquad \begin{cases} -b < x < -a, & z = 0 \\ -a < x < a, & z = h \\ a < x < c, & z = 0 \end{cases} \tag{3}$$

$$\phi_{,z} = \frac{\omega^2}{g}\phi. \qquad -b < x < c, \quad z = d \tag{4}$$

$$\phi_{,x} = 0. \qquad \begin{cases} x = -b, & 0 < z < d \\ x = c, & 0 < z < d \end{cases} \tag{5}$$

$$\phi_{,x} = 0. \qquad |x| = a, \quad 0 < z < h \tag{6}$$

Let fluid region divide into three parts shown in figure 1 and the velocity potentials of the regions I, II and III be ϕ_1, ϕ_2 and ϕ_3, respectively, then the general solutions satisfying (2), (3) and (4) can be expressed as

$$\begin{aligned}\phi_1(x,z) = \sum_{j=1}^{\infty}\Big[&\Big\{A_je^{-ik_jb}e^{-ik_jx} + \left(r_{1j}^{(1)}A_je^{-ik_jb} + t_{1j}^{(3)}B_je^{-ik_jc}\right)e^{ik_jx}\Big\}f_{1j}(z) \\ &+ \sum_{m=2}^{\infty}\left(r_{mj}^{(1)}A_je^{-ik_jb} + t_{mj}^{(3)}B_je^{-ik_jc}\right)e^{k_{mj}x}f_{mj}(z)\Big] \\ &\qquad\qquad (-b \le x \le -a)\end{aligned} \tag{7}$$

$$\begin{aligned}\phi_2(x,z) = \sum_{j=1}^{\infty}\Big\{&\left(C_{1j}\sin k'_jx + D_{1j}\cos k'_jx\right)g_{1j}(z) \\ &+ \sum_{n=2}^{\infty}\left(C_{nj}\sinh k'_{nj}x + D_{nj}\cosh k'_{nj}x\right)g_{nj}(z)\Big\} \\ &\qquad\qquad (-a \le x \le a)\end{aligned} \tag{8}$$

$$\begin{aligned}\phi_3(x,z) = \sum_{j=1}^{\infty}\Big[&\Big\{B_je^{-ik_jc}e^{ik_jx} + \left(r_{1j}^{(3)}B_je^{-ik_jc} + t_{1j}^{(1)}A_je^{-ik_jb}\right)e^{-ik_jx}\Big\}f_{1j}(z) \\ &+ \sum_{m=2}^{\infty}\left(r_{mj}^{(3)}B_je^{-ik_jc} + t_{mj}^{(1)}A_je^{-ik_jb}\right)e^{-k_{mj}x}f_{mj}(z)\Big] \\ &\qquad\qquad (a \le x \le c)\end{aligned} \tag{9}$$

where

$$\begin{aligned}
f_{1j}(z) &= \left[\frac{d}{2}\left(1 + \frac{\sinh 2k_jd}{2k_jd}\right)\right]^{-\frac{1}{2}} \cosh k_jz \\
f_{mj}(z) &= \left[\frac{d}{2}\left(1 + \frac{\sin 2k_{mj}d}{2k_{mj}d}\right)\right]^{-\frac{1}{2}} \cos k_{mj}z \\
g_{1j}(z) &= \left[\frac{d'}{2}\left(1 + \frac{\sinh 2k'_jd'}{2k'_jd'}\right)\right]^{-\frac{1}{2}} \cosh k'_j(z-h) \\
g_{nj}(z) &= \left[\frac{d'}{2}\left(1 + \frac{\sin 2k'_{nj}d'}{2k'_{nj}d'}\right)\right]^{-\frac{1}{2}} \cos k'_{nj}(z-h)
\end{aligned}$$

and

$$\frac{\omega^2}{g} = k\tanh kd = -k_m \tan k_m d \qquad (m = 2,3,4,....)$$
$$= k'\tanh k'd' = -k'_n \tan k'_n d' \qquad (n = 2,3,4,....).$$

Here $A_j, B_j, C_{1j}, C_{nj}, D_{1j}$ and D_{nj} are complex constants, $r^{(1)}$ and $r^{(3)}$ are the reflection coefficients for waves upon the block from region I and III, respectively, $t^{(1)}$ and $t^{(3)}$ are the transmission coefficients. $\{f_{mj}\}$ and $\{g_{nj}\}$ are orthonormal sets over the interval $(0 < z < d)$ and $(h < z < d)$. These equations show that the velocity potentials consist of the coming wave from the wall, reflected wave from the block, transmitted wave from other fluid region, and scattered waves by the block.

Substituting (7) and (9) in (5) and using the orthonormal conditions, a general expression for the determination of the resonant wave numbers, k_j, in a rectangular tank with any internal body can be obtained by

$$t_{1j}^{(1)} t_{1j}^{(3)} = \left(e^{2ik_j b} - r_{1j}^{(1)}\right)\left(e^{2ik_j c} - r_{1j}^{(3)}\right). \tag{10}$$

This resonant condition agrees with the result from the wide-spacing approximation[9]. Because $r_{1j}^{(1)}$ and $t_{1j}^{(1)}$ are identical to $r_{1j}^{(3)}$ and $t_{1j}^{(3)}$ for a symmetric block, respectively, (10) can be reduced as

$$t_j^2 = \left(e^{2ik_j b} - r_j\right)\left(e^{2ik_j c} - r_j\right) \tag{11}$$

where

$$r_{1j}^{(1)} = r_{1j}^{(3)} \equiv r_j$$
$$t_{1j}^{(1)} = t_{1j}^{(3)} \equiv t_j.$$

Having required wave numbers k_j from (11), the resonant frequencies can be calculated by the relation

$$\omega_j = \sqrt{g k_j \tanh k_j d}. \tag{12}$$

3 MODE SHAPES

It is convenient to split velocity potentials by the direction of the coming waves from the wall for the determination of mode shapes, reflection and transmission coefficients. The j-th velocity potentials can be expressed as

$$\phi_{1j}(x,z) = \phi_{11j}(x,z) + \phi_{13j}(x,z) \tag{13}$$
$$\phi_{2j}(x,z) = \phi_{21j}(x,z) + \phi_{23j}(x,z) \tag{14}$$
$$\phi_{3j}(x,z) = \phi_{31j}(x,z) + \phi_{33j}(x,z) \tag{15}$$

in which

$$\phi_{11j}(x,z) = A_j e^{-ik_j b}\left\{\left(e^{-ik_j x} + r_j e^{ik_j x}\right) f_{1j}(z) + \sum_{m=2}^{\infty} r_{mj} e^{k_{mj} x} f_{mj}(z)\right\} \tag{16}$$

$$\phi_{13j}(x,z) = B_j e^{-ik_j c}\left(t_j e^{ik_j x} f_{1j}(z) + \sum_{m=2}^{\infty} t_{mj} e^{k_{mj} x} f_{mj}(z)\right) \tag{17}$$

$$\phi_{21j}(x,z) = \left(C_{1j}^{(1)} \sin k'_j x + D_{1j}^{(1)} \cos k'_j x\right) g_{1j}(z) + \sum_{n=2}^{\infty}\left(C_{nj}^{(1)} \sinh k'_{nj} x + D_{nj}^{(1)} \cosh k'_{nj} x\right) g_{nj}(z) \tag{18}$$

$$\phi_{23j}(x,z) = \left(C_{1j}^{(3)} \sin k'_j x + D_{1j}^{(3)} \cos k'_j x\right) g_{1j}(z) + \sum_{n=2}^{\infty}\left(C_{nj}^{(3)} \sinh k'_{nj} x + D_{nj}^{(3)} \cosh k'_{nj} x\right) g_{nj}(z) \tag{19}$$

$$\phi_{31j}(x,z) = A_j e^{-ik_j b}\left(t_j e^{-ik_j x} f_{1j}(z) + \sum_{m=2}^{\infty} t_{mj} e^{-k_{mj} x} f_{mj}(z)\right) \tag{20}$$

$$\phi_{33j}(x,z) = B_j e^{-ik_j c}\left\{\left(e^{ik_j x} + r_j e^{-ik_j x}\right) f_{1j}(z) + \sum_{m=2}^{\infty} r_{mj} e^{-k_{mj} x} f_{mj}(z)\right\} \tag{21}$$

The reflection and transmission coefficients and mode shapes can be obtained by the velocity potentials satisfying boundary condition (6) and following continuity conditions over the interval ($h < z < d$):

$$\begin{array}{lll} \phi_{1j} = \phi_{2j}, & \phi_{1j,x} = \phi_{2j,x} & (x = -a) \\ \phi_{3j} = \phi_{2j}, & \phi_{3j,x} = \phi_{2j,x} & (x = a). \end{array} \tag{22}$$

Substituting velocity potentials (16), (18) and (20) in (6) and (22), following equations are derived

$$A_j(1-\chi_{1j}^a)e^{-ik_jb_o} = 2i\frac{k_j'}{k_j}\left(C_{1j}^{(1)}\cos k_j'a\Psi_{11j} + \sum_{n=2}^{\infty} C_{nj}^{(1)}\frac{k_{nj}'}{k_j'}\cosh k_{nj}'a\Psi_{1nj}\right) \tag{23}$$

$$A_j\frac{k_{rj}}{k_j}\chi_{rj}^a e^{-ik_jb_o} = 2\frac{k_j'}{k_j}\left(C_{1j}^{(1)}\cos k_j'a\Psi_{r1j} + \sum_{n=2}^{\infty} C_{nj}^{(1)}\frac{k_{nj}'}{k_j'}\cosh k_{nj}'a\Psi_{rnj}\right) \quad (r = 2,3,4,\ldots) \tag{24}$$

$$A_j(1-\chi_{1j}^s)e^{-ik_jb_o} = 2i\frac{k_j'}{k_j}\left(D_{1j}^{(1)}\sin k_j'a\Psi_{11j} - \sum_{n=2}^{\infty} D_{nj}^{(1)}\frac{k_{nj}'}{k_j'}\sinh k_{nj}'a\Psi_{1nj}\right) \tag{25}$$

$$A_j\frac{k_{rj}}{k_j}\chi_{rj}^s e^{-ik_jb_o} = 2\frac{k_j'}{k_j}\left(D_{1j}^{(1)}\sin k_j'a\Psi_{r1j} - \sum_{n=2}^{\infty} D_{nj}^{(1)}\frac{k_{nj}'}{k_j'}\sinh k_{nj}'a\Psi_{rnj}\right) \quad (r = 2,3,4,\ldots) \tag{26}$$

$$C_{1j}^{(1)} = -\frac{e^{-ik_jb_o}}{2\sin k_j'a}\left\{\left(1+\chi_{1j}^a\right)\Psi_{11j} + \sum_{m=2}^{\infty}\chi_{mj}^a\Psi_{m1j}\right\}A_j \tag{27}$$

$$C_{nj}^{(1)} = -\frac{e^{-ik_jb_o}}{2\sinh k_{nj}'a}\left\{\left(1+\chi_{1j}^a\right)\Psi_{1nj} + \sum_{m=2}^{\infty}\chi_{mj}^a\Psi_{mnj}\right\}A_j \quad (n = 2,3,4,\ldots) \tag{28}$$

$$D_{1j}^{(1)} = \frac{e^{-ik_jb_o}}{2\cos k_j'a}\left\{\left(1+\chi_{1j}^s\right)\Psi_{11j} + \sum_{m=2}^{\infty}\chi_{mj}^s\Psi_{m1j}\right\}A_j \tag{29}$$

$$D_{nj}^{(1)} = \frac{e^{-ik_jb_o}}{2\cosh k_{nj}'a}\left\{\left(1+\chi_{1j}^s\right)\Psi_{1nj} + \sum_{m=2}^{\infty}\chi_{mj}^s\Psi_{mnj}\right\}A_j \quad (n = 2,3,4,\ldots) \tag{30}$$

where

$$\begin{array}{llll} b_o &= b - a, & & \\ \chi_{1j}^a &= (r_j - t_j)e^{-2ik_ja}, & \chi_{1j}^s &= (r_j + t_j)e^{-2ik_ja} \\ \chi_{rj}^a &= (r_{rj} - t_{rj})e^{-(ik_j+k_{rj})a}, & \chi_{rj}^s &= (r_{rj} + t_{rj})e^{-(ik_j+k_{rj})a} \\ \chi_{mj}^a &= (r_{mj} - t_{mj})e^{-(ik_j+k_{mj})a}, & \chi_{mj}^s &= (r_{mj} + t_{mj})e^{-(ik_j+k_{mj})a} \end{array}$$

and

$$\begin{aligned} \Psi_{11j} &= \int_h^d f_{1j}(z)g_{1j}(z)dz \\ \Psi_{m1j} &= \int_h^d f_{mj}(z)g_{1j}(z)dz \\ \Psi_{1nj} &= \int_h^d f_{1j}(z)g_{nj}(z)dz \\ \Psi_{mnj} &= \int_h^d f_{mj}(z)g_{nj}(z)dz. \end{aligned}$$

By substituting (27) and (28) in (23) and (24), the coefficients for an antisymmetric potential, $\{\chi_{mj}^a\}$, can be given by

$$\mathcal{X}_{aj} = \mathbf{U}_{aj}^{-1}\cdot\mathbf{V}_{aj} \tag{31}$$

where

$$\mathcal{X}_{aj} = < \chi_{1j}^a \quad \chi_{2j}^a \quad \chi_{3j}^a \quad \cdots \quad \chi_{mj}^a >^T$$

$$\mathbf{U}_{aj} = \begin{bmatrix} (iu^a_{11j}-1) & iu^a_{12j} & iu^a_{13j} & \cdots & iu^a_{1mj} \\ u^a_{21j} & (u^a_{22j}+1) & u^a_{23j} & \cdots & u^a_{2mj} \\ u^a_{31j} & u^a_{32j} & (u^a_{33j}+1) & \cdots & u^a_{3mj} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u^a_{m1j} & u^a_{m2j} & u^a_{m3j} & \cdots & (u^a_{mmj}+1) \end{bmatrix}$$

$$\mathbf{V}_{aj} =< \ -(1+iu^a_{11j}) \quad -u^a_{21j} \quad -u^a_{31j} \quad \cdots \quad -u^a_{m1j} \ >^T$$

in which

$$u^a_{mrj} = \frac{k'_j}{k_{mj}} \Psi_{m1j} \Psi_{r1j} \cot k'_j a + \sum_{n=2}^{\infty} \frac{k'_{nj}}{k_{mj}} \Psi_{mnj} \Psi_{rnj} \coth k'_{nj} a.$$

Similarly, substituting (29) and (30) in (25) and (26), the coefficients for a symmetric potential, $\{\chi^s_{mj}\}$, can be given by

$$\mathcal{X}_{sj} = \mathbf{U}^{-1}_{sj} \cdot \mathbf{V}_{sj} \tag{32}$$

where

$$\mathcal{X}_{sj} =< \ \chi^s_{1j} \quad \chi^s_{2j} \quad \chi^s_{3j} \quad \cdots \quad \chi^s_{mj} \ >^T$$

$$\mathbf{U}_{sj} = \begin{bmatrix} (iu^s_{11j}+1) & iu^s_{12j} & iu^s_{13j} & \cdots & iu^s_{1mj} \\ u^s_{21j} & (u^s_{22j}-1) & u^s_{23j} & \cdots & u^s_{2mj} \\ u^s_{31j} & u^s_{32j} & (u^s_{33j}-1) & \cdots & u^s_{3mj} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u^s_{m1j} & u^s_{m2j} & u^s_{m3j} & \cdots & (u^s_{mmj}-1) \end{bmatrix}$$

$$\mathbf{V}_{sj} =< \ (1-iu^s_{11j}) \quad -u^s_{21j} \quad -u^s_{31j} \quad \cdots \quad -u^s_{m1j} \ >^T$$

in which

$$u^s_{mrj} = \frac{k'_j}{k_{mj}} \Psi_{m1j} \Psi_{r1j} \tan k'_j a - \sum_{n=2}^{\infty} \frac{k'_{nj}}{k_{mj}} \Psi_{mnj} \Psi_{rnj} \tanh k'_{nj} a.$$

Therefore j-th reflection and transmission coefficients, r_j and t_j, can be determined by

$$r_j = \frac{1}{2}(\chi^a_{1j} + \chi^s_{1j}) e^{2ik_j a} \tag{33}$$

$$t_j = \frac{1}{2}(\chi^a_{1j} - \chi^s_{1j}) e^{2ik_j a}. \tag{34}$$

Complex constants $C^{(3)}_{1j}$, $C^{(3)}_{nj}$, $D^{(3)}_{1j}$, and $D^{(3)}_{nj}$ can be given by substituting velocity potentials (17), (19) and (21) in (6) and (22) as follows:

$$C^{(3)}_{1j} = \frac{e^{-ik_j c_o}}{2 \sin k'_j a} \left\{ (1+\chi^a_{1j}) \Psi_{11j} + \sum_{m=2}^{\infty} \chi^a_{mj} \Psi_{m1j} \right\} B_j \tag{35}$$

$$C^{(3)}_{nj} = \frac{e^{-ik_j c_o}}{2 \sinh k'_{nj} a} \left\{ (1+\chi^a_{1j}) \Psi_{1nj} + \sum_{m=2}^{\infty} \chi^a_{mj} \Psi_{mnj} \right\} B_j \tag{36}$$

$$D^{(3)}_{1j} = \frac{e^{-ik_j c_o}}{2 \cos k'_j a} \left\{ (1+\chi^s_{1j}) \Psi_{11j} + \sum_{m=2}^{\infty} \chi^s_{mj} \Psi_{m1j} \right\} B_j \tag{37}$$

$$D^{(3)}_{nj} = \frac{e^{-ik_j c_o}}{2 \cosh k'_{nj} a} \left\{ (1+\chi^s_{1j}) \Psi_{1nj} + \sum_{m=2}^{\infty} \chi^s_{mj} \Psi_{mnj} \right\} B_j \tag{38}$$

where $c_o = c - a$.

Therefore the j-th mode shape, $\phi_{oj}(x,z)$, can be obtained from (13)–(21) as the sum of the components for a tank without a block and for a block, i.e.,

$$\phi_{oj}(x,z) = 2\cos k_j(x+b) f_{1j}(z) + e^{-ik_j b} \sum_{m=2}^{\infty} \left\{ r_{mj} + \frac{t_{mj}}{t_j} \left(e^{2ik_j b} - r_j \right) \right\} e^{k_{mj} x} f_{mj}(z)$$

$$(-b \le x \le -a) \qquad (39)$$

$$\phi_{oj}(x,z) = \frac{1}{2} e^{-ik_j(b-a)} \left\{ \left(C'_{1j} \sin k'_j x + D'_{1j} \cos k'_j x \right) g_{1j}(z) + \sum_{n=2}^{\infty} \left(C'_{nj} \sinh k'_{nj} x + D'_{nj} \cosh k'_{nj} x \right) g_{nj}(z) \right\}$$

$$(-a \le x \le a) \qquad (40)$$

$$\phi_{oj}(x,z) = \frac{1}{t_j} e^{-ik_j(b-c)} (e^{2ik_jb} - r_j) \left[2\cos k_j(x-c) f_{1j}(z) + e^{-ik_jc} \sum_{m=2}^{\infty} \left\{ r_{mj} + \frac{t_{mj}}{t_j}(e^{2ik_jc} - r_j) \right\} e^{-k_{mj}x} f_{mj}(z) \right]$$

$$(a \le x \le c) \qquad (41)$$

where

$$C'_{1j} = -\frac{1}{\sin k'_j a} \left\{ 1 - \frac{1}{t_j} \left(e^{2ik_jb} - r_j \right) \right\} \left\{ (1 + \chi^a_{1j}) \Psi_{11j} + \sum_{m=2}^{\infty} \chi^a_{mj} \Psi_{m1j} \right\}$$

$$D'_{1j} = \frac{1}{\cos k'_j a} \left\{ 1 + \frac{1}{t_j} \left(e^{2ik_jb} - r_j \right) \right\} \left\{ (1 + \chi^s_{1j}) \Psi_{11j} + \sum_{m=2}^{\infty} \chi^s_{mj} \Psi_{m1j} \right\}$$

$$C'_{nj} = -\frac{1}{\sinh k'_{nj} a} \left\{ 1 - \frac{1}{t_j} \left(e^{2ik_jb} - r_j \right) \right\} \left\{ (1 + \chi^a_{1j}) \Psi_{1nj} + \sum_{m=2}^{\infty} \chi^a_{mj} \Psi_{mnj} \right\}$$

$$D'_{nj} = \frac{1}{\cosh k'_{nj} a} \left\{ 1 + \frac{1}{t_j} \left(e^{2ik_jb} - r_j \right) \right\} \left\{ (1 + \chi^s_{1j}) \Psi_{1nj} + \sum_{m=2}^{\infty} \chi^s_{mj} \Psi_{mnj} \right\}$$

4 NUMERICAL EXAMPLES

It is known that the resonant frequencies and mode shapes of a fluid in a tank with a block are varied by the height, width and position of the block as discussed in the previous sections. This study investigates how the resonant frequencies and mode shapes vary with the variation of the dimensions and position of the submerged block through some numerical examples.

Effects of height

The resonant frequencies can be affected by the geometry of a tank and the height of an internal block as shown in figure 2. The resonant frequencies are decreased as the height of a block is increased. The decrease rate for a broad tank is larger than for a tall tank. First resonant frequency for the broad tank is much sensitive to the height of the block, but there is little change in the higher resonant frequencies for $h/d < 0.6$. For $h/d > 0.9$, the resonant frequencies are much rapidly decreased in both tanks. It can be seen that the resonant frequencies are not influenced by the internal body for the tall tanks up to $h/d < 0.8$. The mode shapes of free vibration of free-surface are shown in figure 4. Generally, large h/d decreases the amplitude of mode shapes, but the changes of the mode shapes are small in case of $h/d < 0.5$. When a block is mounted on the center of the bottom, the maximum amplitude of the second mode is developed at the central antinode. If a wide block is placed on the center, the block does not influence in the mode shapes.

Effects of position

Figure 3 shows that resonant frequencies are decreased as b/l approaches to 0.5. In general, there are little changes on the resonant frequencies by the position of the block. Figure 4 shows that small b/l makes great change in the mode shapes and for large $2a/l$, the effect of b/l ratio is decreased. For small b/l less than 0.3, the position does not affect on the mode shapes.

Effects of width

Figure 3 and 4 show that the resonant frequencies are greatly decreased and there are little changes in the mode shapes. For $h/d < 0.5$, the effect of the block can be neglected in the higher than third mode.

5 CONCLUSIONS

The effects of a submerged block on the resonant frequencies and mode shapes for the rectangular liquid containers are investigated. It is known from the results that the resonant frequencies depend upon the height and width of a block and mode shapes depend upon the height and position.

When a block is higher and wider, the resonant frequencies are smaller. Moving the block to the center decreases the resonant frequencies. When the height of a block is larger than 80 percent of the height of liquid, the resonant frequencies are rapidly decreased. If a block is placed near the center, there is little change in the resonant frequencies.

When a block is lower and wider, the variation of mode shapes is smaller. Moving the block to the center makes little change in the mode shapes. The width of a block greatly affects in resonant frequencies, but little influences in mode shapes.

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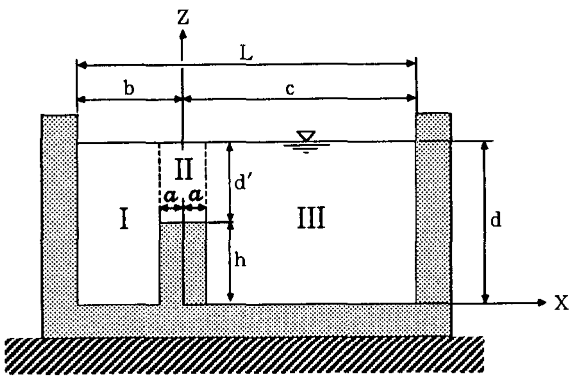
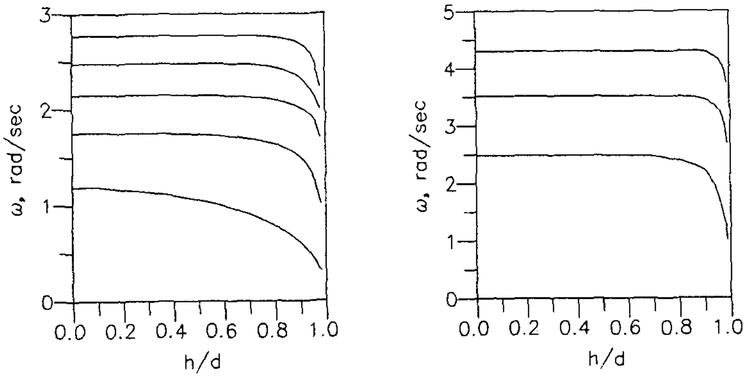
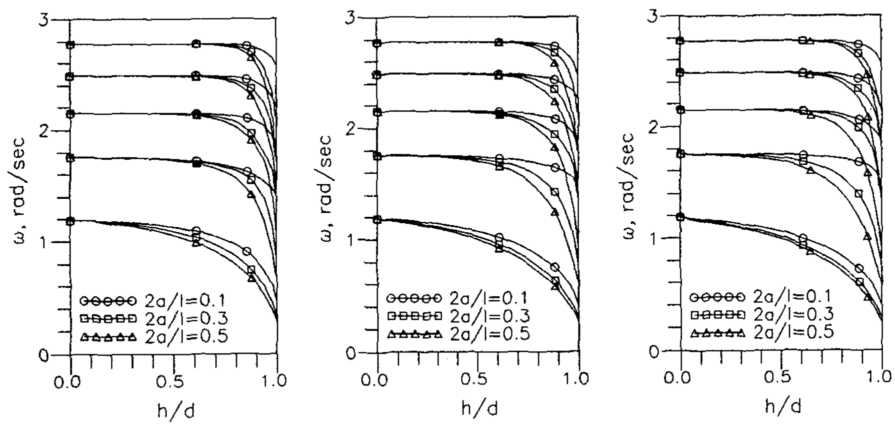


Figure 1: Tank with a submerged block



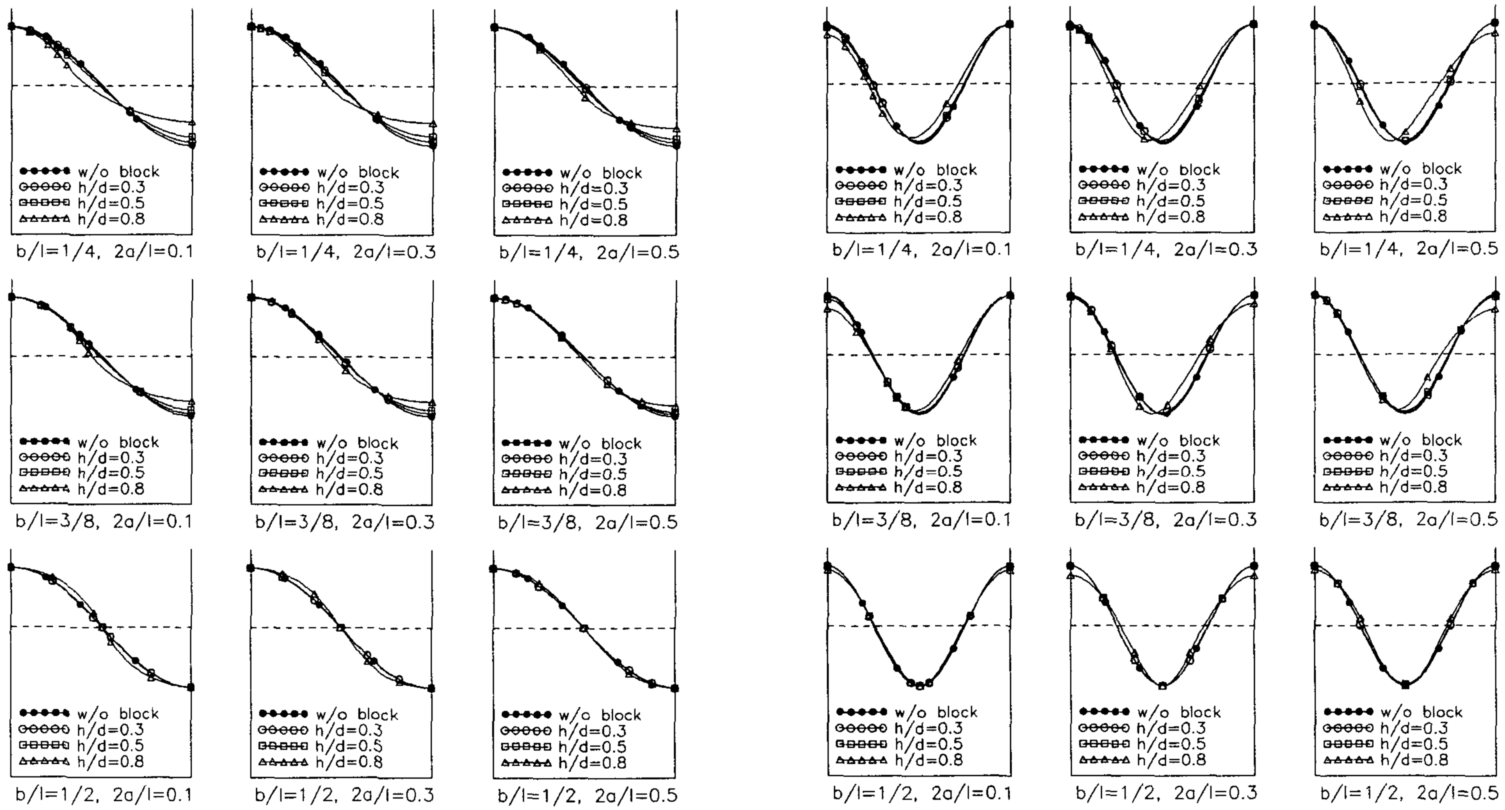
(a) $d/l = 0.5$ ($l = 20,\ d = 10$)   (b) $d/l = 2.0$ ($l = 5,\ d = 10$)

Figure 2: Sloshing frequencies for the variation of the block height ($a/l = 0.1, b/l = 0.5$)



(a) $b/l = 1/4$ (b) $b/l = 3/8$ (c) $b/l = 1/2$

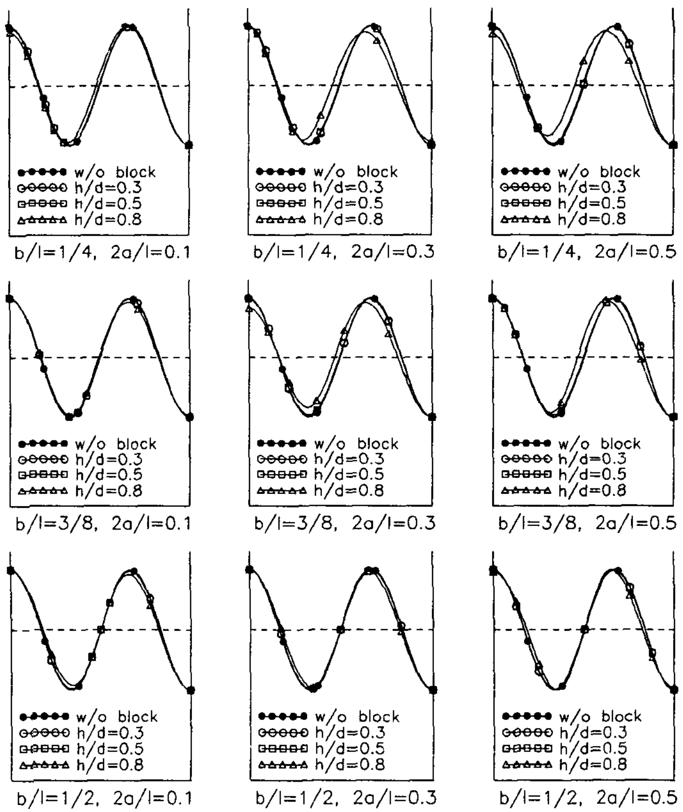
Figure 3: Sloshing frequencies for fluid tanks with a submerged block



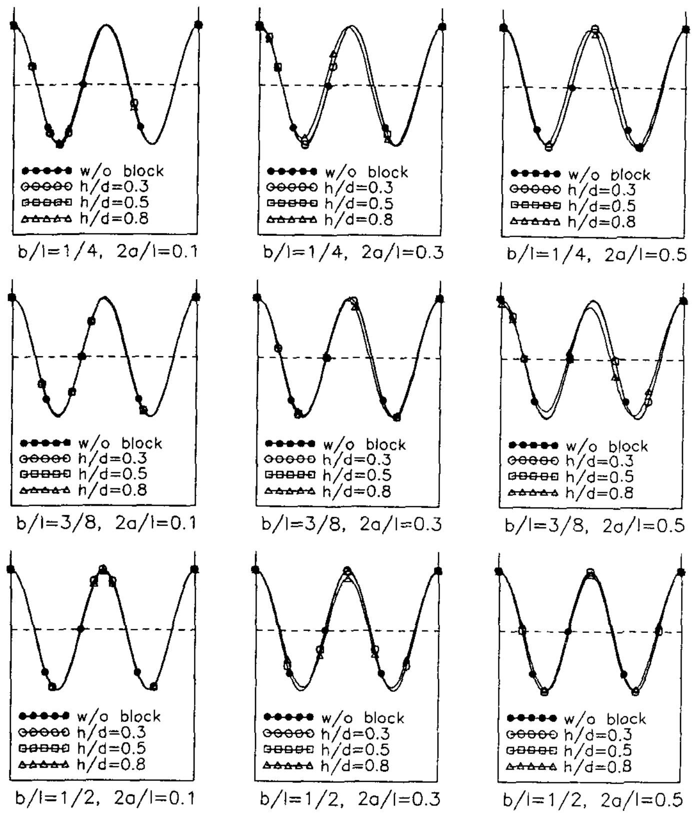
(a) First Mode

(b) Second Mode

Figure 4: Mode shapes for fluid tanks with a submerged block



(c) Third Mode



(d) Fourth Mode

Figure 4: Mode shapes for fluid tanks with a submerged block (continued)