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불연속 최적해의 흔들림 현상과 제어에 관한 연구

Oscillation Phenomena of the discrete Optimum Solutions and control

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ABSTRACT

In the discrete optimum design, occasionally, the solutions oscillate between the feasible and the infeasible resions during the series of redesigns of members with discrete sections. This phenomenon may be caused inherently by the discontinuity of variables of commercially available sections in the database. In this paper, in-depth investigation into the oscillation in the discrete optimization and its control has been conducted. When the structure is optimized through element optimization, the oscillation can be divided into two categories, local and global oscillations. An algorithm which controls these phenomena is suggested and numerical examples demonstrate the oscillation in optimum solutions and the effectiveness of the control strategy suggested here.

1. INTRODUCTION

In most of the previous structural optimization problems[1-3], the solutions (optimal designs) were obtained based on the assumption that design variables are continuous. AS many of the practical structures are composed of members of commercially available sections due to the economical efficiency and standardization, the continuous solutions can not be applied directly to such structures. The design of steel building frames using rolled sections is a typical example of that kind. Therefore, the discrete optimization techniques applied to the structures composed with standard sections have drawn a lot of investigator's efforts lately to obtain the optimal solutions in the specified member sizes for practical applications[4-5].

Most of such techniques, however, are not widely used in the practical problems yet since the stability, convergence and efficiency of the methods can not always be guaranteed when they are applied to the structural optimization problems of a large number of variables. In order to get rid of these difficulties, an optimization technique was

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developed for steel structures which are built with commercially available sections[6]. In this algorithm, the imposed constraints are decomposed into two levels, namely, element (component) level and structural level constraints. The optimal solution is first sought in the problem formulated with the element level constraints. Then, the structural level constraints are imposed.

The solutions obtained by the discrete optimization process may be inherently oscillatable because the intervals of available section variables are not generally equal and the safety margins of sections may be different from member to member. Some of the members may have so little safety margin that the section selected in the next step may fall into the infeasible region. A series of redesigns of these elements may cause the solutions to go back and forth between the feasible and the infeasible regions as the iteration continues.

In this paper, in-depth investigation into oscillation in the structural optimization through element optimization is carried out and the methods of its control are discussed. Oscillation is discussed in two categories, i.e., local and global oscillation.

2. CHARACTERISTICS OF OPTIMUM DESIGN WITH DISCRETE MEMBER SIZES

The optimum structural design problem in this paper may be defined as finding a design variable vector $\boldsymbol{b}$ that minimizes the objective function(the total weight of the structure) in the following form ;

$$\psi_o = \sum_{i=1}^{NE} W_i L_i A_i(b) \tag{1}$$

satisfying

$$\mathbf{h}(b,u) = \mathbf{K}(b)\ \mathbf{U} - \mathbf{P}(b) \tag{2}$$

subjected to element level constraints

$$\psi^s(b,u) \le 0 \tag{3}$$

$$\psi^v(b) \le 0 \tag{4}$$

$$\mathbf{b} \in \mathbf{T} \tag{5}$$

and structural level constraints

$$\psi^d(u) \in 0 \tag{6}$$

where $\boldsymbol{b}$ = design variable vector, NE = total number of elements, W_i, L_i and A_i = weight density, length, and cross sectional area of i-th element, respectively, $\mathbf{h}(b,u)$ = state equation(equilibrium equation in finite element method), $\mathbf{U}$ = state variable vector(n nodal displacements), u = elements of $\mathbf{U}$. $\mathbf{K}$ = (n x n) structural stiffness matrix, $\mathbf{P}$ = load vector(n nodal loads), ψ^s = stress constraints, ψ^d = constraints on the nodal displacements, ψ^v = design variable constraints, such as predetermined flange widths and beam depths, etc., and $\mathbf{T}$ = section table. The design variable vector $\boldsymbol{b}$ in this study must be found in $\mathbf{T}$. It is noted that the element level constraints are the stress and design variable constraints and the structural level constraints include the structural displacements and natural frequencies of the structure.

With the constraints decomposed as above, it was possible to maximize the combined merits of the optimal criteria method and those of the gradient projection method. Wheareas the former is used quite efficiently for structural optimizations under stress and design variable constraints, the latter is useful in obtaining the design directions, i.e., how the current member sizes should be changed to obtain better local behavior of constraint functions, particularly the displacement constraints. An algorithm using above concepts is found in [6]

3. OSCILLATION PROBLEMS IN ELEMENT OPTIMIZATION

3.1 Convergence of General Optimization Process

In the optimization process with discrete variables, it has been generally observed that the convergence process can be divided into three phases: the initial optimization of the objective function (upper bound or lower bound) is the first phase and subsequently, the adjustment phase for the convergence is followed by the final convergence phase. These three phases may appear repeatedly when there are multiple local optimum points and a certain phase may be omitted depending on the initial design and the step sizes in the subsequent redesign. Fig. 1 shows a typical model for the desirable optimization process with discrete variables. The continuous optimum solution (design) may define the boundary between the feasible and the infeasible regions if the objective function is linear and the discrete optimum solution is near the continuous optimum solution. If the optimization process is carried out as shown in Fig. 1 in which the solution has converged from point A to point $B=B'=B''=B'''$ after some adjustments. Occasionally, however, the process dose not converge to a correct solution but gives alternately feasible and infeasible solutions. This oscillation may occur in the discrete optimization process under element level constraints, i.e., the stress and design variable constraints, as defined in [6]. Therefore, caution must be used in the problem of oscillation and control of oscillation becomes necessary during the discrete optimization. Otherwise, either the converged solution can never be obtained or an incorrect solution may be obtained.

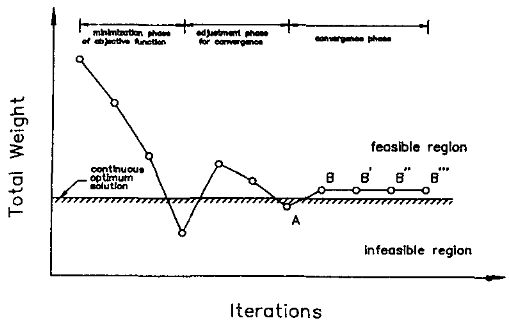

Fig. 1. Example of desirable optimization process

3.2 Local Oscillation Phenomenon

Some of the members in a structure may continually oscillate between feasible and infeasible regions under the imposed element level constraints during the iteration and the designs of such members may alternate between two sections a few steps apart in the section table. These members are called rascal members in this study. When some rascal members oscillate continuously in the convergence phase, the optimal solution can not be obtained and the optimization process will never end. This is called local oscillation in this study, as it is pertinent only to member design. It may also result in oscillation of objective function and the optimum design satisfying the imposed constraints may never be obtained.

To explain local oscillation graphically, two independent design variables, i.e., two individual members in a structure, which are imposed by one stress constraint which is linearly proportional to the design variable are assumed. Such an example of idealized local oscillation is shown in Fig. 2.a where the intersection points of the grid indicate the combination of members assembled with discrete sections in the section table. Here, b_j^i means the ith member in the structure with the jth section in the section table and $\Psi(b^i)$ means the maximum allowable stress for member b^i obtained by the reanalysis after the member design. As members in the indeterminate structure are dependent on each other, any change in member size causes the stiffness redistribution and the allowable stress level is also shifted from Ψ^A to Ψ^C corresponding to the allowable stress level at point A and C, respectively, as shown in Fig. 2.a. Fig. 2.b shows the variation of objective function as the iteration continues.

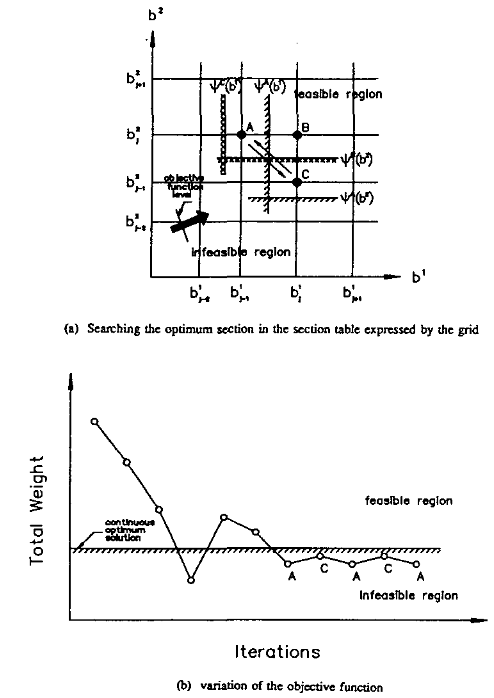

Fig.2. Typical example of local oscillation

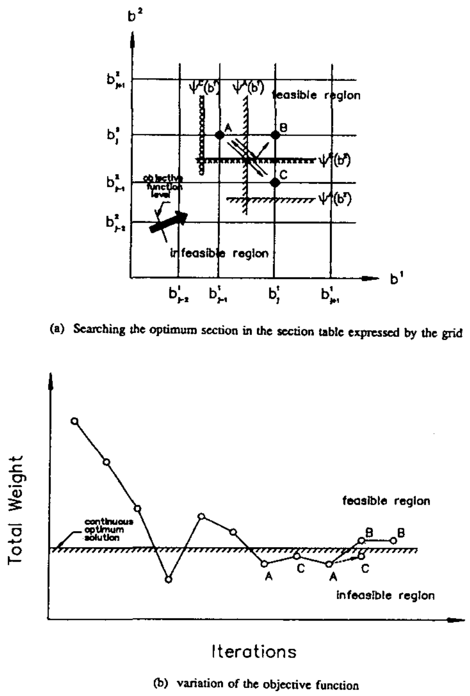

Fig.3. Example of the control of local oscillation

If optimization is reached at point A in Fig. 2.a where the design variable b^1 and b^2 are selected to be the $(j-1)$th and the jth sections in the section table, respectively, point A will be found to be fall into the infeasible region after the new allowable stresses of these members are obtained by the reanalysis. The search for a better solution will change the section of member b^1 from the $(j-1)$th to the jth section in the section table and that of member b^2 from jth to the $(j-1)$th section in the section table, thus making point C a new design. If the allowable stresses are changed from Ψ^A to Ψ^C because of the redistribution of stiffness as the result of reanalysis for point C as shown in Fig. 2.a, point C will fall into the infeasible region and the redesign for better solution at point C will result in changing back to point A again. Therefore, the subsequent optimization process will oscillate perpetually between point A and point C as shown in Fig. 2.b. and the sizes of members b^1 and b^2 are alternately selected at some interval in the section table. As a result, the design satisfying the imposed constraints of all members during the oscillation will never be obtained as shown in Fig. 2.b. This phenomenon is the local oscillation and these members involved are rascal members. Because of this oscillation of rascal members, the objective function may also oscillate. Instead of simple oscillation (A → C → A → C), a more complicated type of oscillation (A → ... → C → ... → A → ... C) will also be possible. The feasible design of this case during oscillation may never be obtained unless some way of controlling oscillation is found.

3.3 The Control of Local Oscillation

If either the member b^2 or b^1 is in the feasible region, i.e., at point A or C, and the solutions alternate continuously between points A and C, holding the jth section at a certain iteration, i.e., not allowing to be changed to the $(j-1)$th section, may bring the rascal members well under control. Expanding the above concept, the local oscillation may be controlled by imposing a lower limit of the section in the section table which any oscillating section will never be allowed to reach again. In other words, once the lower limit is imposed for each rascal member identified, the member is constrained to be designed to have a larger size than the lower limit imposed in the following iteration. Fig. 3 shows how imposing a lower limit can control the local oscillation. Suppose the solution is found to alternate between A and C, and at certain iteration, an improved (optimum) solution is sought from A, C will again be obtained as the next solution in the following iteration if no constraint is imposed, and vice versa. If, however, a constraint (limit) is imposed so that the solution can not reach C again in the following iteration, B will be obtained which is the converged solution instead of C for the next solution.

3.4 Global Oscillation Phenomenon and its Control

When the objective function approaches the converged optimal value, there may be another type of oscillation, i.e., oscillations of the objective function. This may be caused by one rascal member (continuously oscillating) or a group of rascal members (discontinuously oscillating) during iterations. Fig. 4 shows the typical patterns for oscillations discussed in the above paragraph. The oscillation pattern of Fig. 4.a may be caused by one rascal member and that in Fig. 4.b by a group of rascal members oscillating discontinuously. These oscillations are called global oscillation in this paper.

The control of global oscillation is easier than that of local oscillation. In this study, global oscillation is controlled by terminating the iteration at the minimum objective function which has been repeatedly met previously during the iterations. In Fig. 4, optimization process is terminated at point B'.

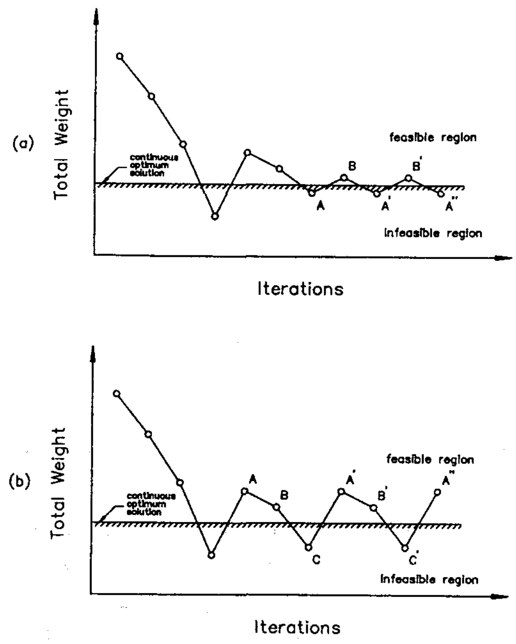

Fig. 4. Examples of global oscilation of the objective function

4. NUMERICAL EXAMPLE

This example shows local oscillation and its control. The structural configuration and loading conditions are shown in Fig. 5. The material properties are as following ; Young's modulus = 2.1×10^6kg/cm^2, weight density = 0.0079kg/cm^3, the allowable stress = 2400kg/cm^2. The standard sections available for the design are listed in Table 1 and the results obtained for this example are given in Table 2 and Fig. 6. The members marked as * in Table 2 are found to be in the infeasible region when reanalyzed.

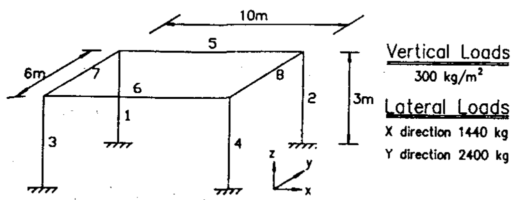

Fig. 5. Eight member frame

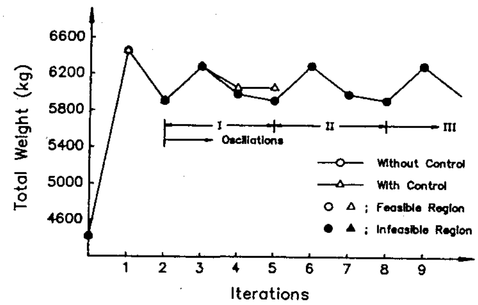

Fig. 6. Optimization for eight member frame

As shown in Table 2, members 1~6 oscillate between the feasible and the infeasible regions. These members are selected alternately at some interval in the sequence of section table (between sections 26 and 28 in columns and sections 76 and 82 in beams) after the second iteration and the patterns of oscillation in the second, third and fourth iterations are repeated again from the fifth iteration, i.e., the second design results are shown identically at the fifth iteration and so on. This is local oscillation, as defined in the earlier sections, and members 1~6 are rascal members.

To bring the oscillation under control, the control option should become effective at the second iteration. After the second design and reanalysis, the rascal members 3~5 in the infeasible region are identified and lower limits are imposed to design these members with section sizes greater than the imposed limits after the third iteration. After the third iteration and reanalysis, another rascal member, i.e., member 6 is in the infeasible region. This member is also brought under control after the fourth iteration. The solutions then converge to the optimum value as shown in Table 2, the converged minimum weight being 6049kg. This optimization process is shown in Fig. 6 and the line with circles indicates the optimization process without control and the line with triangulars the controlled optimization process.

5. CONCLUSIONS

Oscillation of the objective function which occasionally occurs in the discrete optimization process under element level constraints, i.e., the stress and design variable constraints,

Table 1. Available sections

HC Group(columns)				HB Group(beams)			
No	I(cm^4)	A(cm^2)	Z(cm^3)	No	I(cm^4)	A(cm^2)	Z(cm^3)
1	2880.0	51.21	330.0	51	187.0	11.85	37.5
2	4720.0	63.53	472.0	52	413.0	16.84	66.1
3	4980.0	71.53	498.0	53	666.0	17.85	88.8
4	6530.0	83.69	628.0	54	1020.0	26.84	138.0
5	8790.0	82.06	720.0	55	1210.0	23.04	139.0
6	9930.0	84.70	801.0	56	1530.0	29.65	181.0
7	10800.0	92.18	867.0	57	1580.0	23.18	160.0
8	11500.0	104.70	919.0	58	1840.0	27.16	184.0
9	16900.0	107.70	1150.0	59	2690.0	39.01	277.0
10	18800.0	110.80	1270.0	60	3540.0	32.68	285.0
11	20400.0	119.80	1360.0	61	4050.0	37.66	324.0
12	21500.0	134.80	1440.0	62	6120.0	56.24	502.0
13	23400.0	134.80	1540.0	63	6320.0	40.83	424.0
14	28200.0	135.30	1670.0	64	7210.0	46.78	481.0
15	33300.0	146.00	1940.0	65	11100.0	52.68	641.0
16	35300.0	166.60	2050.0	66	11300.0	72.38	771.0
17	40300.0	173.90	2300.0	67	13300.0	83.36	893.0
18	42800.0	198.40	2450.0	68	13600.0	63.14	775.0
19	47600.0	202.00	2670.0	69	18500.0	88.15	1100.0
20	49000.0	178.50	2520.0	70	20000.0	72.16	1010.0
21	56100.0	186.80	2850.0	71	21700.0	101.50	1280.0
22	59700.0	241.40	3030.0	72	23700.0	84.12	1190.0
23	66600.0	218.70	3330.0	73	28700.0	84.30	1290.0
24	70900.0	250.70	3540.0	74	33500.0	96.76	1490.0
25	78000.0	254.90	3840.0	75	33700.0	120.10	1740.0
26	92800.0	295.40	4480.0	76	38700.0	136.00	1980.0
27	107000.0	361.80	5120.0	77	41900.0	101.30	1690.0
28	119000.0	360.70	5570.0	78	46700.0	135.00	2160.0
29	129000.0	424.90	6030.0	79	47800.0	114.20	1910.0
30	142000.0	423.30	6470.0	80	56100.0	157.40	2550.0
31	152000.0	489.00	6950.0	81	56500.0	131.30	2230.0
32	177000.0	554.10	7900.0	82	60400.0	145.50	2500.0
33	187000.0	528.60	8170.0	83	68700.0	120.50	2310.0
34	214000.0	593.70	9130.0	84	71000.0	163.50	2910.0
35	233000.0	612.00	9740.0	85	77600.0	134.40	2590.0
36	242000.0	659.80	10100.0	86	90400.0	152.50	2980.0
37	260000.0	755.40	10900.0	87	102999.0	107.70	3380.0
38	298000.0	770.10	12000.0	88	103000.0	174.50	3530.0
39	331000.0	838.70	13000.0	89	118000.0	192.50	4020.0
40	358000.0	965.70	14100.0	90	137000.0	222.40	4620.0
41	414000.0	942.90	15400.0	91	172000.0	211.50	4980.0
42	433000.0	1024.00	16100.0	92	201000.0	235.50	5760.0
43	472000.0	1185.00	17600.0	93	237000.0	273.60	6700.0
44	551000.0	1214.00	19400.0	94	254000.0	243.40	6410.0
45	737000.0	1488.00	24300.0	95	292000.0	267.40	7290.0
46	–	–	–	96	339000.0	307.60	8400.0
47	–	–	–	97	345000.0	270.90	7760.0
48	–	–	–	98	411000.0	309.80	9140.0
49	–	–	–	99	498000.0	364.00	10900.0
50	–	–	–	100	–	–	–

Table 2. Local oscillation and its control for eight member frame

control	element	iterations								
options	number	initial design	1st	2nd	3rd	4th	5th	6th	7th	8th
without	1	9	28	28	28	*26	28	28	*26	28
	2	14	28	28	28	*26	28	28	*26	28
	3	19	28	*26	28	28	*26	28	28	*26
	4	24	28	*26	28	28	*26	28	28	*26
	5	59	80	*76	82	76	*76	82	76	*76
	6	71	82	76	*76	82	76	*76	82	76
control	7	82	70	70	70	70	70	70	70	70
	8	91	70	70	70	70	70	70	70	70
	weight	4423 kg	6455	5909	6285	5979	5909	6285	5979	5909
with	1	9	28	28	28	26	26			
	2	14	28	28	28	26	26			
	3	19	28	*26	28	28	28			
	4	24	28	*26	28	28	28			
	5	59	80	*76	82	82	82			
	6	71	82	76	*76	82	82			
control	7	82	70	70	70	70	70			
	8	91	70	70	70	70	70			
	weight	4423 kg	6455	5909	6285	6049	6049			

* ; infeasible sections

was defined as in [6]. Besides the minimization algorithm, an oscillation control algorithm is needed to the successful discrete optimization. Oscillation can be divided into two types : local oscillation, or oscillation of the objective function because of continuously oscillating rascal members in convergence phase, and global oscillation, or oscillation of the objective function because of discontinuously oscillating rascal members or one rascal member.

The former can be controlled by imposing a lower limit for the design of the selected members (rascal members), and the latter can be controlled by ending the iteration when the minimum objective function satisfying all the imposed constraints is repeatedly met. Numerical example showed the oscillation control algorithm suggested here can effectively control oscillation of the objective function and can accelerate the convergence of optimal solutions.

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