EVALUATION OF VOLUME VELOCITY OF A LOUDSPEAKER IN A CHAMBER

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ABSTRACT The volume velocity of an acoustic source is important in determining various acoustic parameters. One of the suggested techniques is the internal pressure method incorporating a loudspeaker attached to a chamber wall and a microphone inserted into the cavity. Although the method is easy to handle with a very simple measurement setup, the coupling effects between the dynamic system of the loudspeaker and acoustic field, and the effects of higher order modes introduced by the discontinuities in the acoustic field should be considered for precise result. In this study, higher order modes due to the discontinuities of loudspeaker and microphone boundaries are included and the electro-acoustic coupling effects are compensated for by using the results of two cylinders with different lengths. The volume velocity of a loudspeaker thus obtained agrees very well with that measured by laser sensor.

1. INTRODUCTION

The volume velocity of an acoustic source is considered as being important in the areas of transfer function analysis using the vibro-acoustic reciprocity theorem, in the active noise control, and in the measurement of impedance and absorption of the materials. Frequently, the moving-coil loudspeaker is utilized as an acoustic source for many experiments due to its ease in control, and several methods have been developed to obtain the volume velocity of a loudspeaker heretofore. Among these, Anthony and Elliott[1] suggested a very simple method for measuring the volume velocity by using the acoustic transfer function between the velocity of a loudspeaker located at an end of the enclosing tube and the acoustic pressure measured by a microphone inside the acoustic field. In this method, the best position of microphone is suggested to be $0.43 l_0$ from the loudspeaker end where l_0 means the chamber length. Since the acoustic field can be changed due to the insertion of the microphone and the acoustic coupling between the interior cavity and loudspeaker is not taken into account, the result can contain an appreciable error. In this study, a method is suggested to analyze the acoustic field with a blockage inside the cylindrical chamber.

2. ANALYTIC MODEL

Fig. 1 shows a schematic layout of the chamber for measuring the volume velocity of a loudspeaker, where a microphone is inserted into the cylindrical enclosure. The source is assumed to be a moving piston fluctuating back and forth in a simple harmonic motion. The acoustic field can be expressed by the following Helmholtz integral equation[2]:

$$c(r)p(r) = \int_{S_b} \left[-\mathrm{i}\, k\rho c u(r_b) G(r|r_b) + p(r_b) \partial G(r|r_b) / \partial n_b \right] dS_b \ , \tag{1}$$

where S is the area, the subscript b signifies the boundary surface, $\partial / \partial n_b$ is the normal derivative on the boundary point towards interior acoustic field, and $c(\boldsymbol{r}) = 1$ for $\boldsymbol{r}$ inside the boundary and $c(\boldsymbol{r}) = 0.5$ on the boundary points. If the Green's function satisfies the Neumann boundary condition on the boundaries, Eq. (1) can be rewritten with subdivided surfaces as

$$p(\boldsymbol{r}) = -\mathrm{i}\, k\rho c \sum_{j=1}^{N} \int_{S_j} u(\boldsymbol{r}_j)\, G(\boldsymbol{r}|\boldsymbol{r}_j)\, dS_j \ . \tag{2}$$

The pressure and velocity can be expressed by an orthonormalized eigenfunction set as follows :

$$p(\boldsymbol{r}_j) = \sum_{n=0}^{\infty} p_{jn} \Phi_n(\boldsymbol{r}_j) \quad , \quad u(\boldsymbol{r}_j) = \sum_{n=0}^{\infty} u_{jn} \Phi_n(\boldsymbol{r}_j) \ . \tag{3}$$

By substituting Eq. (3) into (2), $p_{j'm}$ can be obtained as

$$p_{j'm} = -\mathrm{i}\, k\rho c \sum_{j=1}^{N} \sum_{n=0}^{\infty} u_{jn} \int_{S_{j'}} \int_{S_j} \Phi_m(\boldsymbol{r}_{j'}) \Phi_n(\boldsymbol{r}_j) G(\boldsymbol{r}_{j'}|\boldsymbol{r}_j)\, dS_{j'} dS_j \equiv \sum_{j=1}^{N} \sum_{n=0}^{\infty} \rho\, c (T_{j'j})_{mn} u_{jn} \ , \tag{4}$$

where this can be reexpressed in the matrix form as

$$\boldsymbol{P}_{j'} = \rho c \sum_{j=1}^{N} \boldsymbol{T}_{j'j} \boldsymbol{U}_j \qquad \text{for } j' = 1\,,2\,,\ \ldots\,,N\ . \tag{5}$$

In the region A in Fig. 1, all the boundaries except surface 1 are rigid, and the corresponding matrix relation is given by

$$\boldsymbol{P}_1 = \rho c \boldsymbol{T}_{11} \boldsymbol{U}_1 \ . \tag{6}$$

Meanwhile, in the region B, all the boundaries except surfaces 2 and 4 are rigid, and the corresponding matrix relations can be expressed as

$$\boldsymbol{P}_2 = \rho c (\boldsymbol{T}_{22} \boldsymbol{U}_2 + \boldsymbol{T}_{24} \boldsymbol{U}_4) \ \text{ and } \ \boldsymbol{P}_3 = \rho c (\boldsymbol{T}_{32} \boldsymbol{U}_2 + \boldsymbol{T}_{34} \boldsymbol{U}_4) . \tag{7}$$

The continuities of velocity and pressure vectors on the surfaces 1 and 2 are given by

$$\boldsymbol{P}_1 = \boldsymbol{P}_2 \ \text{ and } \ \boldsymbol{U}_1 = -\boldsymbol{U}_2 \ . \tag{8}$$

Rearranging Eqs. (7) and (8), one can derive the following matrix relationship:

$$\boldsymbol{P}_3 = \rho c \{ \boldsymbol{T}_{34} - \boldsymbol{T}_{32} [\boldsymbol{T}_{11} + \boldsymbol{T}_{22}]^{-1} \boldsymbol{T}_{24} \} \boldsymbol{U}_4 \ , \tag{9}$$

Because the pressure measured by the microphone is the averaged value over the microphone membrane face, the measured pressure can be given by

$$p(\boldsymbol{r}_3) = (p_3)_0 \Phi_0(\boldsymbol{r}_3) \ . \tag{10}$$

Consequently, one can obtain the pressure by finding the mode relation coefficient $(p_3)_0$ only.

In Ref. [1], the volume velocity is obtained by measuring the plane wave component ignoring the microphone's blockage effect inside the acoustic field (let it be PWI hereafter). In addition, the plane-wave approach considering blockage effect (PWC) can be introduced as a more refined model than PWI. In order to clearly compare the foregoing analytic methods with each other, a nondimensional parameter is introduced as follows :

$$P_r = \left[p(\boldsymbol{r}_3) / u(\boldsymbol{r}_4) \right] \Big|^{l_A} / \left[p(\boldsymbol{r}_3) / u(\boldsymbol{r}_4) \right] \Big|^{0} \ , \tag{11}$$

where the superscript means the insertion length of the microphone. For the PWI and PWC, the following equations can be easily derived from, respectively.

$$P_r\Big|_{PWI} = \cos(kl_A) \ , \tag{12a}$$

and

$$P_r\Big|_{PWC} = \frac{\sin(kl_0)\cos(kl_A)}{[1-(w_M / w_0)^2 / 2]\sin(kl_0) + (w_M / w_0)^2 \sin[kl_0(1-2L_A)]/2} \ . \tag{12b}$$

Fig. 2 shows that only one mode coefficient at each surface is enough to describe the acoustic field in the plane wave range where $kw_0 < 3.83$. When w_M / w_0 is smaller than 1, the difference between PWI and PWC is considerably small except in the vicinity of natural frequencies. However, one can find in Fig. 3 that the deviation of the present method from PWC increases along with the Helmholtz number kw_0. This is because the evanescent wave generated from surface 4 will fully decay due to the long travel distance, but the one from surface 2 can affect greatly on the measured pressure for large kw_0 and/or for $w_M / w_0 \approx 0.5$.

3. EXPERIMENTS

In Fig. 4, experimental results are compared with predicted ones by using the present theory, and they agree very well in the plane wave region. In this result, it is noted that the effect of microphone insertion into the cavity is not negligible, especially at high frequencies. The volume velocity of the loudspeaker can be estimated by the following formula:

$$V = u(r_4)S_4 = \frac{[p(r_3)/I]\Big|_{EXP}^{l_A}}{[p(r_3)/u(r_4)]\Big|_{BM}^{l_A}} I\Big|_{EXP}^{l_A} S_4 \ , \tag{13}$$

where I is the driving current and the subscript BM indicates the present method. In the foregoing analysis, it can be said that a simple way for volume velocity measurement is to locate the microphone at $l_A = 0$ plane and to use a cylinder with short length. However, even in this cases, the higher modes should be considered and the coupling between the dynamic system of loudspeaker and the acoustic field is unavoidable in the low frequency range. When the microphone is inserted into the acoustic field, the peaks occur at the zeroes of $[p(r_3)/u(r_4)]\Big|_{BM}^{l_A}$ as depicted in Fig. 5. This is due to the longitudinal resonance of annular cavity formed by the microphone and partly due to the small error in the measurements. To sort out the problem, a good measurement location is searched for and the position is found to be $l_A / l_0 \approx 0.4831$.

Because loudspeakers have very small internal impedance in general, one cannot ignore the radiation impedance loaded on the moving surface that the coupling occurs between the dynamic system of a loudspeaker and acoustic field. Fig. 5 illustrates this fact clearly by two examples with varying length. Therefore, the volume velocity estimated from Eq. (13) should be corrected by considering the aforementioned interaction effect. The relationship between the loudspeaker velocity and driving current can be derived as[3]

$$K_f = (Q_U + Z_{SF}S_4)[u(r_4)/I] \ , \ Q_U = Z_M + Z_{SB}S_4 \ , \tag{14}$$

where K_f is the coefficient of driving characteristics, Z_M means the internal mechanical impedance of the loudspeaker, and Z_{SF} , Z_{SB} are the acoustic impedances on front and rear surfaces of the loudspeaker, respectively. Here, the unknown quantity Q_U is the parameter needed to compensate Eq. (13) for the loading effects of the loudspeaker. Q_U can be derived from Eq. (14) for two different chamber lengths as

$$Q_U = \frac{(-Z_{SF}^{I} + Z_{SF}^{II} R) S_4}{1 - R}, \quad R = \frac{(u_4 / I)^{II}}{(u_4 / I)^{I}}, \tag{15}$$

where Roman superscript indicates the value at each chamber. Consequently, the volume velocity V of a loudspeaker mounted on an infinite baffle can be obtained by using Eq. (15) as

$$V = \frac{Q_U + Z_{SF}^{I} S_4}{Q_U + Z_{SF}^{III} S_4} \left(\frac{u_4}{I} \right)^{I} I^{III} S_4 = \frac{Z_{SF}^{I} - Z_{SF}^{II}}{(1/R)(Z_{SF}^{I} - Z_{SF}^{III}) - (Z_{SF}^{II} - Z_{SF}^{III})} \left(\frac{u_4}{I} \right)^{I} I^{III} S_4 . \tag{16}$$

Here, the acoustic impedance Z_{SF}^{III} on an infinite baffle is given by [4]

$$Z_{SF}^{III} = 1 - \mathrm{J}_1(2kw_0)/kw_0 + \mathrm{i} H_1(2kw_0)/kw_0, \tag{17}$$

where J_1 is the first order Bessel function of the first kind and H_1 denotes the first order Struve function. In Eq. (16), if frequencies are located at the resonance of the case I, the volume velocities are simply $(u_4/I)^{II} I^{III} S_4$. In other words, the dips of curve in Fig. 5 are replaced by the case II. In Fig. 6, estimated volume velocity by Eq. (16) is compared with experimental result by using laser sensor and they agree reasonably well. In this figure, it is very clear that the estimated volume velocity without considering higher order modes and interaction effect deviates from the true value very much.

4. CONCLUSION

A measurement technique for volume velocity in a chamber is analyzed for a general testing layout with respect to the geometrical implications and acoustic modes. From the analytical result, it is observed that a considerable error can occur without any proper compensation when a measuring microphone is inserted into the cylindrical chamber. In order to compensate the coupling effect, two chambers with different lengths are employed, and the results for two chambers are combined to yield the true volume velocity. With the proposed "boundary mode" method, one can estimate the volume velocity of an acoustic source in precision without *a priori* information on the characteristics of the loudspeaker.

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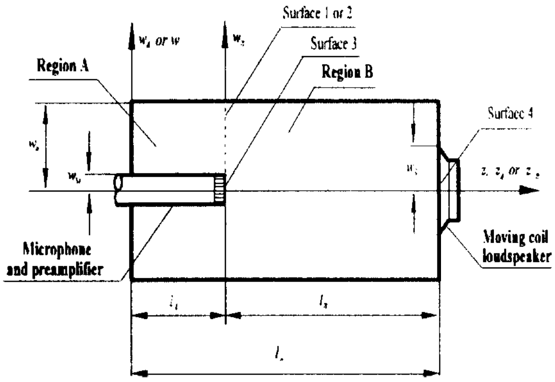

Fig. 1. Geometric description of measurement system for volume velocity.

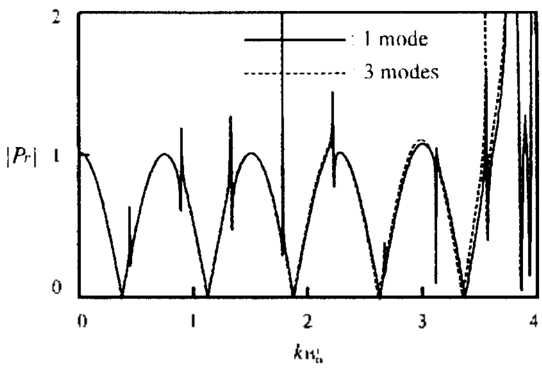

Fig. 2. Predicted $|P_r|$ varying the included surface mode numbers in computation
($l_0 / w_0 = 7.05$, $l_A / l_0 = 0.57$,
$w_M / w_0 = 0.175$, $w_S / w_0 = 0.825$).

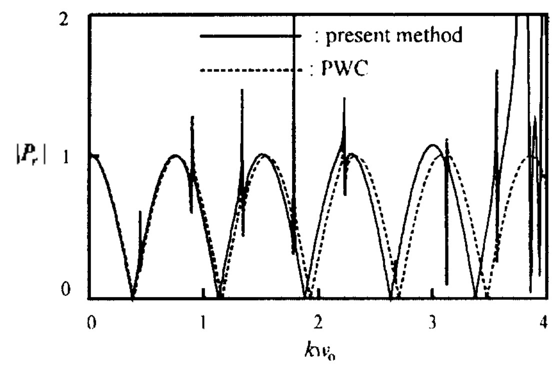

Fig. 3. Comparison between the results of PWC and the present method
($l_0 / w_0 = 7.05$, $l_A / l_0 = 0.57$,
$w_M / w_0 = 0.175$, $w_S / w_0 = 0.825$).

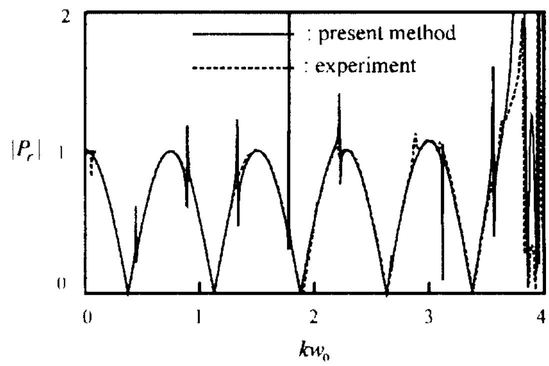

Fig. 4. Comparison between the predicted and experimental results ($l_0 / w_0 = 7.05$, $l_A / l_0 = 0.57$,
$w_M / w_0 = 0.175$, $w_S / w_0 = 0.825$).

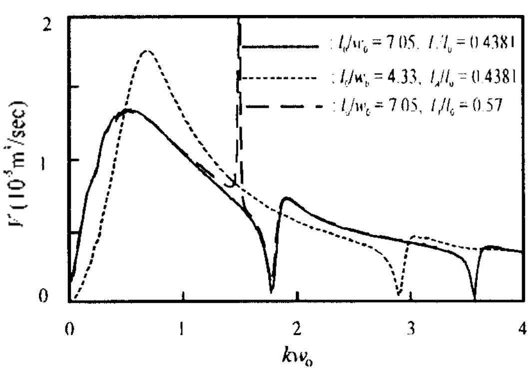

Fig. 5. Estimated volume velocity without considering the interaction effect
($w_s / w_0 = 0.825$).

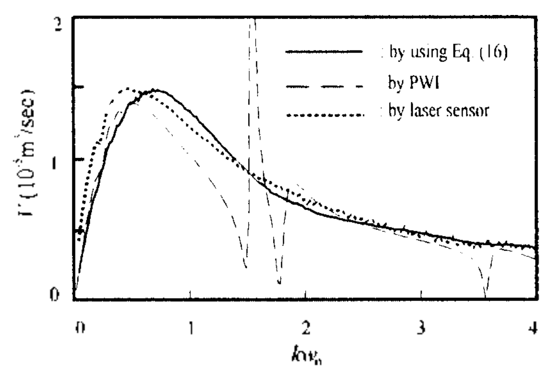

Fig. 6 Comparison of the estimated volume velocities with experimental result
($l_0 / w_0 = 7.05$, $l_A / l_0 = 0.4381$, $w_S / w_0 = 0.825$).