THEORY OF BACKGROUND NOISE CANCELLATION ON PREDICTION OF RESPONSE PROBABILITY DISTRIBUTION FOR AN ARBITRARY SOUND WALL SYSTEM AND ITS APPLICATION TO ACTUAL SOUND WALL SYSTEMS

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ABSTRACT In the actual situation of measuring the environmental noise, it is very often that only the resultant phenomenon fluctuation contaminated by the additional noise of arbitrary distribution type can be observed. Furthermore, the observed data is usually given in a sound level form on dB scale based on the non−linear transformation of sound pressure. Therefore, for the purpose of estimating only the undisturbed objective ouptut response, some estimation method is necessary to reasonably remove the effect of the above additional noise. In this paper, first, a mathematical model of arbitrary sound insulation systems is introduced in the form of a linear system on intensity scale, by using the well−known additive property of energy quantities. Next, some estimation method of the output response under the existence of background noise is derived. Then, based on the expression of the above estimation method, a new prediction method of only the output response probability function form for arbitrary sound insulation systems without a background noise is proposed by use of observed data contaminated by a background noise. Finally, the effectiveness of the proposed method is confirmed experimentally too by applying it to the actual various type sound wall systems.

1. INTRODUCTION

In the actual measurement of environmental noise, the desired signal is very often contaminated by the additional noise of an arbitrary distribution type and it is only this resultant signal that can be measured [1]. Therefore, for the purpose of determining solely the uncontaminated, objective output response, it is necessary to find some method of removing the effects of the additional noise. Furthermore, the observed data are usually given in a sound level form (dB scale) based on the logarithmic type non−linear transformation of sound pressure.

In this paper, first, a method of evaluating the output response probability of the system in the presence of background noise is found without introducing any artificial error criterion like the least−squares method. More specifically, a mathematical model of arbitrary sound insulation systems is introduced by using the well−known additive property of energy [2] or intensity quantities as a linear system based on intensity. Based on the above probability expression, a new estimation procedure is developed by which the uncontaminated output response probability function form for arbitrary sound insulation systems can be derived from the observed data contaminated by a background noise. Then, based on the proposed background noise cancellation method, the prediction problem of the system response probability with the other kinds of stochastic input of arbitrary distribution type and the contamination of the same background noise is discussed as its application example. Finally, the practical effectiveness of our new method is confirmed experimentally too by applying it to the observed response data of various types of sound wall systems.

2. THEORETICAL CONSIDERATIONS

2.1 A Generalized Mathematical Model for Arbitrary Insulation Systems Based on an Intensity Scale

Based on the additive property of sound intensity, the arbitrary sound insulation systems on an intensity scale can be described by the following linear system model :

$$\xi_k = \sum_{i=1}^{N} a_i x_{k-i+1} , \tag{1}$$

where ξ_k and x_{k-i+1} are the system output and input at two discrete times k and $k-i+1$, respectively. Here, the acoustic system whose order N and parameter a_i are known in advance is considered. Furthermore, let us consider the observation mechanism based on the non−linear transformation as follows :

$$y_k = f(\xi_k) , \tag{2}$$

$$z_k = \xi_k + \sum_{i=1}^{N+1} a_i v_{k-i+1} , \tag{3}$$

where $f(\cdot)$ denotes the mechanism of non−linear transformation measurement. Also, z_k and y_k are two kinds of observed data with and without the existence of background noise. Furthermore, v_i (i=1,2,⋯N) and v_i (i=N+1) denote the sound intensities of background noises added on the input and output sides, respectively.

2.2 Estimation of System Response Probability After Background Noise Cancellation

First, we derive the synthetic probability density function (abbr. p.d.f.) of the stochastic system response, in the presence of background noise, after the non−linear transformation in Eqs.(2) and (3). Then, based on this synthetic probability expression, it becomes possible to estimate analytically a p.d.f. of the objective output response under no existence of background noise. More specifically, we introduce an arbitrary function $\psi(z_k)$ which plays the role of a certain kind of the catalytic operation in the decomposition the above synthetic expression for the p.d.f. Then, we examine the expectation value of this arbitrary function :

$$I = \langle \psi(z_k) \rangle = \int_{-\infty}^{\infty} \psi(z_k) p_z(z_k) dz_k , \tag{4}$$

where $p_z(z_k)$ is a p.d.f. of z_k. Here, it seems to be natural to assume that the ith (i=0,1,2,⋯)

successive derivatives of $\psi(z_k)$ and/or $p_z(z_k)$ tend to zero at the boundary region $z_k \to \pm\infty$. After substituting Eq.(3) into $\psi(z_k)$ and expanding this in a Taylor's series form, under the above natural boundary condition, $\psi(z_k)$ can be rewritten as follows :

$$\psi(z_k) = \sum_{n=0}^{\infty} [(\sum_{i=1}^{N+1} a_i\ v_i)/n!](d/d\ \xi_k)\ \psi(f(\xi_k)) \ . \tag{5}$$

Accordingly, after substituting Eq.(5) into Eq.(4) and successively integrating by parts, the expectation I of the arbitrary function $\psi(z_k)$ can be obtained, under the above natural boundary condition, as follows :

$$I = \int_{-\infty}^{\infty} \psi(y_k)\{\sum_{n=0}^{\infty}(-1)^n/n![(1/(df^{-1}(y_k)/dy_k))d/dy_k]^n$$

$$\cdot[<(\sum_{i=1}^{N+1} a_i v_i)^n \mid f^{-1}(y_k)>p_y(y_k)]\}\ dy_k \tag{6}$$

Thence, after the variable transformation ($y_k \to z_k$) and comparison of the definition of the expectation of the arbitrary function in Eq.(4) with Eq.(6), the p.d.f. $p_z(z_k)$ of z_k can be derived as follows, :

$$p_z(z_k) = p_y(z_k) + \sum_{n=1}^{\infty}(-1)^n/n![(1/(df^{-1}(z_k)/dz_k))d/dz_k]^n$$

$$\cdot[<(\sum_{i=1}^{N+1} a_i\ v_i)^n \mid f^{-1}(z_k)>p_y(z_k)] , \tag{7}$$

or

$$p_y(z_k) = p_z(z_k) - \sum_{n=1}^{\infty}(-1)^n/n![(1/(df^{-1}(z_k)/dz_k))d/dz_k]^n$$

$$\cdot[<(\sum_{i=1}^{N+1} a_i\ v_i)^n \mid f^{-1}(z_k)>p_y(z_k)] . \tag{8}$$

When the input and the background noise are statistically independent of each other, we obtain directly the following expression of the output p.d.f. :

$$p_z(z_k) = p_y(z_k) + \sum_{n=1}^{\infty}(-1)^n \sum_{n_1+n_2+\cdots=n} (a_1^{n_1}\cdot a_2^{n_2}\cdots a_{N+1}^{n_{N+1}})/(n_1!\cdot n_2!\cdots n_{N+1}!)$$

$$\cdot <(v_1^{n_1}\cdot v_2^{n_2}\cdots v_{N+1}^{n_{N+1}})>[(1/(df^{-1}(z_k)/dz_k))d/dz_k]^n p_y(z_k) \ . \tag{9}$$

Consequently, based on the above synthetic probability expressions Eqs.(7) and (8), it is possible to estimate analytically only the undisturbed p.d.f. $p_y(y_k)$ of the objective output y_k in the absence of background noise for the arbitrary sound insulation systems. That is, after substituting the expression of Eq.(7) into the first term in the expansion expression of the right hand side of Eq.(8) and successively repeating the same procedure, the following expression of $p_y(y_k)$ can be derived :

$$p_y(y_k) = p_z(y_k) - \sum_{n_1=1}^{\infty} A_{n_1} [(1/(df^{-1}(y_k)/dy_k))d/dy_k]^{n_1}$$

$$\cdot [<(\sum_{i=1}^{N+1} a_i v_i)^{n_1} \mid f^{-1}(y_k)>] p_z(y_k) \quad + \quad \cdots$$

$$+ (-1)^s \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \cdots \sum_{n_s=1}^{\infty} \prod_{k=1}^{s} A_{n_k} [(1/(df^{-1}(y_k)/dy_k))d/dy_k]^{n_k}$$

$$\cdot [<(\sum_{i=1}^{N+1} a_i v_i)^{n_k} \mid f^{-1}(y_k)>] p_z(y_k) \quad \cdots \tag{10}$$

with $A_{n_k} = (-1)^{n_k} / n_k !$. Similarly, for the special case when input and background noise are statistically independent, Eq.(10) is easily rewritten as follows :

$$p_y(y_k) = p_z(y_k) - \sum_{n_1=1}^{\infty} A_{n_1} <(\sum_{i=1}^{N+1} a_i v_i)^{n_1}> [(1/(df^{-1}(y_k)/dy_k))d/dy_k]^{n_1} p_z(y_k) \; + \; \cdots$$

$$+ (-1)^s \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \cdots \sum_{n_s=1}^{\infty} \prod_{k=1}^{s} A_{n_k} <(\sum_{i=1}^{N+1} a_i v_i)^{n_k}>$$

$$\cdot [(1/(df^{-1}(y_k)/dy_k))d/dy_k]^{n_k} p_z(y_k) \quad \cdots \quad . \tag{11}$$

Therefore, the p.d.f. expression for the output response of the sound insulation systems without background noise can be estimated from the observed data after the non−linear transformation of data which include background noise.

2.3 Prediction of the System Response Probability with the Other Stochastic Input and the same Background Noise (Application)

In this section, we consider the problem of predicting the p.d.f. of the output response for a sound insulation system excited by an arbitrary stochastic input, based on the output p.d.f. for the system excited by a specific reference input with contamination by a background noise.

The relationship between the system output p.d.f. $p_y(y_k)$ for the arbitrary stochastic input and the system output p.d.f. $p_{y0}(y_k)$ for the specific reference input can be given in the following expression :

$$p_y(y_k) = \sum_{l=0}^{\infty} (-1)^l / l! \cdot B_l \, (d/dy_k)^l \, p_{y0}(y_k) \, , \tag{12}$$

where, $B_0 = 1$, $B_l = <y_k^l> - \sum_{j=0}^{l-j} {}_lC_j <y_{0k}^{l-j}> B_j$.

A well−known unified p.d.f. expression of Gram−Charlier A series type can be adopted as the p.d.f. of $p_{y0}(y_{0k})$:

$$p_{y0}(y_{0k}) = \sum_{n=0}^{\infty} A_n H_n((y_{0k} - \mu_{y_{0k}})/\sigma_{y_{0k}}) N(y_{0k};\mu_{y_{0k}}, \sigma^2_{y_{0k}}), \tag{13}$$

where $\mu_{y_{0k}} = <y_{0k}>$, $H_n(\cdot) = (-1)^n exp\{(\cdot)^2/2\}\cdot(d/d(\cdot))^n exp\{-(\cdot)^2/2\}$, (14)

$$\sigma^2_{y_{0k}} = <(y_{0k} - \mu^2_{y_{0k}})>, \quad A_n = <H_n\{(y_{0k} - \mu_{y_{0k}})/\sigma_{y_{0k}}\}>/n! \tag{15}$$

and

$$N(y_{0k};\mu_{y_{0k}}, \sigma_{y_{0k}}) = (2\pi)^{-1/2}/\sigma_{y_{0k}} \cdot exp\{-[(y_{0k} - \mu_{y_{0k}})/\sigma_{y_{0k}}]^2/2\}. \tag{16}$$

This can be rewritten as follows:

$$(d/dy_{0k})^l p_{y0}(y_{0k}) = (-1)^l(\partial/\partial\mu_{y_{0k}})^l[\sum_{n=0}^{\infty} A_n \sigma^n_{y_{0k}}(\partial/\partial\mu_{y_{0k}})^n N(y_0;\mu_{y_{0k}}, \sigma^2_{y_{0k}})]$$

$$= (-1)^l(\partial/\partial\mu_{y_{0k}})^l p_{y0}(y_{0k}). \tag{17}$$

Thence, the system output p.d.f. $p_y(y_k)$ for the arbitrary stochastic input can be expressed in the parameter differential form of the system output p.d.f. $p_{y0}(y_k)$ for the reference stochastic input as follows:

$$p_y(y_k) = \sum_{l=0}^{\infty} B_l/l!\cdot(\partial/\partial\mu_{y_{0k}})^l p_{y0}(y_k). \tag{18}$$

As was shown previously in Eq.(9), in the natural case when the input and the background noise are mutually independent, the relationship between $p_z(z_k)$ and $p_y(z_k)$ is given as follows:

$$p_z(z_k) = \sum_{n=0}^{\infty}(-1/b)^n S_n D_z^n p_y(z_k) \tag{19}$$

with $b=10/\ln 10$, $D_z^n \equiv \overbrace{d/dz_k \cdot exp(-b\cdot z_k)[d/dz_k \cdot exp(-b\cdot z_k)(d/dz_k \cdot exp(-b\cdot z_k)\{\cdots\})]}^{n \text{ times}}$

and

$$S_n = \sum_{n_1+n_2+\cdots=n} (a_1^{n_1}\cdot a_2^{n_2}\cdots a_{N+1}^{n_{N+1}})/(n_1!\cdot n_2!\cdots n_{N+1}!)\cdot<10^{\sum_{i=1}^{N+1} n_i\cdot v_{k-i+1}/10}>.$$

After substituting Eq.(18) into Eq.(19), the following equation can be easily derived :

$$p_z(z_k) = \sum_{l=0}^{\infty} B_l/l!\cdot(\partial/\partial\mu_{y_{0k}})^l[\sum_{n=0}^{\infty}(-1/b)^n S_n D_{z0}^n p_{y0}(z_{0k})]$$

$$= \sum_{l=0}^{\infty} B_l/l!\cdot(\partial/\partial\mu_{y_{0k}})^l p_{z0}(z_k). \tag{20}$$

In summary, it is possible to predict theoretically the output response p.d.f. $p_z(z_k)$ for a sound insulation system excited by an arbitrary stochastic input with the contamination by the background noise, based on the output response p.d.f. $p_{z0}(z_k)$ for the same system excited by a reference input and also with contamination by the same background noise. Here, the expansion coefficient B_l reflects hierarchically the lower and higher order statistics with respect to y and y_0 .

3. EXPERIMENTAL CONSIDERATIONS

For the purpose of confirming experimentally the effectiveness of our newly developed method, two examples of sound insulation systems – (A) a double wall with a sound bridge and (B) a non–parallel double wall were examined. Their system order N and system parameters a_i were determined in advance by the previously reported evaluation method [3]. The results of cumulative density function (abbr. c.d.f.) only for two cases (A) and (B) are shown in Figs. (1) and (2). Here, the 1st or the 2nd approximations correspond respectively to cases when employing only the 1st or the 1st and 2nd terms in the above theoretical expansion expression. The good agreement between theoretically and experimentally derived values is shown in Figs. (1) and (2).

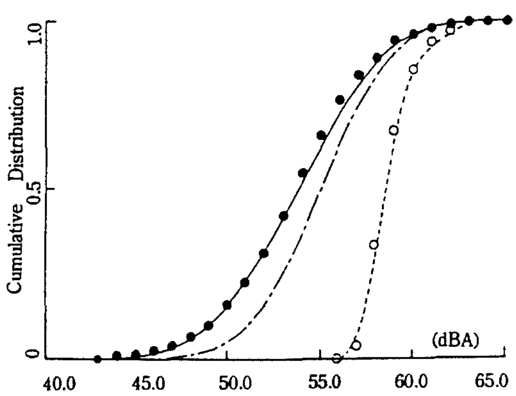

Fig.1 The comparison between theory and experiment for the output probability distribution form. The observed and fitted curve for c.d.f. $Q_z(z_k)$ of z_k are shown as (○ and ············) and the true and estimated curves for c.d.f. $Q_y(y_k)$ of y_k are shown as (●, –·–·–·–· (1st approx.) and ——— (2nd approx.)).

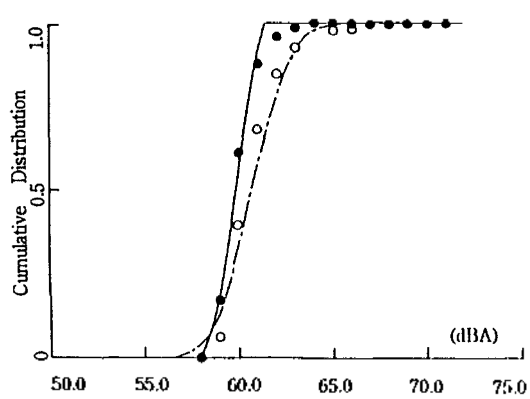

Fig.2 The comparison between theory and experiment for the output probability distribution form. The observed data is shown as (●) and c.d.f. $Q_{z0}(z_k)$ of z_k and predicted curves for c.d.f. $Q_z(z_k)$ of z_k are shown as (○, –·–·–·– (1st approx.) and ——— (2nd approx.)).

4. CONCLUSIONS

We have developed new methods by which the output response probability functions of the sound insulation systems in the absence of a background noise may be derived from the observed data contaminated by a background noise. Then, as one of application examples, the prediction problem of the system response probability with the other type stochastic input and the same background noise has been discussed. Finally, the effectiveness of our methods has been confirmed experimentally too by applying them to two types of the actual sound insulation wall systems – a double wall with sound bridge and a non–parallel double wall.

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