Experimental Testing of Feature Structures and Unification

Michiko Nakano
School of Education, Waseda University, Tokyo 169-50 *
CSLI, Stanford University, Stanford CA 94305 4115

Abstract

This paper presents two experiments where feature structures and unification provide an explanatory framework for what has been called illusory conjunctions in visual perception. Feature Structures and Unification has been successfully applied to computational analyses of natural languages. However, this efficient computational technique has not been experimentally tested among human subjects. This is an attempt to show some psychological validity for the notion of feature structures and unification.

1 Introduction

This paper examines some psychological plausibility of feature structure representation and unification, in the area of letter perception. Our visual system involves two successive stages of processing visual inputs: preattentive and attentive stages. In the preattentive stage, simple features are processed rapidly and in parallel over the entire visual field. Our visual system in the first stage processes retinal images through several separate channels, i.e., luminance, color, motion, binocular disparity and texture (for review, see van Essen and Maunsell, 1983; Zeki and Shipp, 1988). These channels process a retinal image almost independently from each other. In the second attentive stage attention integrates the outputs from the channels and localizes a particular visual object in the visual field. In order to segregate figure from background and to recognize what a given object is, the outputs coming from several channels need to be combined. It is in the second stage that attention plays an important role in integrating the outputs which different channels in the first stage processed.

As this brief account suggests, the psychological function of attention may be likened to the operation of unification and the outputs coming from various channels can be regarded as values as in feature-value logic. In the present experiment, features are types which are transferred from the higher order knowledge. In this paper we investigate this possibility experimentally in relation to illusory conjunctions.

The feature integration theory in visual perception outlined above has been proposed by Treisman and Gelade (1980), Treisman(1988), etc. The theory gives a plausible explanation to the role of attention. Treisman regards attention as integrating primitive features of an object, so that we can perceive an object as the result of integrating such features. In order to demonstrate the integrative role of attention experimentally, Treisman and her colleagues have repeatedly used an experimental procedure known as the elicitation of illusory conjunctions, where by integration irrelevant features, we actually see an illusory object which is not presented on the display.

The psychological function of attention proposed by Treisman may be defined as Seligman's perspectives in Situation Theory (ST): for definitions, see Nakano, 1992_a and Nakano, 1992_b. The main difference between Seligman's perspectives and the present approach is that Seligman's perspectives do not involve unification. Instead, Seligman emphasizes the functional roles of perspectives, but fails to see them as a more general operation of integration which yields a cognitive unit. Here we regard attention as triggering off the operation of unification.

The version of ST taken up previously was originally conceived as modelling visual scenes (see Barwise, 1981; Seligman, 1990). We have seen elsewhere that

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most of Seligman's definitions of perspectives are translatable into the corresponding notions in a Boolean valued model. In a recent version of ST, the theory of channels is introduced into the framework(see Selligman,1993; Barwise, Gabbay and Hartonas,1994, etc.). The metaphysical picture of channels given by Selligman(1993) does not differentiate physiological channels from classification-based featural knowledge at the higher level. Since the theory needs to be elaborated further, the present paper does not pursue some possible connections between the channel theory on one hand and physiological channels, typed features (both implicit and explicit classifications in the knowledge domain), values (the outputs from the psysiological channels) on the other. The latter terminologies are used in the present paper.

There are several ways of characterizing Unification(see Kay, 1985, Kaplan, 1983, Ait-Kaci, 1984, Kasper and Rounds, 1986, Goguen, 1989, Carpenter, 1993, Kogure, 1993, etc.). The present interpretation is based on a Boolean-valued model, but the essential features follow the standard feature structures. Briefly, unification is a unifier which can specify a least upper bound but the elements belonging to a cognitive unit must be compatible. The compatibility checking is defined as the existence of an element which is an greatest lower bound for all the elements belonging to the same cognitive unit. Thus, in the present Boolean-valued view, iff there is an maximal ideal and a ultrafilter for a given cognitive unit, all the elements (all the feature–value pairs) are unifiable. How do we know that something forms a cognitive unit? We assume any knowledge must be already stored in cognition and be retrievable at our disposal. This assumption does not prevent our agent from learning or inferring a new thing. Once it is learned or inferred, it is a part of his/her knowledge. We also assume that a part of one's knowledge is fed back in recognizing a visual object. That is, typed features (classification knowledge) are fed back in visual perception to aid the task of visual perception; wihtout any knowledge one can only perceive a collection of physical values, which is not a usual understanding of letter/object perception. On the other hand, actual physical values such as luminance, color, motion, texture and binocular disparity are, we may assume, fed forward to be matched with typed features. These feed-forward and feed-back mechanisms can be defined as maximal ideals and ultrafilters. Therefore, unification and feed forward and feed–back mechanisms are jointly occurring and defined as an concomitant operation in this paper.

In the next section, we give the present experimental rationale and some definitions and we also illustrates how the present framework can explain illusory conjunctions. In the third section, a few predictions derived from the present view are presented. In the final section, we discuss the results.

2 A Basic Framework and Experimental Procedures

The purpose of this section is to give an illustrative account of Treisman's attentive integration of features in terms of Unification in a Boolean valued model. Treisman's experiments deal with the linkage between the preattentive stages of visual perception in which the physical properties (values) of stimuli are processed preattentively without being localized, and according to our knowledge attention integrates those physical values and localize them as a cognitively meaningful unit. It is important to note that 'dimensions' and 'features' in psychological terms roughly correspond to typed features (types) and values (tokens in some cases) in logic respectively, although the definitions in experimental psychology tend to be more task specific. Garner(1974) defined dimension as a set of mutually exclusive values for any single stimulus and the word feature to refer to a value on a dimension. For instance, a line can be both yellow and green (values on different dimensions), but it cannot be both vertical and horizontal (values on the same dimension). As these examples indicate, the referents of these psychological definitions depend on experimental tasks. In order to avoid confusion, we use the word 'features' to refer to typed features and 'values' to denote physical outputs from separable physiological channels.

It is common to distinguish two or more levels of processing; intensities, wavelengths, retinal locations and binocular disparities are coded early at one level. These pieces of information are then combined, together with some spatial and structural relationships between them, and transformed at the same time to represent functional properties needed for a cognitive agent to identify what the object is, including surface color, size, reflectances, orientation, spatial frequency, the direction of motion, temporal position, texture and so on. According to Marr's theory, the transition comes in two stages: one between the primal sketch and the $2^{1/2} - D$ sketch and the other between a viewer-centred and an object-centred representation. Since we are concerned with behavioral tests, there is not much point in pursuing the routes of information flow. For the subjective experience of seeing an object, it may be sufficient for usto refer to luminance, color, texture, motion and binocular disparity, as in Treisman and Gormican(1988), and Cavanagh, Arguin and Treisman(1990). This assumption is justifiable, since there is a growing body of evidence that a separate analysis of each module recursively operates on a single common representation, while each module consists of a number of separate submodules each of which yields primitive features so that the primal sketch (a single common representation) is represented as a neural event in the form of conjunction of all the primitive features: see Marr (1982) and Treisman and Sato (1990).

This brief overview suggests two things. First, the hierarchical structure with recursive operations is in keeping with a Boolean-valued model. Second, our experimental task is a letter detection : K and D. In order to say which lines are primitive constitiuents for the visual input, we do not need to go into detailed physsiological specification, as long as our putative features are compatible with the primal sketch or the stimulus properties. Further, the primitive features tend to be processed in parallell: see papers cited above. *The features identified by the parallell processing criterion are those that migrate independently to form illusory conjunctions* (Treisman and Peterson, 1984; Treisman and Schmidt, 1982). So, as a preliminary investigation to check our assumption, by assuming that K consists of a vertical bar '|' and a wedge '<'; and D, a vertical bar '|' and the mirror image of C, we ran pilot experiments. The results showedthat these features could be considered as a set of primitives used by the visual system, according to Treisman's theory. For this reason, we assume here that the letters, 'K' and 'D', can be represented as two typed features: in 'K' a vertical bar '|' and a wedge '<' and in 'D' a vertical bar '|' and the mirror image of C. These constituents need to be unified at the early stage of perception. In fact at the early stage of visual perception model it is often assumed that constituent features are perceived separately (see McClelland and Rumelhart, 1981 and 1986, etc.). There are many anatomical and physiological discoveries that many separate visual areas specialize in coding different properties (Zeki, 1981;van Essen and Maunsell, 1983; Maunsell and Newsome, 1987). For these reasons, Treisman experimentally tested her proposal of free-floating or pooling hypothesis of separately processed features. This hypothesis enabled her and her colleagues to explain the formation of illusory conjunctions. Let us illustrate illusory conjunction in the present experimental settings.

Since Treisman and Gelade(1980) report that responses distribute bimodal, and Treisman and Gormican(1988) adopt a signal detection theory, we have also adopted the elicitation technique of the signal detection theory in the following two experiments. At the onset of the K experiment, a digit (7, 8 or 9 randomly chosen by the computer) was displayed at the central position of the CRT and the subjects reported back the digit they saw. Six kinds of stimuli were used:

(1) <<<;(2) | < |;(3) | < X ;
(4) < K <; (5) | K |;(6) | K X .

These stimulus sets were displayed vertically on the CRT, but for the sake of typographic reasons they were presented horizontally here. The stimuli were placed in the left side or in the right side of the central digit in order to examine the anisotropy, i.e., the figure superiority of the left side of visual field or the letter superiority of the right side of visual field. By implication, we can infer whether the subjects saw the stimuli as figures or letters. The exposure time was 71 milliseconds in order to minimize the effect of eye movements. Throughout the experiment, the target letter to be detected was consistently 'K'. Fifteen undergraduates took part in the experiment. There were 480 trials.

In the D experiment, there were four kinds of stimulus sets:

(1) |) | ;(2) |) O; (3) | D |; (4) | D O,
where) represents the mirror image of C.

These stimulus sets were displayed on the CRT vertically, as in K experiments. At the onset of the experiment, a fixation point was displayed. When the subject is ready to start a trial, she pressed the space bar and after 500 ms a cue (arrow) was displayed for 83 ms. Then, the stimulus set was displayed for 50 ms either on the right side of the cue or on the left side. This is to examine the anisotropy effect, as in the K experiment and also to examine the effect of divided attention. The target letter to be detected was consistently 'D'. There were two kinds of trials. In the valid cue trials, the arrow in the centre of the CRT indicates the correct direction of position in which the stimulus set was displayed. On the other hand, in the invalid cue trials, the arrow in the centre of the CRT indicates the incorrect direction of position where the stimulus set was not presented. In Treisman's feature integration theory, attention focuses onthe specific set of features to be integrated and localized. For this reason, in the divided attention task which invokes spatial attention, the subjects would report more illusory conjunctions, recombining constituent features which are present at the different locations in the display. Five undergrduates took part in the experiment. There were 1200 trials in all.

According to the signal detection theory there are four kinds of responses.

Misses This stands for a subject's failure of detecting the target in spite of the presence of the target on the display. Misses can be obtained from the stimulus sets (4), (5) and (6) in the case of K experiment, and (3) and (4) in the D experiment.

Correct Rejections The subject correctly detects the absense of the target on the display. These responses can be obtained from the stimulus sets (1), (2) and (3) in the K experiment, and (1) and (2) in the D experiment.

Hits The subject correctly detects the presence of the target on the display. Hits may be obtained from the stimulus sets (4), (5) and (6) in the K experiment, and in the D experiment,(3) and (4).

False Alarms The subject erroneously detects the target in spite of the absence of the target on the display. These responses may be observed in the cases of the stimulus sets (1), (2) and (3) in the K experiment; and in the D experiment, (1) and (2). Illusory conjunctions are the cases of false alarms in the signal detection theory.

The experimental paradigm of illusory conjunctions appears to give evidence that unification together with the representation of feature structures (compatible elements of feature-value pairs) is a cognitively plausible operation among human subjects. We have some reasons for this claim.

1. As indicated above, the features identified by the parallel processing criterion may be considered to be primitive according to Treisman's operational definition.
2. The primitive features are separately analized by the visual system at the early stage. So, the stimulus 'K' and 'D' are decomposed into the respective features of the vertical bar '|' and the rotated wedge '<' and of the vertical bar '|' and the mirror image of 'C'. These decomposed features are floating or pooled in the master map of attention, according to Treisman; they are ready to be unified and localized by attention. Recall that the outcome of this decomposition is, in terms of visual perception, equivalent to the stimulus sets which do not contain the targets, i.e., sets (1), (2) and (3) in the K experiment and (1) and (2) in the D experiment: see above. So, when unification operates erroneously on these sets where the targets are absent but the constituent features are visually presented, the subjects will see an illusory 'K' or 'D'. The occurrences of illusory conjunctions thus appear to support the operation of unification of primitive features.

Therefore, if we could observe the occurrences of illusory conjunction, we will be able to interpret the four kinds of response patterns in the signal detection theory as follows. We will regard Hits as the cases of sucessful unification, Misses as those of unification failures, Correct Rejections as correct non-application of unification and False Alarms as the errorneous applications of unification.

The Misses have been called "feature errors" which requires some clarification, since the conjunction of '<<' or '))' does not sugget 'K' or 'D'. However, '<' in '<<' and ')' in '))' can be significant distinctive features for a letter 'K' or 'D' respectively, since out of 26 alphabtical letters, 15 letters have '*I*' and '<' and ')' can differentiate 'K' or 'D' from the other 15 letters. When the exposure duration is extremely brief (71 ms in the 'K' experiment and 50 ms in the 'D' experiment) the presence of these distinctive markers may be sufficient for some subjects to recognize 'K' or 'D'. The process may be similar to our visual understanding of a subjective contour, when we extrapolate the missing line segments in the contour. Just like the extrapolation for a subjective contour, the distinctive features may evoke the missing bar(i.e., '<' as a '|-evoker' and ')' as a '|-evoker'), causing an illusory conjunction to be formed. This may explain the formation of illusory conjunctions in the cases of Misses; '{[<],[<]}' may be subjectively seen as [|, <] which is unifiable as 'K'. Likewise, '{[)],[)]}' may be subjectively seen as [|,)]which is unifiable as 'D'.

Posner, Petersen, Fox and Raichel (1988), and Petersen, Fox, Posner, Mintun and Raichele (1988) showed data from positron emission tomography concerning the localization of an attention system for visual spatial information. According to their data, early visual processing is done in striate visual cortex which interacts with visual word forms in extrastriate occipital cortex. Presently, although the general occipital areas are identified, there are not physiological bases for '<-extractor', '| extractor' and ')-extractor'. These are cover terms for the outputs from several channels whose physical values are fed-forward. However, the nature of '|-evoker' is different from these extractors, since it is the knowledge of the visual shape of the letters that can evoke the missing bar on the part of the visual attention system. Thus, it is necessary for us to postulate and accommodate some such feed-back mechanism from our knowledge base into our framework. McClleland and Rumelhart's lexical model also proposes abundant and precise feedback among feature, letter and word levels. The notion of feed-back and feed-forward mechanisms in the neural system has also been attested by physiologists (e.g. Damasio and Damasio, 1992, Crick and Asanuma, 1986, etc.).

The present definitions of feed-back and feed-forward mechanisms accords with the definition of unification here. Since the definitions are presented below(see also Nakano, 1993), it may be sufficient to indicate that feed-forward mechanism is defined as maximal ideal and feed-back is defined as ultrafilter and that unification is the joint operation of feed-forward and feed-back mechanisms. Since major lexical processing models propose that lexical processing is automatic and parallel, rather than concious and serial, it is not necessary for us to have any specific proposal on the question of which mechanism, feed-back or feed-forward, operates earlier in visual perception.

3 Some Predictions

The present approach can offer some predictions. We will measure three kinds of entropy: relative entropy[$I(S\&R)$], joint entropy[$H(S,R)$] and channel entropy [$I(S)$]. We use the following equations.

$$I(S\&R) = H(R) - H(R|S) \quad (1)$$

$$= \sum_{ij} p(s_i, r_j) \log\{\frac{p(s_i, r_j)}{p(s_i)p(r_j)}\}.$$

$$H(S,R) = H(R) + H(S) - I(S\&R) \quad (2)$$

$$= -\sum_{ij} p(s_i, r_j) \log p(s_i) - \sum_{ij} p(s_i, r_j) \log q(r_j)$$

$$+ \sum_{j} q(r_j) \sum_{ij} p(s_i|r_j) \log\{\frac{p(s_i|r_j)}{p(s_j)}$$

where S stands for input at an information source; R, output (the received signal); I, relative entropy; H, entropy; $p(s_i, r_j)$, joint entropy of S and R; $p(s_i)$, input probability at an information source; and $q(_j)$, output probability at an information receiver.

Many thories of visual perception claim that visual channels are separate and independent, although the process of operations is in parallel. If so, relative entropy would approach zero, for the following reason. Equation (1) can find out the degree of independence between S and R. Here we regard S and R as the input and output to a unknown number of channels. If S and R are independent of each other, $p(s_i, r_j) = p(s_i, r(_j)$. Then,

$$\log p(s_i, r_j)/p(s_i)q(r_j) = \log 1.0 = 0.0.$$

That is, if S and R are independent, the value of relative entropy will be 0.0. In this sense, relative entropy suggests the extent of independence between S and R. The relative entropies would influence the ways in which channel entropes can be evaluated.

Equation (2) represents the joint enropy of input and output and, as the right hand side shows, the joint entropy measures the sum of the entropy of the input to the channels and the entropy of the output from the channels, excluding the overlapping portion of the two. For this reason the joint entropy is sometimes called net entropy. When R is independent of S, $H(S,R)$ can be used to measure the amount of informtion (the reduction of uncertainties). The net entropy may differentiate the subjective difficulties among the experimental conditions.

We can also make some predictions about the channel entropy in relation to the well-known algorithms for feature structure representation. Since feature structures can be regarded as disjunctive normal forms which is known to be *NP-complete* , several algorithms are proposed: Ait-Kaci, 1984; Kaper and Rounds, 1989, etc. As the above illustration indicates, the present experimental demonstration may be regarded as the case of Karttunen's value disjunctions (i.e., those disjunctions used to specify the value of a single feature). But the entire description can be interpreted as a set of structures, each of which contains no disjunction. In this case, according to Kasper, Ait-Kaci's union/find algorithm has the time complexity of $O(n \log n)$ where n represents the length of the sum of mutually disjoint sets. According to Ait-Kaci, the union/find sequence has a computation time cost of order at most $O(nF^{-1}(n))$. The computational difficulty of disjunctive normal forms is as hard as the problem of travelling salesman in the plane, and Karp's algorithm can perform at worst in almost linear time $O(n \log n)$. As these cases indicate, the cardinality of the disjoint sets reflects the computational complexity. The channel entropy also reflects the number of channels used, according to Shannon and Weaver (1963). For this reason, it is interesting to see whether $O(n \log n)$ can be reflected in the channel entropies we will obtain. The present Boolean valued approach can predict the speedier time cost than $O(n \log n)$. The channel entropy would be $\log n$. The reason is as follows. The present Boolean approach is based on Rasiowa and Sikorski's work (1950) but for the sake of simplicity I refer to Levy's version, Jech's version or as before; see Nakano, 1992_a, 1992_b and 1993.

First of all, let us recall our experiment and our explanatory framework for the experiment in particular. The features are regarded as types as opposed to tokens, since features must possess attributive criteria in the form of classifications in logic (partitions or equivalent classes) wnich can account for various physical variations fed forward from the visulal input, such as the differences of sizes, orientation or colors of bars, rotated wedges, or the mirror images of C. There are at least two different levels of presentation: whether the actual transmission of information in neurons is a chemical or an electrical event, at this physical level the event must be evaluated numerically, and on the other hand, at the level of higher order knowledge which enables a cognitive agent to recognize an object on the display, we will require an abstract and perhaps logical representation. Furthermore, the higher order knowledge must contain some information about the correspondance between the physical values fed forward from the visual input and the featural classificatory information, so that the physical values are processed as belonging to a feature, and that a bundle of features should be recognized as a visual object. The present approach appears to give some theoretical framework for this mechanism. The key notion here is regular open sets which is clopen, i.e., closed and open. The following definitions derived from Rasiowa and Sikorski's work along with Scott's insights may clarify the present position. First of all, we note that the present Boolean-valued model is based on the following basic framework, in which we regard our knowledge of any domain as partially ordered.

Definition 1 (A Basic Framework) *Let $B = < B, +, \cdot, -0, 1 >$ be a complete Boolean algebra. Let $B_0 = B - \{0\}$. $\mathcal{P} = \langle B_0, \leq \rangle$. We define topology on $\mathcal{P}$. The Boolean algebra B' whose elements are regular open sets of $\mathcal{P}$ is isomorphic to B.*

Scott's program involves continuous lattices, but the present approach is similar to his notion of data types as continuous lattices. The present definition of $B_0 = B - \{0\}$ accords with Scott's treatment of undefined elements, since

$$\{0\} = \phi = \text{a null set} = \bot .$$

Let the domain of a set be D. Let us also suppose that

$$D_0 = D \cup \{\bot\}, \textit{ and } D \cap \{\bot\} = \phi.$$

$\bot$ is a collection of undefined values, which means in a Boolean-valued model that

$$\{\bot\} \cdot b_i \in D = \phi = \mathbf{0},$$

which satisfies the above assumption. We also notice that

$$\{\bot\} + b_i \in D = D = \mathbf{1}.$$

But, with respect to any s and $y \subseteq D_0$, we define the inclusion relation, $\subseteq$, such that

$$x \subseteq y \rightleftharpoons x = \bot \text{ or } x = y.$$

Then, we get a partially ordered structure $(D_0, \subseteq,)$ which means that y is a greatest element or a least upperbound of x. In both cases, it is a complete Boolean algebra but in the latter, it requires the underlying continuity as its framework, since a least upper bound is not strictly an element of the domain D. $\{\bot\} + b_i \in D = D = D_0$, which contains $\{\top\}$. Since there are many points in the neibourhood of x, x may not be uniquely determined; it may be evaluated in many ways. In order to accommodate theses possible contradictory or incompatible values, we need to include $\{\top\}$ for this reason as well. It follows that in Boolean terms,

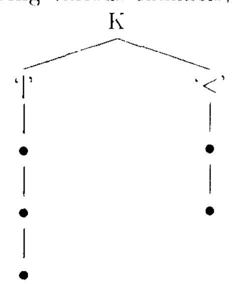
$$D_1 = D_0 \cup \{\top\} = D \cup \{\perp, \top\} = \mathbf{1}$$

and at the same time,

$$(\perp \neq \top), \text{ and } \{\perp, \top\} \cap D = \phi = \mathbf{0}.$$

So, we can understand that the data types represented as continuous lattices can be represented in terms of the present Boolean-valued algebra.

We have seen briefly that Scott's treatment of data types as lattices can be considered in terms of the present Boolean-valued model in its bare skelton. Borrowing his insights, what we are doing here is to rephrase naively Marr's idea of visual perception, i. e., grouping processes operate recursively at different scales on a single representation. For instance, the |-extractor is the result of layered grouping operation involving various channels, as the following tree suggests:



We have regarded the letter detection task not only as a physical event where the visual stimuli transmit some measurable information, but also as a conceptual event in which the knowledge of constituent features plays a part. At present we are concerned whether the physical event can be measured in theory in terms of continuous function of some sort. It is known that the Boolean-valued algebra has *automorphism*, i.e., the isomorphism of a Boolean algebra onto itself. Thus, we can establish self-reflexive domain, as in Scott's progam:

$$(D \to D) = D.$$

$$\text{Hence, if } (D_n \to D_n) = D_n,$$

$$\text{then, } D_{n+1} = (D_n \to D_n) = D_n.$$

The reflexive domain enables us to obtain a continuous domain, since

$$D = D_1 =, \cdots, D_n.$$

Furthermore, Russell's paradox inherent in the reflexive domain can be avoided by the closure opeation in the Boolean-valued algebra: see Nakano, 1992_a. Excluding the respective three nodes which are features belonging to the knowledge domain, the above figural tree can be evaluated by a partial graph of $\mathbf{P}_\omega$, since the power set of a set of natural numbers (ω) is represented by a Boolean-valued model:

$$f : \mathbf{P}_\omega \to \mathbf{P}_\omega$$

$$= f(x) = \bigcup\{f(e_n) | e_n \subseteq x\},$$

where e_n stands for a finite subset of ω.

For these reasons, we can say that even if several channels are needed to process '|' and '<' and the physical information going through each channel is computed separately, those values are integrated ultimately as a single value to be matched with the feature which is fed back from our knowledge source.

Now we are ready to look at our definitions of Feed-forward and Feed-back mechanisms.

Definition 2 (Definition: Feed-Forward) *The following three conditions must be satisfied.*

$$\forall b \in B(b \in I \vee -b \in I). \tag{3}$$

$$\forall b \in B \neg(b \in I \wedge -b \in I). \tag{4}$$

$$I \subsetneq J \to I = J \vee J = B. \tag{5}$$

In the above definition, the first and second clauses satisfy the requirements for strict ordering which according to Karp(1972) is needed for computability and definability. These clauses relates to Jech's separative property as well.

Definition 3 (Definition:Feed-back) *The following three conditions must be met for some information to be fed back from the higher order knowledge-base.*

$$r \in F(r \leq p \wedge r \leq q). \tag{6}$$

$$p \in F,\ p \leq q \to q \in F. \tag{7}$$

$$\forall F'((F' \mathit{\ is\ a\ filter}) \wedge F \subseteq F' \to F = F'). \tag{8}$$

The first clause represents the compatibility which is one of the two conditions for unification. When the numerical evaluation of a physical event including visual perception is at issue, the value for r can determine the threshold for a classification. It is also possible for r to be any feature which may suggests a group of (complex) feature-value pairs. It is important for us to note that the present definition of feed-back contains the specification of compatibility.

At the end of this section, we will examine one more important point. That is, the ultimate physical value fed forward from the physical stimuli can be compatible with the feature which is fed back from the knowledge source in terms of a Boolean algebra. The point is proved by Scott(1976) which summarizes our discussion.

$$(\mathcal{F} \bigcup \mathcal{B})_{\Sigma} = \mathcal{B}_{\Pi}. \quad (9)$$

$$\mathcal{B}_{\Sigma} = (\mathcal{F} \bigcap \mathcal{B})_{\Sigma}. \quad (10)$$

$$\{x | f(x) = g(x)\}, \quad (11)$$

where $f(x)$ denotes the computation of a finite set $\mathcal{F}$ and $g(x)$, a Boolean algebra.

Our notion of unification takes place at the point when equation (11) holds for each feature-value pair. For these reasons, since the experimetnal stimuli of two letters are decomposed into two constituent features respectively, we will predict that the complexity of computing the letter identification task requires $O(\log n)$, which will be reflected on the channel entropy, since it estimates what is happening inside the channels as a whole.

4 Results and Discussion

As indicated in §2.0, the computational complexity of feature structures is as hard as *NP-cmplete*. If Kasper's alogarithm is psychologically realistic, the channel entropy would be $O(2\log 2)$ and At-Kaci's union/find alogarithm would predict $O(2\log 2)$ as well. On the other hand, the present approach would predict that the channel entropy would be $\log 2$.

First of all, we note that the data obtained were elicited on the basis of the classical signal detection theory. Under this assumption the channel entropy would not be $n \log n$. If $O(n \log n)$ is tenable under the assumption of the signal detection theory, the following two conditions must be met. The second equation shows one of the assumption in the signal detection theory: the sum of the probabilities of Hits and Misses is equal to 1.0; and the sum of the probabilities of Correct Rejections and False Alarms (Illusory Conjunctions) is also equal to 1.0. The first equation shows that the entropy for a pair of probabilities with respect to Hits and Misses on one hand and with Correct Rejections and False Alarms on the other needs to be expressed as $n \log n$.

$$P_1 \log P_1 + \cdots + P_{2n-1} \log P_{2n-1} + P_{2n} \log P_{2n} \quad (12)$$
$$= n \log n, \text{where } n \in \omega.$$
$$P_1 + P_2 = \cdots = P_{2n-1} + P_{2n} \quad (13)$$
$$= 1.0, \text{where } n \in \omega.$$

However, we can see that there are no positive probabilties which can satisfy the two equations. Suppose there is a set of probabilities which are necessarily positive:

$$\mathcal{P} = \{P_{2n} | n = 1, \cdots, \infty.\}$$
$$\text{Let } n \log n \text{ be } f(x) \text{and } 0 < x.$$
$$\text{Then, } f'(x) = \log n + 1.$$
$$\text{When } 0 < x < 1 \text{ , } f(x) < 0.$$
$$\forall P_i \in \mathcal{P} \text{ and } 0 < P_i < 1.0.$$
$$\text{Therefore, } P_i \log P_i < 0.0.$$
$$\text{Thus, } \sum_{i=1}^{2n} P_i \log P_i < 0.0.$$
$$\text{When } n \geq 1, \; n \log n \text{ must be greater than } 0.0.$$
$$\text{Thus, } \sum_{i=1}^{2n} P_i \log P_i < 0 \leq n \log n.$$
$$\Longleftrightarrow \sum_{i=1}^{2n} P_i \log P_i < n \log n.$$

This is contradictory to the condition which must be met under the assumptions of the signal detection theory. The discussion shows that there is no positive probability which satisfies the above two conditions. This suggests that the signal detection theory is not compatible with the union/find alogarithm or that the two assumptions are not likely to be realized at the same time in real life.

Tables (1), (2), and (3) in Appendix represent the results of the experiments. Since the present paper is mainly concerned with feature structures and unification, we will omit psychological discussions concerning the binocular disparity and the net entropies here. For the purpose of the present paper, it may be sufficient to point out that the invalid cues yielded more misses and illusory conjunctions than the valid cues, and that what has been called additive item effects" (i.e., the presence of the items which facilitates the formation of illusory conjunctions auch as X and O in the present experiments) ecilted the same amount of illusory conjuctions in the two experiments. Since in both experiments, the values for $I(S\&R)$ are sufficiently low, channel entropies are computed accordingly. In the case of the K experiment, the channel entropies turned out to be $\log 2.0$. In the D experiment, apart from the second pair, the remaining pairs yielded $\log 2.0$. Since the experimental conditions of the D experiment were different from the K experiment, we ran the D experiment again in the manner of the K experimental conditions. As Table 3 shows, this uncued experiment turned out to be $\log 2.0$, replicating the results of the K experiment. In the present study, we dealt with the fourteen cases and except for the one case, the channel entropies were estimated as $\log 2.0$. The results thus show some evidence to confirm the present interpretation of feature structures and unification.

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Table 1 represents the results of K Experiment and Table 2, those of D Experiment.

Table 1: K Experiment(uncued)

<table>
<tr><th>Stimuli</th><th>Anisotropy</th><th>Hits</th><th>Ill. Con.</th><th>$I(S\&R)$</th><th>$H(S,R)$</th><th>$I(S)$</th></tr>
<tr><td><<<&<K<</td><td>Right</td><td>0.57</td><td>0.11</td><td>0.126</td><td>1.209</td><td>0.692</td></tr>
<tr><td><<< & <K<</td><td>Left</td><td>0.8</td><td>0.17</td><td>0.213</td><td>1.123</td><td>0.691</td></tr>
<tr><td>| < | &|K|</td><td>Right</td><td>0.77</td><td>0.39</td><td>0.076</td><td>1.325</td><td>0.693</td></tr>
<tr><td>| < | &|K|</td><td>Left</td><td>0.89</td><td>0.42</td><td>0.131</td><td>1.207</td><td>0.693</td></tr>
<tr><td>| <X &|KX</td><td>Right</td><td>0.68</td><td>0.4</td><td>0.040</td><td>1.343</td><td>0.693</td></tr>
<tr><td>| <X &|KX</td><td>Left</td><td>0.86</td><td>0.6</td><td>0.044</td><td>1.232</td><td>0.693</td></tr>
</table>

Table 2: D Experiment

<table>
<tr><th>Stimuli</th><th>Anisotropy</th><th>Ill. Con. or Misses</th><th>$I(S\&R)$</th></tr>
<tr><td>|) |</td><td>Right</td><td>0.142(valid) 0.167(invalid)</td><td>0.0457</td></tr>
<tr><td>|) |</td><td>Left</td><td>0.142(valid) 0.133(invalid)</td><td>-0.0006</td></tr>
<tr><td>|) O</td><td>Right</td><td>0.150(valid) 0.400(invalid)</td><td>-0.0276</td></tr>
<tr><td>|) O</td><td>Left</td><td>0.158(valid) 0.600(invalid)</td><td>-0.0747</td></tr>
<tr><td>| D |</td><td>Right</td><td>0.083(valid) 0.267(invalid</td><td>0.0248</td></tr>
<tr><td>| D |</td><td>Left</td><td>0.070(valid) 0.267(invalid)</td><td>0.0343</td></tr>
<tr><td>| D O</td><td>Right</td><td>0.100(valid) 0.400(invalid)</td><td>-0.0450</td></tr>
<tr><td>| D O</td><td>Left</td><td>0.100(valid) 0.267(invalid)</td><td>0.0632</td></tr>
</table>

Table 3: D Experiment: Channel Entropy

<table>
<tr><th>Stimuli</th><th>Anisotropy</th><th>$I(S)$ (Divided Attention)</th><th>$I(S)$(Uncued)</th></tr>
<tr><td>|) | &| D |</td><td>Right</td><td>0.6931</td><td>0.6931</td></tr>
<tr><td>|) | &| D |</td><td>Left</td><td>0.7140</td><td>0.6931</td></tr>
<tr><td>|) O & | D O</td><td>Right</td><td>0.6931</td><td>0.6931</td></tr>
<tr><td>|) O & | D O</td><td>Left</td><td>0.6931</td><td>0.6931</td></tr>
</table>