

# Continuous Hitting by a Flexible Link Hammer with Neural Networks Generating Input Pattern

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**Abstract** - This paper proposes a continuous hitting by a flexible link hammer. This hammer system is used only the first mode of vibration for a desired hitting. The input of the hammer driver for a continuous hitting is obtained from numerical solutions of two sets of non-linear simultaneous equations which satisfy the hitting conditions. Being too complicated, these numerical calculations are not useful for on-line processing. Therefore, the multi-layered neural networks are applied to the generation of the input patterns of the hammer driver. The trained network outputs agree well to the numerical solutions.

## 1. INTRODUCTION

In recent years many researchers take an interest in the control of flexible manipulators. Such manipulators have energy efficient and achieve high speed motion in comparison to ones with rigid links. However, the end-effector of the manipulator vibrates due to flexibility of the link. Therefore, there are many reports on suppression of the vibration of the end-effector[1].

On the other hand, a flexible manipulator is applicable to various operations with a contact[2]. For example, a flexible manipulator will catch a baseball, hammer a nail into a board, or collect space debris while attached to a moving space vehicle[3]. For these operations, the last link of the manipulator is chosen to be flexible while the other links are assumed rigid[4]. If the heavy payload is attached to the tip of the last flexible link, the manipulator can be used as a hammering robot[5]. Since a flexible link vibrates complicatedly with many modes, it is necessary for a proper hit to suppress the second mode or higher modes so that only the first mode in the dynamical motion remains. By the suppression of higher modes, the robot can flap an object with a specified velocity[5]. However, in this intermittent hitting, the first mode of vibration is generated strongly at setting of the initial posture. The suppression of this vibration wastes the energy of the driver and

the setting time. And many hitting operations are not an intermittent but continuous.

This paper describes a continuous hitting by a flexible link hammer. The aim of this paper is to decide the input patterns of the hammer driver for the continuous hitting. First, a relative deflection of a flexible link hammer is obtained as a function of the pulses of an angular acceleration which drives the link. The expression of the relative deflection can easily derive two sets of non-linear simultaneous equations for the hitting conditions that the hammer does not collide with an object twice in one hitting cycle and can flap an object with a desired hitting velocity. The generated time and width of the pulses are determined from the numerical solutions of the simultaneous equations so that a continuous hitting can be achieved. However, it is difficult for this method to find out the minimum value of the driving energy with on-line processing. Therefore, the multi-layered neural networks are applied to the generation of the input patterns. The neural networks acquires the relationship between an initial velocity of the hammer head and the parameters of the input pattern of the driver. The trained neural networks can generate the input pattern of the driver for the initial velocity and are effective for the on-line proceeding.

## 2. RELATIVE DISPLACEMENT OF A FLEXIBLE LINK HAMMER

A flexible link hammer of length  $l$  is driven in an X-Y horizontal plane as shown in Fig. 1. One end of the flexible link is clamped at DC motor axle. The x-y relative coordinate system, the x-axis of which corresponds to the link without any deflection, is rotated with the motor. When the motor is driven by an angular acceleration  $\ddot{\theta}(t)$ , the head of the hammer vibrates with several modes complicatedly. Therefore, the hammer can not flap an object with a specified velocity. So this hammer system make use of only the first mode of the vibration by suppressing the second mode or higher modes sufficiently. The relative deflection  $y_1(x,t)$  of the first mode of the link is represented by

$$y_1(x,t) = a_1(t) \phi_1(x) \quad (1)$$

where  $\phi_1(x)$  is the eigen-function of the first mode and the coefficient  $a_1(t)$  satisfies the following equation.

$$a_1(t) + \Omega_1^2 a_1(t) = -b_1 \ddot{\theta}(t) \quad (2)$$

where  $\Omega_1$  is a natural angular frequency of first vibration mode and  $b_1$  is an inner product  $(x, \phi_1(x))$ .

For one-cycle of the hitting operation, the base of the link is driven by an angular acceleration which consists of  $N$  pulses as

$$\ddot{\theta}(t) = \sum_{j=1}^N \frac{c_j}{T_j} \{u(t-T_j) - u(t-T_j - T_j)\} \quad (3)$$

where  $u(t)$  is a unit step function,  $c_j$  and  $T_j$  are the  $j$ -th angular velocity and the rise time, respectively. An example of (3) is illustrated in Fig. 2(a). And Fig. 2(b) shows an angular velocity  $\dot{\theta}(t)$ . It is assumed that an initial deflection of the link at  $x=l$  is

$$y_1(l,0) = 0 \quad (4)$$

and an initial velocity of the hammer head is

$$\dot{y}_1(l,0) = v_0 \quad (5)$$

After the  $n$ -th pulse is inputted into the base of the link, the relative deflection of the first mode in  $T_n + T_{r,n} \leq t < T_{n+1}$  ( $n \leq N-1$ ) or  $T_n + T_{r,n} \leq t$  ( $n=N$ ) is given from (1) and the solution of (2) by using (4), (5) as

$$y_1(l,t) = \sum_{j=0}^n z_j \sin \Omega_1(t - \tau_j) \quad (6)$$

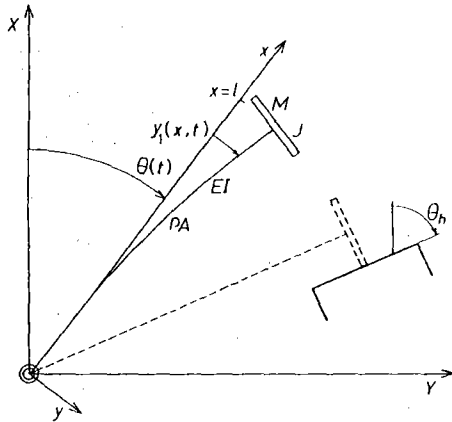


Fig. 1 Motion of a flexible link

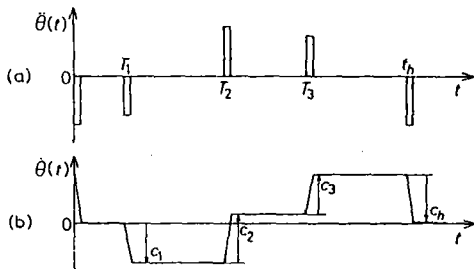


Fig. 2 Time charts of the angular velocity and acceleration

where

$$z_0 = v_0 / \Omega_1 \quad (7)$$

$$z_j = \frac{-c_j b_1 \phi_1(l)}{\Omega_1} \times \frac{\sin(\Omega_1 T_j / 2)}{\Omega_1 T_j / 2}, \quad (j=1, 2, \dots, n) \quad (8)$$

$$\tau_0 = 0 \quad (9)$$

$$\tau_j = T_j + T_{r,j} / 2 \quad (j=1, 2, \dots, n) \quad (10)$$

It is seen from the right side of (6) that the relative deflection is the sum of the same frequencies of the sine waves even though each sine wave has a different amplitude and phase. Therefore equation (6) can be rewritten as

$$y_1(l,t) = Y_n \sin(\Omega_1 t - \phi_n) \quad (11)$$

where

$$Y_n = \sqrt{(z_0 + \sum_{j=1}^n z_j \cos \Omega_1 \tau_j)^2 + (\sum_{j=1}^n z_j \sin \Omega_1 \tau_j)^2} \quad (12)$$

$$\phi_n = \tan^{-1} \frac{\sum_{j=1}^n z_j \sin \Omega_1 \tau_j}{z_0 + \sum_{j=1}^n z_j \cos \Omega_1 \tau_j} \quad (13)$$

The angle  $\phi_n$  is defined the lagged phase. When an initial angle  $\theta_0$  of the base of the link is  $\theta_0$ , the angle  $\theta(t)$  is given from (3) for  $T_n + T_{r,n} \leq t < T_{n+1}$  ( $n \leq N-1$ ) or  $T_n + T_{r,n} \leq t$  ( $n=N$ ) as

$$\theta(t) = \theta_0 + \sum_{j=1}^n c_j(t - \tau_j) \quad (14)$$

### 3. INPUT PATTERN FOR THE CONTINUOUS HITTING

#### 3.1 HITTING CONDITIONS

It is considered that the hammer hits an object periodically by using the flexibility of the link and reaction at a hitting. The object face is directed with an angle  $\theta_0$  about  $X$ -axis and its weight is much heavier than the hammer head. One-cycle of the hitting is executed after the  $N$ -th angular acceleration pulse is inputted into the link. In order to use the elastic energy of the link, a hitting time  $t_h$  is adapted to such a time as the relative velocity of the hammer head becomes a maximum value. The hitting time is obtained from the differentiated equation (11) with  $n=3$

$$\dot{y}_1(l,t) = \Omega_1 Y_n \cos(\Omega_1 t - \phi_n) \quad (15)$$

as

$$t_h = (2\pi + \phi_n) / \Omega_1 \quad (16)$$

As soon as the hammer hits the object, the angular acceleration  $c_h = -(c_1 + c_2 + \dots + c_n)$  as shown in Fig. 2(b) is inputted in order to stop the rotation of the motor. Then the hammer starts on the next hitting cycle. When the hammer flaps the object with a specified velocity  $v_h$ , the hammer head gets a following initial velocity  $v_0$  and starts a new movement.

$$v_0 = e v_h \quad (17)$$

where  $e$  is a coefficient of rebound between the hammer head and the object. However, the hammer head collides with the object twice in one hitting cycle due to this motion. Therefore, the hammer has to be swung up quickly by the first angular acceleration pulse. After the first pulse is inputted, the absolute deflection of the hammer head is given from (11) and (14) with  $n=1$  as

$$y_n(t) = \{\theta_0 + c_1(t - \tau_1)\} l + Y_1 \sin(\Omega_1 t - \phi_1) \quad (18)$$

In order to avoid the twice collision in one hitting cycle, the maximum value of the absolute

deflection  $y_n$  has to be minus quantity. The time  $t_0$  for the maximum value of  $y_n$  satisfies the following equation.

$$c_1 \ell + \Omega_1 Y_1 \cos(\Omega_1 t_0 - \phi_1) = 0 \quad (19)$$

When  $t=t_0$ , the absolute deflection becomes the limit of the twice hitting. This condition is obtained from (18) into which  $t_0$  is substituted.

$$\{ \theta_n + c_1(t_0 - \tau_1) \} \ell + Y_1 \sin(\Omega_1 t_0 - \phi_1) = 0 \quad (20)$$

Therefore, the condition of the first pulse can be obtained by solving the simultaneous equations (19) and (20) with the parameter  $t_0$ . Under this condition, the flexible link hammer has to flap an object with desired velocity  $v_n$  as shown with the dotted line in Fig. 1. The velocity of the hammer head is the sum of two velocity components. One is an absolute velocity due to the rotation of the x-y coordinate system and the other is a relative velocity due to the motion of the flexible link. Then the condition of the hitting velocity is represented by

$$\ell \sum_{j=1}^N c_j + \dot{y}(\ell, t_h) = v_n \quad (21)$$

Since the relative deflection  $y_1$  becomes 0 at  $t=t_h$ , the angle of the link has to be

$$\theta(t_h) = \theta_n \quad (22)$$

### 3.2 INPUT PATTERNS

In this section, the input pattern of the hammer driver will be derived. The values in (3), namely  $N, c_j, T_j, T_{j1}$  ( $j=1, 2, \dots, N$ ), have to be fixed for the continuous hitting. The necessary number of the pulses  $N$  is three in order to satisfy the hitting conditions. And rise times  $T_{j1}$  are determined from the motor driving system as

$$T_{j1} = 2.19 \times 10^{-3} c_j \quad (23)$$

The other values corresponding to the initial velocity  $v_0$  of the hammer head are obtained by solution of two sets of non-linear simultaneous equations.

The values of  $c_1$  and  $T_1$  to avoid the twice collision in one cycle are obtained from (19) and (20) for several coefficient of rebound  $e$  by using the Newton method. Figure 3 shows the boundary lines of twice collision in one hitting cycle when  $v_n = 247$  [cm/sec],  $\theta_n = 0$  [°]. The parameters of the first mode of the flexible link hammer are  $\Omega_1 = 1.86 \times 2\pi$ ,  $b_1 = 243.78$  and  $\phi_1(\ell) = 0.1244$ . The lower part of the each line is an effectual region for the first pulse. For example, when  $e=1$  and  $T_1=0.12$  [sec], the angular velocity  $c_1 = -3.14$  will be selected to avoid the twice collision. The values of  $c_2, c_3, T_2$ , and  $T_3$  are obtained based on the condition of the first pulse. By using (14) and (16), the hitting conditions (21), (22) can be rewritten as

$$\{ v_n - (c_1 + c_2 + c_3) \ell \}^2 - (\Omega_1 Y_3)^2 = 0 \quad (24)$$

$$\sum_{j=1}^3 c_j(t_h - \tau_j) = 0 \quad (25)$$

When  $e=0.5$ ,  $c_1 = -3.14$ , and  $T_1 = 0.12$  [sec], Fig. 4 shows the numerical solutions of the simultaneous equations (24), (25) using the parameters  $T_2$  and  $T_3$ . The generated time  $T_2$  is varied from 0.14 to 0.30. The marks  $\circ$  in Fig. 4 show the minimum point of the driving energy ( $c_2^2 + c_3^2$ ) of the link for each parameter  $T_3$ . In this case, the minimum energy point exists in

the curved line of the solutions with  $T_3 = 0.51$  [sec]. The minimum energy points were investigated for the several coefficient of rebound  $e$ . This investigation leads that  $T_3$  is almost equal to the time when  $y_1(\ell, t) = 0$  after the second pulse. Therefore, the generated time  $T_3$  is chosen as

$$T_3 = (\pi + \phi_2) / \Omega_1 \quad (26)$$

The components of the angular acceleration is reduced to  $c_2, c_3$ , and  $T_2$ . When the time  $T_2$  is parameter, the angular velocity  $c_2$  and  $c_3$  for several values of  $e$  are obtained from the non-linear simultaneous equations (24) and (25) into which  $T_3$  of (26), the fixed  $c_1$  and  $T_1$  are substituted. The angular velocity  $c_1$  and its generated time  $T_1$  are set to  $-3.14$  and  $0.12$  [sec] respectively in order to avoid the twice

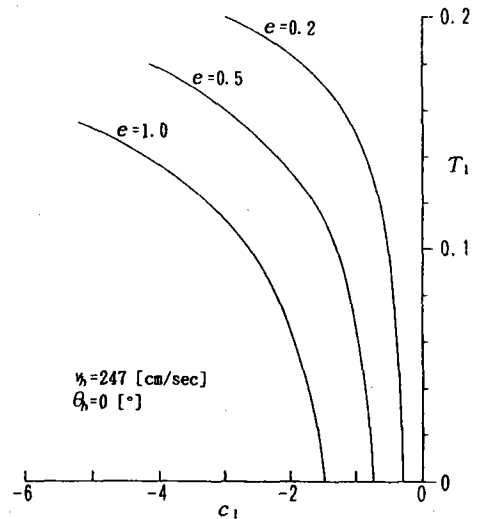


Fig. 3 The boundary lines of collision with an object twice in one hitting cycle

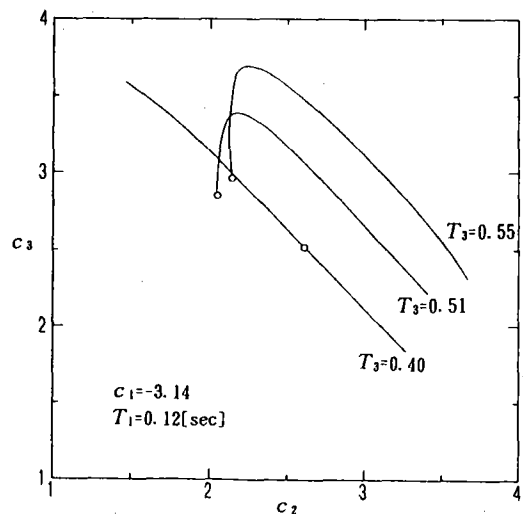


Fig. 4 Numerical solutions of  $c_2$  and  $c_3$

collision in one-cycle for the any coefficient of rebound. The generated time  $T_2$  should be fixed so that the driving energy becomes the minimum. If this input pattern is used, the flexible link hammer can hit an object continuously with the smallest driving energy.

#### 4. APPLICATION OF THE NEURAL NETWORKS FOR INPUT PATTERN GENERATOR

The input patterns of the hammer driver are obtained by solutions of two sets of non-linear simultaneous equations as shown in the previous section. Since this method uses the numerical calculation, it is unsuitable for an on-line processing. And further the pulses for low consumption of the driving energy are fined out by the search for the minimum point of  $(c_2^2+c_3^2)$ . Therefore, the multi-layered neural networks apply to this problem. The multi-layered neural networks have the nonlinear functional mapping property. The networks are trained to generate the input patterns with low consumption of energy. Figure 5 shows the structure of the neural networks for on-line processing. The each neural network consists of 1 input unit, 4 units in the hidden layer, and 1 output unit. An input-output function of an unit in the input layer is a linear function and the others are respectively a sigmoid function as

$$f(\chi) = 1 / (1 + \exp(-a \cdot \chi)), \quad (0 < f(\chi) < 1) \quad (27)$$

where  $a$  is the slope of the function. The error back-propagation algorithm is used for learning of the neural networks[6]. The weights  $w_{ij}$  of connection are changed at  $k$ -th step as

$$\Delta w_{ij}(k) = \eta \delta_i q_j + \alpha \Delta w_{ij}(k-1) \quad (28)$$

where  $\eta$  is a learning rate,  $\delta_i$  is an error term for unit  $i$ ,  $q_j$  is the output of unit  $j$ , and  $\alpha$  is a decay factor. The training data are produced by the method of chapter 3 with  $c_1 = -3.14$  and  $T_1 = 0.12$ [sec]. The data at  $e = 0.41, 0.5, 0.6,$  and  $0.7$  are given for iterative learning with  $\eta = 0.7, \alpha = 0.8,$  and  $a = 0.3$ . The solid lines in Fig.6 are the numerical solutions for several  $e$  and the white marks show the outputs of the neural networks. The trained neural networks can be generated the output value close to the learned value in about  $e \leq 0.80$ . This result shows that the neural network is useful for the generation of the input patterns of the hammer driver.

#### 5. CONCLUSION

The operation of the continuous hitting was discussed for utilization of an elastic energy of the flexible link hammer. Three pulses of the angular acceleration are required for the continuous hitting. The generated time and width of the pulses are determined from two sets of non-linear simultaneous equations which are obtained by the hitting conditions. However, this calculation is not useful to on-line processing. Therefore, the multi-layered neural networks are applied to the generation of the input patterns of the hammer driver. The output values of the trained networks show that the method is effective for the on-line generation of the input patterns.

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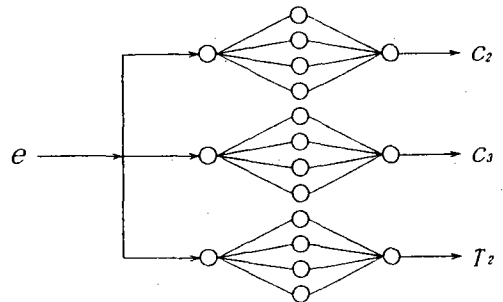


Fig. 5 The structure of the neural networks

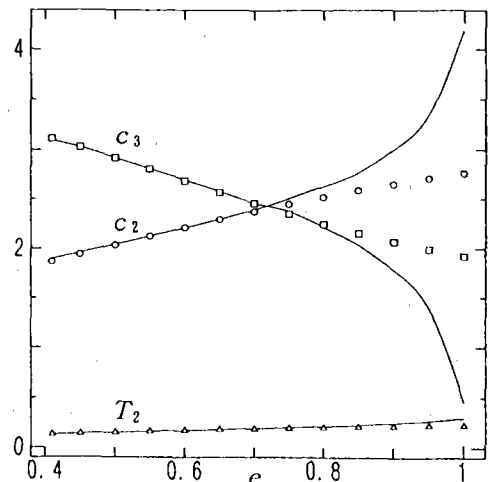


Fig. 6 Outputs of the neural networks