Reduced Marking in Petri Nets Analysis

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Abstract

We propose a method to reduce the marking expression about the places when we model a discrete-event system using the Petri net. The net with reduced marking expression has the same dynamic behavior as the original model. The number of the places can be reduced by the number of the resource places of the Petri net, and consequently the net can be significantly simplified, still preserving the dynamic properties of the net.

1 Introduction

The construction of reachability trees is one of the basic techniques used in the analysis of the behavior of Petri nets. The tree represents the reachability set, i.e., all reachable markings from the initial marking. Starting with the initial marking, we graph the results as the tree. The number of the elements of the marking equals the number of the places of the net.

When, for example, an FMS is modeled in Petri nets, some places (which we will call the resource places: RP) indicate the availability status of the resources such as the robots or parts, and the others (which we will call state places) represent the states such as the operational status of the robot.

Eliminating the resource places from the net makes the size of the marking expression be shorter, and the original Petri nets get reduced, without changing the properties of the Petri nets. But the structure of the transitions are invariant and the number of reachable markings is not changed.

2 Property of Resource Place

A Petri net is a 5-tuple, $PN = \{P, T, I, O, M_0\}$, where P is a finite set of places, T is a finite set of transitions, $I : P \times T \rightarrow \mathbf{N}$ is an input function, $O : P \times T \rightarrow \mathbf{N}$ is an output function. A marking M is a vector, M_0 being initial marking. The number of tokens in a component $p \in P$ is $M(p)$. $\mathbf{N}$ indicates the set of natural numbers. A transition $t \in T$ is said to be M-enabled iff

$$M(p) \geq I(p,t),\ \forall p \in \cdot t \qquad (1)$$

$$M(p) \leq K(p) - O(t,p),\ \forall p \in t\cdot \qquad (2)$$

where $\cdot t$ and $t\cdot$ are called the preset of t, (i.e., input places) and the postset of t (i.e., output places), respectively. $K(p)$ is the maximum number of tokens that p can hold at any time. An enabled transition can fire and the firing yields a new marking with

$$\overline{M}(p) = M(p) - I(p,t) + O(p,t),\ \forall p \in P. \qquad (3)$$

The set of all possible markings from M_0 in a net Z is denoted by $L(Z, M_0)$ or simply L(Z). A marking M_n is said to be reachable from a marking M_0 (denoted by $M_0[\sigma > M_n)$ if there exists a sequence of firings ($\sigma = t_1 t_2 \cdots t_n$) that transforms M_0 to M_n. The reachability tree from M_0 in a net Z is denoted by $RT(M_0)$. $\sharp RT(M_0)$ indicates the number of markings in a net Z. Let $M_i(p_1, p_2, ..., p_n)$ be the ith marking representation of the places $(p_1, p_2, ..., p_n)$ in $L(Z, M_0)$.

We divide the place set P into resource place(RP) set R and state place set S.

$$P = \{R, S\},\ R \cap S = \phi$$

$$R = \{r_i \mid i \in \mathbf{N}\}$$

$$S = \{s_j \mid j \in \mathbf{N}\}.$$

Definition 1 *A resource circuit C is a directed closed path containing only one resource place, where there is no directed closed path including the same resource place in its interior.*

Definition 2 *The set F_i of the associated places(AS) is the set of the places encountered in a resource circuit excluding the resource place r_i, i.e., $F_i = \{f_{ij} \in S \mid i, j \in \mathbf{N}\}$.*

For r_i, we make up the associated pair (r_i, F_i). In Fig.1, the Petri net model of an FMS is shown. There are three resource circuits(C_1 C_2 C_3) in Fig. 2, where

$$C_1 = r_1t_1s_1t_2s_2t_3s_3t_4r_1$$

$$C_2 = r_2t_1s_1t_2r_2t_3s_3t_4r_2$$

$$C_3 = r_3t_1s_1t_2s_2t_3s_3t_4r_3$$

and the sets of associated places for each resouce circuit are

$$F_1 = \{s_1, s_2, s_3\} \ for \ r_1$$

$$F_2 = \{s_1, s_3\} \ for \ r_2$$

$$F_3 = \{s_1, s_2, s_3\} \ for \ r_3$$

We assume the set F_i associated with r_i is nonempty.

3 Elimination of Resource Places

A marking can be regarded as a state of the system at each time. Token distribution in the resource circuit changes from one node(i.e., place) to neighbor sequentially, along the directed arc.

The total number of tokens in each resource circuit C is invariant by transition firings. There is no extinction or generation of token in any resource circuit. If we section a resource circuit into two parts, RP node and AS nodes, the marking distribution has a manner of exclusive occurrence, in r_i or in F_i. If we eliminate the resource places in the Petri net, we can get the reduced marking expression by the following theorem.

Theorem 1 *Let $RT'(M_0')$ denote the reachability tree or graph whose nodes(markings) are constructed with only the set S. Then, there is no topological difference between $RT(M_0)$ and $RT'(M_0)$.*

proof) We show this theorem in two cases, i) sage net and ii)general case. We consider a resource circuit which consists of a resource place r_i and its set F_i of the associated places.

i)To simplify the proof, let us assume $\forall M_0$, $M_0(r_i) = 1$ and $M_0(F_i) = 0$(zero vector) in a resource circuit. If $M(r_i) = 0$, then $\exists|\ j$, such that $M(f_{ij}) = 1$ and $M(r_i, f_{i1}, f_{i2}, ..., f_{ij}, ..., f_{in}) = (0, 0, 0, ..., 1, ..., 0)$, where $n = |F_i|$. Therefore, there are n unique possi ble states(markings), since $j = 1, 2, ..., n$ and they are all mutually independent. It is easily shown that the number of markings in (r_i, F_i) is the same as that of F_i.

ii)Let the set of all the possible distinct markings in C be

$$L(C) = \{M_1, ..., M_m\}.$$

where m is the number of distinct markings. Each marking is the marking of C with the associated pair of (r_i, F_i), i.e., $M(r_i, F_i)$. If we let the initial marking M_0 of r_i be $M_0(r_i) = n$, then $M_0(F_i) = 0$. For some marking $M_\alpha \in L(C)$ if $M_\alpha(r_i) = k$ for $\alpha = 1, 2, ..., m$, $k = 0, 1, ..., n$, then $M_\alpha(F_i)$ is a unique state(the distribution of $n-k$ tokens in associated places) which is different from $M_\beta(F_i)$, $\beta = 1, ..., m$, $\beta \neq \alpha$. The marking set of the subcircuit C' where the resource place is eliminated also has m distinct markings.

$$L(C') = \{M_1', ..., M_m'\}$$

Let $M_Z = M(R, S)$ and $M_Z' = M(S)$ be marking of net Z and the corresponding reduced marking expression, respectively. Extending the above result to the entire net Z , we obtain the following

$$\#RT(M_0) = \#RT'(M_0')$$

Thus, the elimination of $M(R)$ can not affect the number of states that could exist . Next we show that RT' is an isomorphic representation of RT for μ: $M_Z \rightarrow M_{Z'}$, which maps nodes of original reachability graph RT into the ones of reduced marking reachability graph RT'. Let $L(Z)$ be the marking set of a net Z and let σ be flow relation in $L(Z)$. For the mapping μ: $M_Z \rightarrow M_{Z'}$, $M_Z \in L(Z) \Leftrightarrow \mu(M_Z) \in L(Z')$ and $M_\alpha\ \sigma\ M_\beta \Leftrightarrow \mu(M_\alpha)\ \sigma\ \mu(M_\beta)$ for $\alpha, \beta \in \mathbf{N}$. This implies that RT and RT' are isomorphic for some bijection μ. RT is equivalent to RT' because their relationship(mapping) μ is bijective. RT and RT' have an isomorphic representation to each other. □

By Theorem 1, RTcan be replaced by RT' topologically. For an FMS, there can be many RPs in Petri net models even for the moderate size of systems. From this point of view, we will use the marking without RPs in order to simplify the expression. Actually we can see what the state of the resource is, only from AS nodes, without RP node which describes whether the resource is being used or not. Only with the marking of AS nodes, it is possible to identify the states of resource, process, load, unload, idle(absence of token in associated places), etc.

In the following Fig. 3, we provide an example, for an initial marking $M_0(r) = 2$, $M_0(F) = 0$, where $F = \{s_1,\ s_2\}$, and all possible markings of this circuit are

$$M_0 = (2\ 0\ 0)$$

$$M_1 = (1\ 1\ 0)$$

$$M_2 = (1\ 0\ 1)$$
$$M_3 = (0\ 1\ 1)$$
$$M_4 = (0\ 2\ 0)$$
$$M_5 = (0\ 0\ 2)$$

where $M_i, i = 1, 2, ..., 5$ is the marking of the places(r, s_1, s_2). Even if we drop out RP set(r) in marking representation, we still have six possible states:

$$M'_0 = (0\ 0)$$
$$M'_1 = (1\ 0)$$
$$M'_2 = (0\ 1)$$
$$M'_3 = (1\ 1)$$
$$M'_4 = (2\ 0)$$
$$M'_5 = (0\ 2)$$

where M'_i is the marking of the places (s_1, s_2). To handle the reduced markings M' is more convenient than to handle M due to the decreased size of the places in the marking expression.

4 Reduced Marking Expression

Petri nets can be scaled down by eliminating RPs in a net model. If we get rid of RPs and relevant arcs(their ingoing and outgoing arcs) from the entire model, we get the reduced net(RN) Z', which keeps the information of a system. There may exist not only closed net but also open net. We exclude the situation of open reduced net in this paper. We regard RP nodes as redundant information, and construct the reduced Petri net with reduced nets. RN generation takes the following steps. For a resource circuit, step 1: eliminate resource place, and its input and output arcs to obtain reduced net, step 2: impose the bound $K(M_0(r))$ to associated places, but $\forall s_j \in F$, $\sum_j M(s_j) \leq M_0(r)$ where $n = |F|$, and step 3: adjust interrelationship between processes using inhibitor arc. In Petri net model, the mutual exclusion situations that share the common resource(s) can occur. The definitions on mutual exclusion are referred in [7] . Draw inhibitor arc(s) from AS of a process to input transitions of other processes. The RNs for mutual exclusions are shown in Fig. 4.

Note that the transitions of the original net are all conserved in the reduced net. None of transitions (i.e., events) are dropped away. The model keeps the original dynamics. The dynamic behavior is unchangeable for RN. Fig. 5(a) is prototype of C. t_1 is enabled iff the enabling conditions are satisfied. t_1 of RN counterpart Fig. 5. (b) cannot be enabled when $M(p_2)$ reaches its bound. Therefore, whatever the situations in p_1 may be, there is no difference in state transformation for Fig. 5. (a) and (b). We can say Fig. 5 (a) and (b) have the same dynamic behavior.

Fig. 6 shows a Petri net model for material handling system in [5]. In this model, there are eight distinct positions, each having special sense that can be modeled as a single resource, which is represented as the place s_1 through s_8, respectively. The availability of positions is modeled by the places r_1 through r_8. For example, place r_1 represents the availability of the first position. RP set $\{r_1, r_2, \cdots, r_8\}$ plays a role of flag, whether corresponding places are free or not. There doesn't exist any relation between the elements of the RP set. Fig. 8 shows the reachability tree of the net in Fig. 7 and all the nodes M can be replaced by another expression eliminating RPs from original reachability tree. Note that the number of nodes of RP eliminated representations is equivalent to that of Fig. 8, and flow relations between nodes is identical to each other. The original and reduced markings are

$$M_0 = (1\,0\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0\,0\,1\,1\,1\,0\,0\,1\,0)$$
$$M_1 = (0\,1\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0\,1\,0\,1\,1\,0\,0\,1\,0)$$
$$M_2 = (1\,0\,0\,0\,1\,1\,0\,1\,0\,0\,0\,1\,0\,1\,1\,1\,0\,0\,1\,0)$$
$$M_3 = (1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,1\,0\,0\,0\,1\,1\,0\,0\,1\,1)$$
$$M_4 = (0\,1\,0\,0\,1\,1\,0\,1\,0\,0\,0\,1\,1\,0\,1\,1\,0\,0\,1\,0)$$
$$M_5 = (1\,1\,0\,0\,1\,1\,0\,0\,0\,1\,1\,0\,0\,0\,1\,1\,0\,0\,1\,1)$$
$$M_6 = (1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,1\,0\,0\,1\,1)$$
$$M_7 = (0\,0\,1\,0\,1\,1\,0\,1\,0\,0\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0)$$

$$M'_0 = (1\,0\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0)$$
$$M'_1 = (0\,1\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0)$$
$$M'_2 = (1\,0\,0\,0\,1\,1\,0\,1\,0\,0\,0\,1)$$
$$M'_3 = (1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,1\,0)$$
$$M'_4 = (0\,1\,0\,0\,1\,1\,0\,1\,0\,0\,0\,1)$$
$$M'_5 = (1\,1\,0\,0\,1\,1\,0\,0\,0\,1\,1\,0)$$
$$M'_6 = (1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,0\,1)$$
$$M'_7 = (0\,0\,1\,0\,1\,1\,0\,1\,0\,0\,0\,0)$$

We present complexity indicator V that is the sum of the number of elements of P, T, I, and O for a finite net. Let $V_e = V - V'$ be the volume to indicate how much the complexity of a Petri net is reduced. Removing of the resource places and their relevant arcs produces the reduced Petri net for the model in [5], yielding $V_e = 70 - 46 = 26$. The reduced net shown in Fig. 7 has the same dynamic behavior as the one in Fig. 6.

5 Conclusion

We showed that the reduced marking expressions and the reduced net can be obtained by the concept of the resource circuit. But the reduced net has the same transition properties as the original net. However, to get the reduced reachability graphs of the net, further study is needed.

References

[1] A. E. Kostin, "Deterministic Algorithm for Determining Reachability of Marking in a Petri Net," Avtomatika Vychislitel'naya Tekhnika, Vol. 24, No.2, pp16-22, 1990.

[2] P. Huber, A. M. Jensen, L. O. Jepsen, and K. Jensen, "Towards reachability trees for high-level Petri nets, " Lecture Notes in Computer Science, Advances in Petri Nets, Springer-Verlag, Vol.188, pp215-232, 1984.

[3] W. Reisig, Petri Nets, An Introduction, EATCS, Monographs on Theoretical Computer Science, Springer-Verlag, 1985.

[4] Tadao Murata, "Petri Nets:Properties, Analysis and Applications," Proceedings of IEEE, Vol. 77, No. 4, April 1989.

[5] MengChu Zhou, Kevin McDermott, and Paresh A. Patel, "Petri Net Synthesis and Analysis of a Flexible Manufacturing System Cell," IEEE Trans. on SMC, Vol. 23, No. 2, March 1993.

[6] DianLong Guo, Frank DiCesare, and MengChu Zhou, "A Moment Generating Function Based Approach for Evaluating Extended Stochastic Petri Nets," IEEE Trans. AC, Vol.38, No. 2, Feb. 1993.

[7] MengChu Zhou and Frank DiCesare, "Parallel and Sequential Mutual Exclusions for Petri Net Modeling of Manufacturing Systems with Shared Resources," IEEE Trans. Robotics and Automation, Vol. 7, No. 4, Aug., 1991.

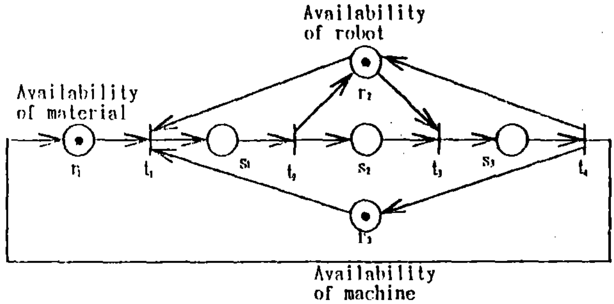

Fig. 1. An example of FMS.

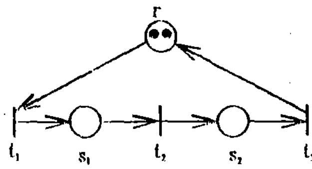
Fig. 3. A resource circuit

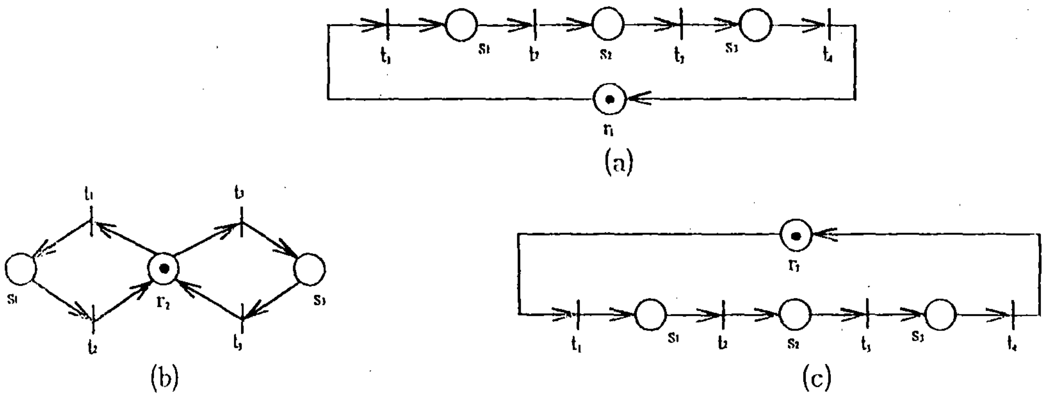
Fig. 2. Resource circuits of the model in Fig. 1. (a) C_1 (b) C_2 (c) C_3

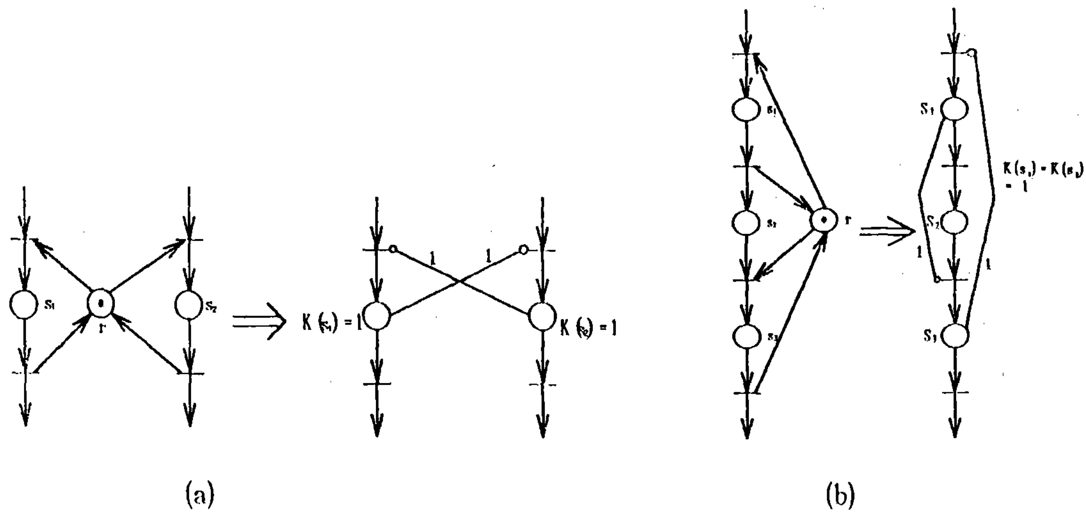
Fig. 4. The mutual exclusions (a) 2-PME (b) 2-SME

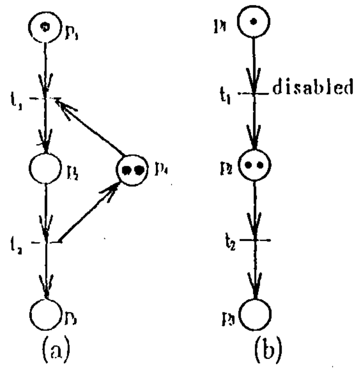

Fig. 5. (a) Part of a model.
(b) RP reduced counterpart.

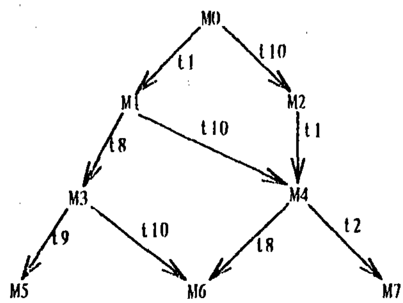

Fig. 8. Reachability tree for the model in Fig. 5.
(up to the 4th layer)

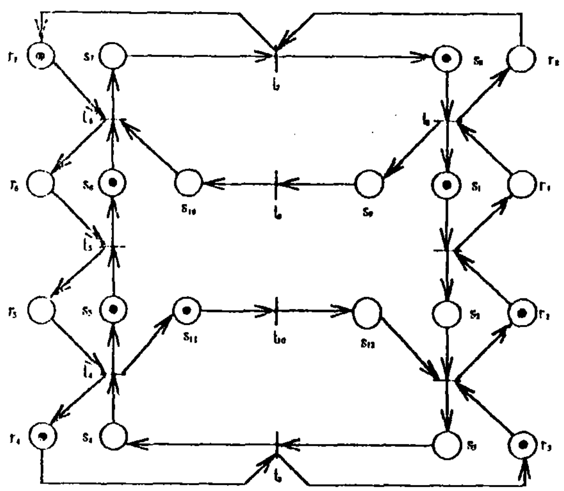

Fig. 6. Petri net for the material handling system.

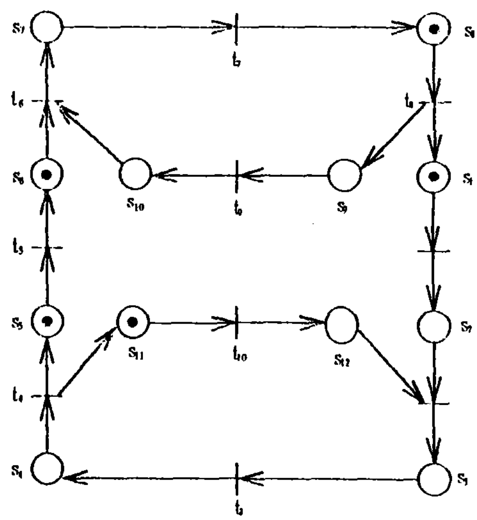

Fig. 7. RPN for the material handling system.