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System Identification using Multiple Decimation Method and Design of PID-ATC

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Abstract - LSM(Least-Squares Method) has inherent limitation that precise system identification over wide frequency band is difficult , especially at low frequency band. In this paper we propose to use decimation, a spectrum analysis method widely used in signal processing.

The merits of decimation are the flexibility of selection of the frequency band concerned and the function of LPF(Low Pass Filter). In this paper, frequency-domain is divided into separate frequency bands which will be combined into full frequency-domain by using MDM(Multiple Decimation Method). In this way, free selection of sampling frequency for each band is possible and the low frequency oscillation modes of LSM are avoided.

I. Introduction

System identification which determines system dynamics from input-output data is the important system technique that is used in modeling of controller design , adaptive control, abnormal diagnosis, measurement,etc.

System identification is widely divided into traditional identification method (represented by frequency response method) which obtains system dynamics from experiment and parametric identification method (represented by LSM) which has been actively researched in time domain since 1960's.

Parameter identification method of input-output model or state variable model etc. was based on time series analysis. But it is important that consider error rearing of frequency-domain and identification at time-domain model[1]. Because parameter error of time-domain model dosen't directly correspond with estimated error of frequency characteristic.

System identification consists of assumption of model structure, unknown parameter identification from input data , verifying validation of model.

Especially, in respect that the purpose of system identification is controller design, the modern controller design fields are studying new fields called such as H_∞ optimal control theory and robust control as attempt to combine both advantages of frequency-region and time-region, and recently brings about recognition for the importance of frequency characteristic, which was looked down upon in the past [2,3].

Frequency response means steady-state response for sinusoidal input. Frequency response method which can be used for interpretation and design of control system is to research about the responses that result from the input signal with the frequency varied through the region that we are interested in [4].

In this paper we propose new identification method using decimation as the conception of frequency division region [5]. Decimation is the application algorithm of FFT(Fast Fourier Transform) which changes high sampling frequency into low sampling frequency in order to identify the system and the algorithm reduces the amount of computation and it has the adventage as the structure which makes possible improvement of convergence velocity and parallel processing [6].

And also this algorithm can improve identification accuracy in low frequency region and oscillation mode, which is often noticed as the problem of LSM [7]. We use MDM(Multiple Decimation Method) which synthesizes the frequency for the whole region as division frequency region using the adventages of decimation. We obtain the transfer function applying this algorithm to the controlled object and apply it to the real plant as the transfer function of the model. And we realize auto tuning and considerate the performance of this algorithm through the simulation.

Thus this paper consists of two structures as follows.

(1) The proposal of identification method for modeling of the unknown system: the estimation procedure of transfer function based on decimation and frequency response data.

(2) The proposal for controller design method according to frequency region division: PID(Proportional Integral Derivative)-ATC(Auto Tuning Controller) design using M-circle.

One way, Byun has been announced estimation method of transfer function using decimation[8], and PID-ATC uses to design at frequency-domain[9].

II. Background

This paper consists of two parts as shown below. First, one is system identification , and secondly one is PID-ATC design method using identified information of the system. Fig. 1 shows the whole flow-chart of the paper.

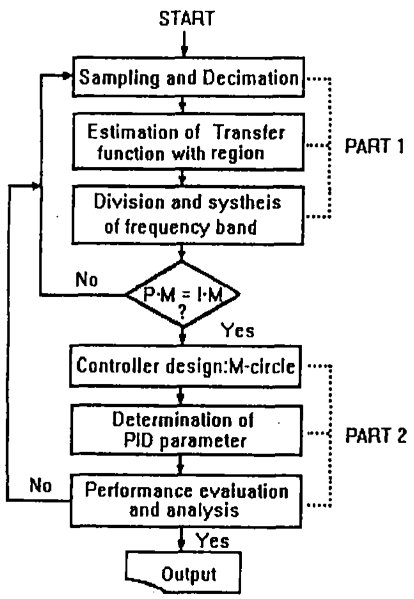

Fig. 1 Flow-chart of the whole.

At first, we sample the given input, let it pass through LPF according to decimation order. And we resample the output and then obtain the transfer function for each frequency region. We search following ITSE (Integral of Time multiplied Square Error) method with performance index of PM(Plant Model) and IM(Identification Model) after we synthesize the obtained transfer functions. And if we have the allowed error, go to the controller design process.

Controller design is to design PID parameters using design variables(ω, M_p) of M-circle , and the above process is repeated until satisfactory performance is obtained through performance evaluation and analysis.

PART 1. The estimation procedure of Transfer function using Decimation

2.1-1 Statement of the problem

In this paper, we use decimation which was applied to improve computational efficiency of 'DFT(Discrete Fourier Transform) in FFT (Fast Fourier Transform)' proposed as system identification method by Colley and Tukey in 1965 [10].

Decimation has the filtering characteristic of the lowpass filter by changing from high sampling frquency into lower sampling frequency by adjustment samping ratio of input and output without loss sampling theorem and resamples smoothed signals in the low frequency.

At first, we explain decimation principles by low filtering. In this paper, we assume that object of identification is SISO(single input single output) in continuous-time system and $y(n)$ is the output.

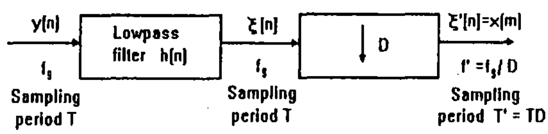

Fig. 2 Principle of Decimation.

Let the signal $y(n)$ pass through lowpass filter $h(n)$ and the output from h(n) be ξ(n) as shown in Fig. 2. Let the sampled signal from ξ(n) with the interval of D be $\xi'(n) \triangleq x(m)$. The passband of h(n) is $|w| \le \pi/D$ and f_s is sampling frequency. Also f' is sampling frequency following decimation order.

Thus, decimation means to change sampling ratio of input and output, and defines as follows.

$$\xi'(n) = \xi(n),\ n = 0, \pm D, \pm 2D, \ldots$$
$$= 0, \text{ etc.}$$

ξ'(n) can be rewritten as

$$\xi'(n) = \xi(n)\left[\frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi kn/D}\right] \quad \cdots\cdots\cdots\cdots (1)$$

By the relation of input-output in Fig 2.

$$x(m) = \sum_{n=-\infty}^{\infty} h(Dm-n)\, y(n) \quad \cdots\cdots\cdots\cdots (2)$$

$x(m) = \zeta'(Dm) = \zeta(Dm)$

Based on the procedure above

$$X(z) = \sum_{m=-\infty}^{\infty} \zeta'(Dm) z^{-m} = \sum_{m=-\infty}^{\infty} \zeta'(m) z^{-m/L}$$

$$= \sum_{m=-\infty}^{\infty} \zeta(m)\left[\frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi km/D}\right] z^{-m/D}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1}\left[\sum_{m=-\infty}^{\infty} \zeta(m) e^{j2\pi km/D} z^{-m/D}\right]$$

$$= \frac{1}{D}\sum_{k=0}^{D-1} E(e^{-j2k\pi/D} z^{1/D}) \quad \cdots\cdots\cdots\cdots\cdots (3)$$

$E(z) = H(z) \cdot Y(z)$ according to equation (2).

On account of $z = e^{j\omega T}$, it follows that

$$X(e^{j\omega'}) = \frac{1}{D}\sum_{k=0}^{D-1} H(e^{j(\omega'-2\pi k)/D}) Y(e^{j(\omega'-2\pi k)/D}), \qquad (4)$$

$$\omega' = 2\pi f T'$$

Aliasing is removed since $H(e^{j\omega})$ has lowpass characteristic. Hence, only first term is derived from equation(4).

$$X(e^{j\omega'}) \approx \frac{1}{D} Y(e^{j\omega'/D}), \quad |\omega'| \le \pi \quad \cdots\cdots (5)$$

If $\xi(n)$ is substituted with $(-1)^n \xi(n)$ for decimation by bandpass filter, the same relation as shown in equation(5) be derived for each bandpass.

2.1-2 System Identification using Decimation.

The effect of noise power out of the bandpass is removed and SNR (Signal to Noise Ratio) is improved remarkably since the region is extended as the full band and identified when it is needed to obtain the frequency characteristic only for such a low band ($0 \le \omega < \pi/D$) as known in equation (5).

But it is impossible to get the precision degree as described above only with low band or band filtering and this method shows strong feature for low SNR in comparison to another method[8] applying LSM after taking transformed expression including difference form of output signal. It is because corelation matrix condition of input-output signal is improved a lot (band-density just as power spectrum of input-output is wide and minimal eigenvalue increases) and it has robustness for feedback error of input-output data.

Moreover the amount of computation of algorithm can be reduced remarkably since data were sampled with the interval of D by decimation. But if D select too big, accuracy degree can be damaged according to decrease of data number. In order to improve it we can use effectively the whole data if we use multiple decimation.

Fig. 3 shows the procedure of the system identification of this algorithm.

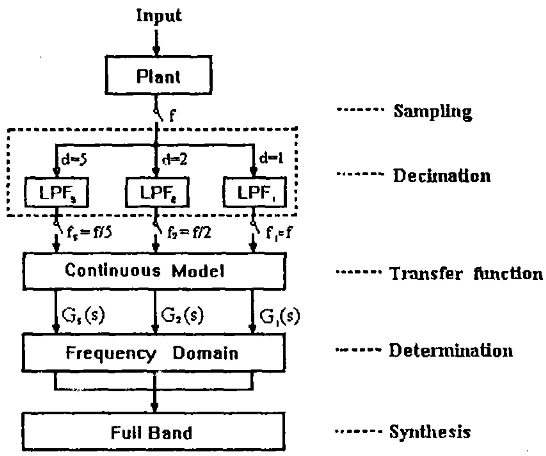

Fig. 3 Flow-Chart of Algorithm using Decimation

At first, each output data which passed through the system for input are sampled. And it is inputted to lowpass filter. Then output data of each frequency band is obtained by resampling with sampling period $f_d = f_s/d$. Transfer function is estimated with the output data as response data for structure determination. That is, it is the synthesized transfer function for the full band.

2.1-3 Transfer Function Estimation.

We can produce transfer function at continuous-time with discrete-time data obtained through the previous decimation.

(1) Frequency Response Data : G(jw)
(2) Inverse Fourier Transform : Step Response
(3) Sampling : Impulse Response
(4) Order Determination of Model : SVD
(5) Determination of Discrete-time Model: DTM
(6) Transformation into Continuous-time Model: CTM
(7) Transfer function Estimation: TF Synthesis

Table 1. Estimation Procedure of Transfer Function.

If we formulate the above process it has the relations as follows.

(1) Frequency Response Data : The G(jw) in

frequency-domain is computed according to the method synthesizing transfer function from response data in discrete-time.

(2) Inverse Fourier Transform : G(jw) is changed to time-domain by inverse Fourier transform. In order to match with the minimum implementation algorithm in discrete-time, we obtain step response with numerical integration after product 1/jw over it.

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{G(j\omega)}{j\omega}e^{-j\omega t}d\omega \quad \cdots\cdots(6)$$

(3) Sampling : Impulse response train $\{g_0, g_1, \cdots, g_N\}$ is calculated from increment of sampling data $\{y_0, y_1, \cdots, y_N\}$ for the obtained step response.

$$g_0 = y_0$$

$$g_i = y_i - y_{i-1} \quad (i = 1, 2, \cdots, N) \quad \cdots\cdots (7)$$

(4) Order Determination of Model : We distinguish with SVD (Singular Value Decomposition) for rank of Hankel matrix H_1 coming from $\{g_i\}$,and then determine order n.

$$H_1 = \begin{bmatrix} g_1 & g_2 & \cdots & \cdots & g_{N-\mu} \\ g_2 & g_3 & \cdots & \cdots & g_{N-\mu+1} \\ \cdot & \cdot & \cdots & \cdots & \cdot \\ g_\mu & g_{\mu+1} & \cdots & \cdots & g_{N-1} \end{bmatrix}$$

SVD
$=U\Sigma V \quad \cdots\cdots (8)$

$$\Sigma = \mathrm{diag}\ \{\sigma_1, \sigma_2, \cdots, \sigma_n, \cdots, \sigma_\mu\}$$

$$(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \gg \varepsilon \geq \sigma_{n+1} \geq \cdots \geq \sigma_\mu)$$

(5) Determination of DTM(Discrete-Time Model): we obtain discrete-time model (A,B,C).

$$A = \tilde{\Sigma}^{-\frac{1}{2}}\tilde{U}^T H_2 \tilde{V}^T \tilde{\Sigma}^{-\frac{1}{2}}$$

$$B = \tilde{\Sigma}^{\frac{1}{2}}\tilde{V}[1\ 0 \cdots 0]^T \quad \cdots\cdots (9)$$

$$C = [1\ 0 \cdots 0]\tilde{U}\tilde{\Sigma}^{\frac{1}{2}}$$

단, $\underset{\mu \cdot n}{U} = [\tilde{U} : U^1] \in C^{\mu \cdot \mu}$; $\quad \underset{<-n->}{}$

$$\underset{(N-\mu)\cdot N-\mu)}{V} = \begin{bmatrix} \tilde{V} \\ -- \\ V^1 \end{bmatrix}^n \in C^{(N-\mu)\cdot(N-\mu)}$$

$$\tilde{\Sigma} = \mathrm{diag}\ \{\sigma_1, \sigma_2, \cdots, \sigma_n\}$$

$$H_2 = \begin{bmatrix} g_2 & g_3 & \cdots & \cdots & g_{N-\mu+1} \\ g_3 & g_4 & \cdots & \cdots & g_{N-\mu+2} \\ \cdot & \cdot & \cdots & \cdots & \cdot \\ g_{\mu+1} & g_{\mu+2} & \cdots & \cdots & g_N \end{bmatrix} \quad \cdots\cdots (10)$$

(6) Transformation into CTM(Continuous-Time Model) : we transfer discrete-time model (A,B,C) into continuous-time model(A_c, B_c, C_c).

a) $\tilde{A} = T^{-1}AT = \mathrm{diag}\{\lambda_1, \lambda_2 \ldots \lambda_n\}$

$\tilde{B} = T^{-1}B$ (T: diagonal matrix of A)

$\tilde{C} = CT$

b) $\eta_i = \frac{\alpha_i + j\beta_i}{\tau}$, $\lambda_i = e^{\alpha_i + j\beta_i}$, (τ : sampling period)

c) $A_c = \mathrm{diag}\{\eta_1, \eta_2, \ldots \eta_n\}$, $B_c = (A - I)^{-1}A_c B$

$C_c = C$

(7) Synthesis of TF(Transfer Function) according to State Space Model.

$$G(s) = C_c(sI - A_c)^{-1}B_c + y_0 \quad \cdots\cdots (11)$$

2.1-4 Division and Synthesis of Frequency Band.

Reliable value of estimations can be computed at the band under 1/5T in frequency band as explained in the previous section, and frequency band including decimation order d is estabilished as shown in Fig. 3.

At first, reliable identification band is the band under 1/5 $T_5 = 1/25T$ since sampling period is $T_5 = 5T$ at d=5. So frequency characteristic is computed by substitution to $z^{-1} = \exp^{(-j\omega T_5)}$ as estimation value of transfer function at d=5 for the band of $0 \sim 1/25T$.

Secondly, reliable identification band is the band under $1/5T_2 = 1/10T$ since sampling period is $T_2 = 2T$ at d=2. Here is computed frequency characteristic based on estimation value of d=2 for the band of $1/25T \sim 1/10T$.

Finally, we compute frequency characteristic using estimation value based on original output data which is not decimated for the band of $1/10T \sim 1/2T$. The estimation values of frequency response characteristic at $d = 1, 2, 3$ are synthesized and it is defined as estimation value of frequency response for full frequency band.

PART 2. PID-ATC design using M-circle.

This paper consists of determination of PID coefficients applying M-circle to transfer function of the model obtained from each band. Fig. 4 is block diagram of the system of the whole algorithm .

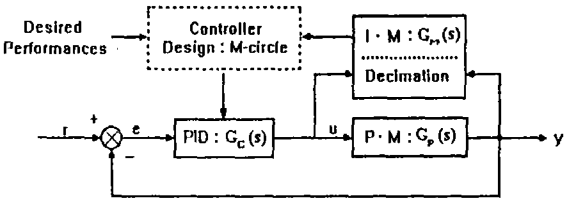

Fig 4. Block diagram of control system

Where r : input signal

y : system output

e : input-output error

u is the input for plant control and controls the input for plant-control, controls $G_P(s)$ as unknoun plant P · M(Plant Model) using I·M (Identification Model) $G_M(s)$ obtained in PART 1 according to decimation order.

The whole structure of this algorithm is determined using inner loop (feedback structure of plant) and outer loop (decimation). And we design controller prameters applying M-circle to the transfer function of the model.

We use PID controller which has played an important role for the last half century has firm and effective characteristic in the various process. Especially, PID controller has simple structure, decrease of the amount of computation is possible , and it can be included in micro-processor. But it takes long time to frequently modification of coefficients so that the optimal performance can be carried out in the system which has the change of processing condition and nonlinear time variable feature and it controls on the level of maintaining stability of process gain.

Automatic establishment of the parameters of PID controller has been studied by Ziegler - Nichols(1942), and tuning technique has been studied by Cohen - Coon(1953) and Astrom - Hagglund(1982).

ATC(Auto Tuning Controller) has ability to establish automatically parameters confronting feature of control object or driving condition. PID-ATC which establishs automatically PID parameters such as proportional gain (K), integral time (T_i), derivative time (T_d) can bring automization of process operation, decrease of modification time lack problem of specialist and can solve personal difference of modification results, etc.

In this paper, variables of controller design are M_P, ω of M-circle and proportional gain, integral and derivative time are obtained using them. The design procedure of PID coefficients is shown as follows.

2.2-1 M_P value

$G = G_C G_P$ is loop transfer function multiplying transfer function of controller by transfer function of process. G_s is closed loop transfer funtion and M is defined as follows.

$$G_s = \frac{G}{1+G} \ , \ \left| \frac{G}{1+G} \right| = M \qquad \cdots\cdots\cdots\cdots\cdots (12)$$

M_P is the biggest value of M in Nyquist plot. M_P can be related to the characteristic of other systems and can be computed approximatly from ζ (relative damping ratio)

$$M_P = \frac{1}{2\zeta\sqrt{(1-\zeta^2)}}, \zeta \le 1/\sqrt{2}, d = e^{-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad \cdots\cdots (13)$$

2.2-2 Design method

Design procedure is shown as follows, Fig. 5 shows M-circle design method.

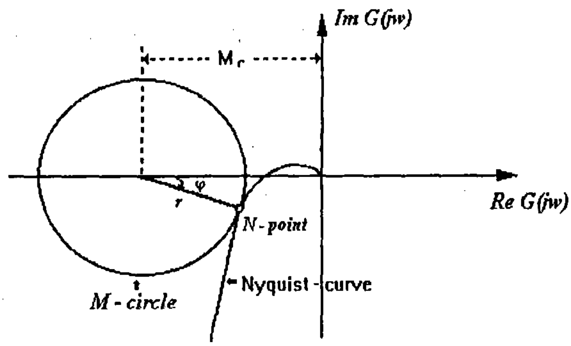

Fig. 5 Design method using M-circle

Here, M_r is center distance of M-circle, φ is angle connected a point of contact of M-circle and Nyquist-curve, r is diameter of M-circle.

1) Computation of $G_P(j\omega)$ and $G'_P(j\omega)$

$$G_P(j\omega) = a + jb \quad , G'_P(j\omega) = c + jd \qquad \cdots\cdots (14)$$

2) Computation of $G_C(j\omega)$ and $G'_C(j\omega)$:

$$G_C(j\omega) = K\left[1 + j(\omega T_d - \frac{1}{\omega T_i})\right]$$

$$G'_c = jK\left[T_d + \frac{1}{\omega^2 T_i}\right] \qquad \cdots\cdots\cdots\cdots\cdots (15)$$

3) Computation of Controller parameter :

$$K = \frac{-r\sin\varphi}{b + a\omega T_d - 1/\omega T_i}$$

Where , T_i and T_d are given as follows

$$\tan\varphi = \frac{c - d(\omega T_d - \frac{1}{\omega T_i}) - b(T_d + \frac{1}{\omega^2 T_i})}{d + c(\omega T_d - \frac{1}{\omega T_i}) + b(T_d + \frac{1}{\omega^2 T_i})} \quad \cdots\cdots\cdots(16)$$

Nyquist-curve meets M-circle at the point N if (17) is satisfactory.

$$N = -M_r + r\cos\varphi - jr\sin\varphi \quad \cdots\cdots\cdots (17)$$

Compensated Nyquist plot passes through N if (18) is satisfactory.

$$G_P(j\omega)G_C(j\omega) = -M_r + r\cos(\varphi) - jr\sin(\varphi) \quad \cdots\cdots (18)$$

Eventually three conditions are determined in order to choose four parameters (K, T_i, T_d, φ) for the control.

① $\omega T_i = \alpha \ \ (3 \le \alpha \le 6)$

② $\omega T_d < 0.8$ (The observation is the sinusoidal signal within 10 %.)

③ $\gamma = \tan^{-1}(\omega T_d - \frac{1}{\omega T_i})$ (Phase movement is below $\pi/3$)

Since closed loop poles move forward zeros , we need $T_i > T_d$ condition. With the exception, considerable item determines T_i/T_d relation.

2.2-3 Design variables

Design variables are frequency (ω) and M_P value. Even if the system has M_P values with good damping ratio, some problems can happen in design variable determination when design variables are choosed very small. Because the big circle includes some frequency of Nyquist plot if M-circle has the large diameter. Design of M-circle with large diameter can be more sensitive. Thus M_P is determined within 1.3~1.5 .

III. Simulation Results

First, we confirm identification precision degree transfer function of given plant and model transfer function using decimation in order to verify validation of the algorithm. Table. 2 shows determination of frequency-domain and identification-domain according to decimation. Fig. 6 is resultant graph using it.

Fig. 6 shows that the characteristic of the given plant model is similar to that of the transfer function for frequency region passed through decimation. Thus accuracy of identification can be verified. Eventually it is possible to divide frequency response data for the region according to decimation degree and control synthesis transfer function.

Fig. 7 and Fig. 8 show simulation result for control system, which is the result for PID controller design for plant model $G_P(s)$ and identification model $G_M(s)$.

Experimental objects have 4 order delay system. And it is expressed such transfer function as follows.

$$G_P(s) = \frac{K}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$$

Where, K : proportional gain

T_1, T_2, T_3, T_4 : time constant.

As a result of simulation, we can find control of transfer function for the region through decimation is performed with better precision because of more accurate identification in low and high pass region.

Item \ Order	Identification-band(Hz)		Frequency-domain(rad/sec)	
	Computation domain	Real identification band	Computation domain	Real frequency domain
d=5	$f_5/5=f_s/25$	$0 \sim f_s$	$0 \sim 2\cdot\pi\cdot f_s/25$	$0 \sim 25.1327$
d=2	$f_2/5=f_s/10$	$f_s/25 \sim f_s/10$	$2\cdot\pi\cdot f_s/25 \sim 2\cdot\pi\cdot f_s/10$	$25.1327 \sim 62.8318$
d=1	$f_1/5=f_s/2$	$f_s/10 \sim f_s/2$	$2\cdot\pi\cdot f_s/10 \sim 2\cdot\pi\cdot f_s/2$	$62.8318 \sim 314.1593$

Table 2. Domain Determination for Decimation Order.
(Sampling frequency f_s= 100 Hz)

IV. Conclusions

There have been a lot of studies for selection method of the optimal sampling frequency. In the other hand, most of them have taken the method which only determines the optimal sampling frequency. By comparison to it, the method proposed in this paper which takes multiple decimation enables to determine different sampling frequency per each divided frequency band and increases much the degree of freedom for the selection.

In this paper we found dynamics for unknown plant as estimation of transfer function using decimation, and designed PID controller found plant dynamics for the frequency region and synthesized them. And the controller design has good performance in low region as well as in high region.

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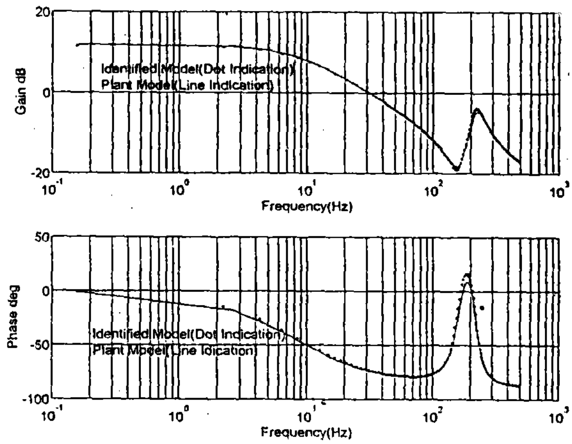

Fig 6. Frequency response between Identification model and Plant model.

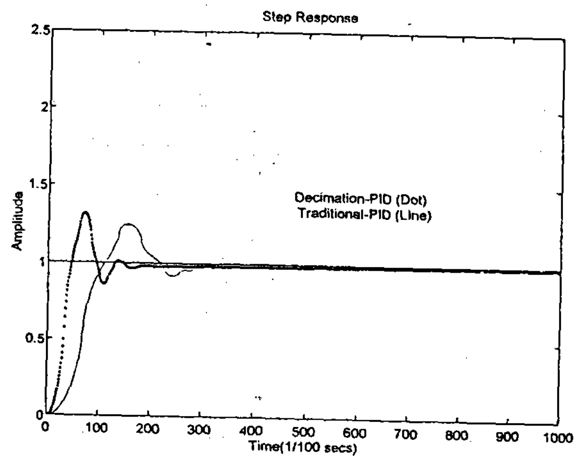

Fig 7. Step response of PID-ATC.

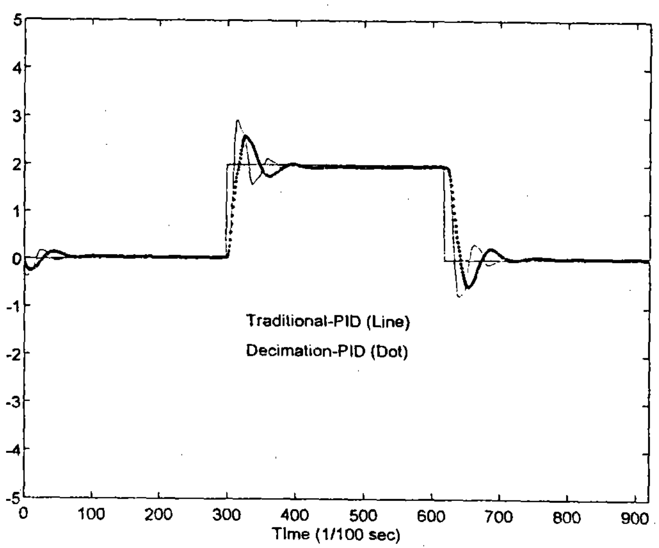

Fig 8. Pulse response of PID-ATC.

(4-order delay system : when K=1, $T_1=1, T_2=0.2, T_3=0.05, T_4=0.01$)