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Synchronic Relations with a Time Constraint

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Abstract

Three synchronic variables (Deviation Time, Fairness Time, Synchronic Time) are defined for Timed Place Petri Nets (TPPN). These parameters show the dependency between the firing of transition subsets in the time domain by different values. The approaches in this paper can be used to find synchronic relations in Stochastic Petri Nets. This paper presents how to decide the minimum resources required to a Flexible Manufacturing Cell using Synchronic Time concept.

I. Introduction

A synchronic variable for generalized Petri Nets is a measure for the degree of event dependency, an upper bound on the relative frequency of event occurrences. Based on this interpretation, synchronic variables between two events or two sets of events are important properties when designing or analyzing dynamic behaviors of systems.

To measure the dependency between the firing of transition subsets, synchronic relations are used. The synchronic relations are defined a behavioral sense, for a marked Petri Net or in a structural sense, for any initial marking [3]. However, in this paper only the behavioral synchronic relation is considered.

Synchronic variables for Generalized Petri Nets are studied by many researchers. The firing Deviation Bound is the most basic concept. A related property is the Synchronic Lead. Fairness Bound and Synchronic Distance are constructed by making Deviation Bound and Synchronic Lead, respectively, symmetric [3].

Until recently, the synchronic relations are defined without a time concept. Therefore, the results can not be applied to Timed Petri Nets or Stochastic Petri Nets. Synchronic relations defined by the previous researchers are mainly used in qualitative analysis, not in quantitative analysis.

This paper introduces three synchronic relations in time domain, and formulates them. One of the above synchronic relations is used to analyze a simple manufacturing system. The rest of the paper is organized as follows. In Section II, a Timed Petri Net is reviewed briefly, including some notation. In Section III, synchronic relations in time domain are described intuitively. In Section IV, Deviation Time, Fairness Time, and Synchronic

Time are defined formally. In Section V, the Synchronic Time is applied to Flexible Manufacturing Cell. In Section VI, a conclusion is given and future research is discussed.

II. Definition of Timed Placed Petri Nets

We assume that the reader is familiar with ordinary Petri Nets. The model studied in this paper is a Timed Place Petri Net, in which certain delays are associated to it's places.

The Timed Placed Petri Net (TPPN) used in this paper is formally defined as follows.

Definition : Timed Place Petri Nets (TPPN)

A TPPN N is defined by a 6-tuple, N=(P, T, I, O, M_0, D), where:

(1) $P=\{p_1, p_2, ..., p_m\}$, is a finite set of places, $m \geq 0$.

(2) $T=\{t_1, t_2, ..., t_n\}$, is a finite set of transitions, $n \geq 0$, such that $P \cap T = \emptyset$.

(3) $I: T \rightarrow \mathbb{N}^m$ is an input function, $\mathbb{N}$ is a non-negative integer.

(4) $O: T \rightarrow \mathbb{N}^m$ is an output function.

(5) $M_0 \in \mathbb{N}^m$ is an initial marking.

(6) $D=\{d_1, d_2, ..., d_m\}$, is a firing delay for each place.

To understand the contents of this paper easily, some basic notations are described as follows.

$[M(\tau)^T] = [m_1(\tau), m_2(\tau), m_3(\tau), \ldots, m_m(\tau)]$

: Marking of the TPPN at the time τ.

$[X(\tau)^T] = [x_1(\tau), x_2(\tau), x_3(\tau), \ldots, x_n(\tau)]$

: A firing vector of the TPPN at the time τ.

σ : A firing sequence

$\bar{\sigma}$: A firing count vector

T_i : Subset of T

$L(N, M_0)$: The set of all firing sequences from M_0

III. Basic Concepts of Synchronic Relations

Before introducing the formal definitions of Synchronic Time (ST), Deviation Time (DT), Fairness Time (FT) in TPPN, an example to explain the basic concepts intuitively is shown in Figure 1.

The example shown in Figure 1 (The system was shown in [12]) describes the changing of four seasons. The synchronic distance between $\{t_1, t_2\}$ and $\{t_3, t_4\}$ shown in Figure 1(a) is calculated by adding the place s shown in (b), such that ${}^\bullet s=\{t_1, t_2\}$and $s^\bullet=\{t_3, t_4\}$. Then the capacity of tokens of the place s at least should be 2, when $M_0(s)=0$. Because, whenever t_1 and t_2 fire, each transition puts a token into the place s, and by subsequent firing of t_3 and t_4, the two tokens in the place s are removed again, therefore, the range of the number of tokens in the place s will never exceed 2. This maximum range of the number of tokens in the place s is called the synchronic distance between $\{t_1, t_2\}$ and $\{t_3, t_4\}$.

If the initial marking of the net shown Figure 1(a) is changed to $M_0(P)=[0, 1, 0, 0]^T$ and the initial marking of the place s is still 0, then the behavior of the original net is changed. The original net is live and bounded, but the modified net is no longer live. To be consistent with the behaviors of the original net, initially two tokens are placed in the place s. This does not affect the maximum range of the number of tokens in the place s. The synchronic distance between $\{t_1, t_2\}$ and $\{t_3, t_4\}$ remains 2. The synchronic distance is considered the maximum range of the number of tokens in the place s while maintaining the behaviors of the original net.

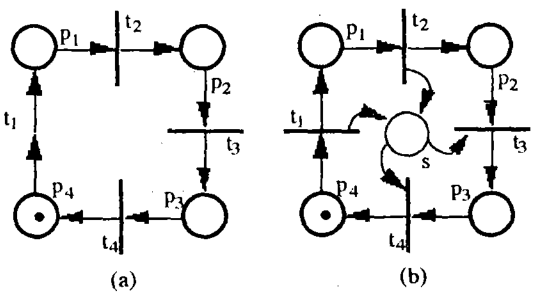

Figure 1: A synchronic distance.

Apply the above concepts to Timed Place Petri Nets. We assume that the delay of the four places (p_1, p_2, p_3, p_4) are the same with 3 unit time. The above net convert to the TPPN with D = {3, 3, 3, 3}. Then, Synchronic Time (ST) between {t_1, t_2} and {t_3, t_4} is described as follows.

ST (T_1, T_2); Synchronic Time between

T_1={t_1, t_2} and T_2={t_3, t_4}.

= The minimum token delay in the place s to return to its initial condition.

With changing the time, the variation of the number of tokens of the place S is shown in Table 1.

IV. Formal Definitions of Synchronic Relations

This section presents the formal definitions of Deviation Time (DT), Fairness Time (FT), and Synchronic Time (ST). Before introducing the formal definitions of Deviation Time and Fairness Time, Deviation Bound and Fairness Bound are described. The reason is that the concept of Deviation Time of a Timed Place Petri Net comes form the Deviation Bound of an Ordinary Petri Net. Deviation Bound between T_i and T_j is defined as follows [3]. Also, the definition of Fairness Bound is given.

Definition : Deviation Bound

$DB(T_i, T_j)=\sup\{\bar{\sigma}(T_i) \mid \sigma\in L(N, m), m\in R(N, m_0), \bar{\sigma}(T_j)=0\}$.

Definition : Fairness Bound

$FB(T_i, T_j)=\max\{DB(T_i, T_j), DB(T_j, T_i)\}$.

To apply the deviation concept to Timed Place Petri Nets, Processing Duration Time (PDT) is required.

Definition : Processing Duration Time

PDT: Maximum Processing Time until Deadlock Situation

Using the processing duration time, the Deviation Time is defined as follows.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13
# of tokens in s	0	0	0	1	1	1	2	2	2	1	1	1	0	0

Table 1: The Token Variation with respect to Time

The number of tokens varies from "0" to "2" as the same with the synchronic distance. However, the delay time of the place s to return to its initial condition (zero token in s) is 12. In this case, ST (T_1, T_2)=12. In another case, ST (t_1, t_3) = 6.

Definition : Deviation Time (DT)

$DT(T_i, T_j) =\sup\{PDT \mid \sigma\in L(N, m), m\in R(N, M_0), \bar{\sigma}(T_j)=0\}$

Properties of Deviation Time

1) Non-negativity: $DT(T_i, T_j) \geq 0$.

2) Non-symmetry: $\exists <N, M_0>$ such that $DT(T_i, T_j) \neq DT(T_j, T_i)$.

Definition : Fairness Time

$FT(T_i, T_j)=\max\{DT(T_i, T_j), DT(T_j, T_i)\}$.

V. Applications to Factory Automation

To illustrate the application of Synchronic Time, a simple system is shown in Figure 2. The Flexible Manufacturing Cell consists of a machining center, a tool inspection system, and a storage for tools. There is one process which use the machining tools.

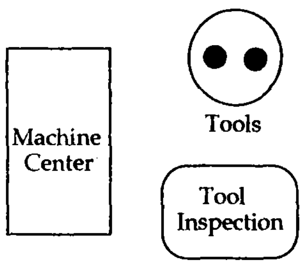

Figure 2. A Simple System

Figure 3 represents the TPPN model of this system, where:

- p_1: Available machines for processing
- p_2: Number of machines of processing busy
- p_3: Tools in inspection
- p_4: Available tools

The place p_3 can be thought of as an inspection where the tools are check for wear, and replaced if necessary. The following delays are associated with the model:

d_1: Load /unload delay for processing (1 unit time)

d_2: Time needed to carry out processing (2 unit time)

d_3: Time needed to transport the tools (3 unit time)

d_4: Delay due to tool inspection (4 unit time)

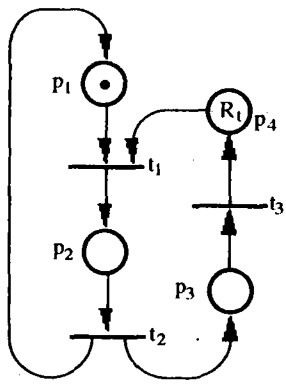

Figure 3. Timed Placed Petri Net Model

We try to find out the minimum number of required tools maintaining the maximum performance of the processing. We assume that p_1 has one token. The maximum performance of the processing can be realized, if there are a large number of tools. But, unnecessary tools means cost increasing. Here, we try to find out how to minimize cost.

The maximum firing rate of t_1 in Figure 3 is 0.33/unit time. The firing of the transition t_1 and t_3 are closely related to the system performance.

$$ST(t_1, t_3) = 9 \text{ unit time}$$

As seen in the above, the maximum firing rate (maximum performance) is 1/3. In other words, the transition t_1 should fire to maintain the maximum system performance. We can easily conclude that three ($ST(t_1, t_3)/3$) tokens are required in the place p_4.

VI. Conclusions

In this paper, synchronic relations with a time concept are presented. Deviation Time (DT), Fairness Time (FT), and Synchronic Time (ST) are explained and defined for Timed Place Petri Nets. The concepts can be used for analyzing system performance and controlling of factory automation. A simple example, which is to decide the minimum number of tools, is shown to verify the usefulness of Synchronic Time. However, the definitions of DT, FT, and ST need to be more refined.

Synchronic Time can also be applied to detect or avoid deadlock situation and to characterize concurrency against sequential behavior in certain circumstances. In an automated manufacturing system, resources and information are shared among several processes. This sharing should be controlled or synchronized to insure the correct operation of the overall system in time domain. For future research, the concepts described in this paper will be applied to predicting the time of deadlock situation in automated manufacturing systems.

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