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Geometric Position Determination algorithm and Simulation in Satellite Navigation

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Abstract This paper presents a new algorithm to determine the receiver position in satellite navigation for GPS (Global Positioning System). The algorithm which is based on vector analysis is able to obtain simultaneously the receiver position and the direction vector which is from the receiver position to a satellite. In its first calcuation stage, it does not require the complex initial value which is used in the previous works and affects the accuracy of the observed receiver position. Furthermore, the algorithm tells us whether a selected configuration among the visible satellites is good or poor for the accuracy. Comparing the algorithm with the previous method, the effectiveness of the algorithm is verified through the experimental simulations.

Index Terms – GPS, navigation, vector analysis, position determination, new algorithm

1 Introduction

This paper presents a new algorithm to determine the receiver position in satellite navigation for GPS (Global Positioning System).

The receiver position is obtained by measuring the travel time of the satellite signal. However, the satellite clock and the receiver clock are slightly different in general, and these clocks are never perfectly synchronized. Thus, the pseudorange which depends on the clock bias is obtained instead of the true range. From the fact, there are four unknown quantities ; three point coordinates of the receiver position and the clock bias, so it is need to select four satellites among the visible satellites. On its selection, the configuration of the selected satellites affect the accuracy of the obtained receiver position, and GDOP (Geometric Dilution of Precision) is used to verify the accuracy [1]. GDOP is an important factor evaluating for the satellite configuration, i.e., a low GDOP value should mirror good satellite configuration, so obtain the receiver position in high accuracy, and vice versa. The GDOP value is calculated by the direction vector, but the direction vector is unknown previously.

The useful algorithm to get the receiver position is based on the linearization method [1] concerned with a set of nonlinear algebraic equations, and it is generally solved using the iterative method, e.g., Newton-Raphson method. However, the method tells us no information about the accuracy of the receiver position obtained from a certain combination from the visible satellites. Thus the GDOP value needs to calculate the direction vector from the obtained receiver position, and the value may not have high accuracy. Furthermore, the initial value which is required in the method affects the accuracy, but it is difficult to determine that previously.

To avoid the problems, an algorithm which is based on vector analysis is presented. The algorithm does not require the initial value as mentioned above, and it obtains both the receiver position and the direction vector from the receiver position to a satellite, simultaneously. Furthermore, the proposed algorithm tells us whether a selected combination among the visible satellites is good or poor for the accuracy. Comparing it with the linearization method, we verify the effectiveness of that through some experimental simulations, and it is pointed out that the proposed algorithm has the relation to the GDOP value in its iterative situation.

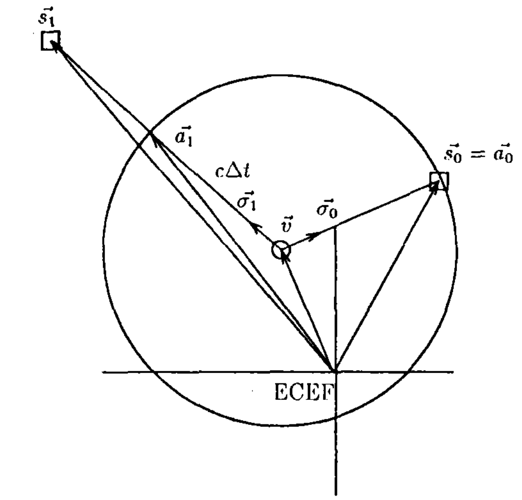

Fig.1 Geometric representation of the proposed method.

2 Problem Statement

The geometric interpretation between the satellite position and the receiver position is shown in Fig.1. The denotation " $\rightarrow$ " represents a position or a direction vector, and the origin of the position vector is the center of Earth, and they are expressed ECEF ; Earth - Centered - Earth - Fixed coordinates.

As shown in Fig.1, the receiver position to be calculated by using the given values of the satellite positions and the pseudoranges is expressed as the center of a sphere with a radius of range being the clock bias, which is a virtual object but plays an important role in the proposed algorithm. The number of the visible satellite we can observe is generally about 10 at most. The symbolic notations in Fig.1 are shown below

- $\vec{s_i}$: the satellite position vector
- $\vec{v}$: the receiver position vector
- $\vec{a_i}$: the auxiliary position vector
- $\vec{\sigma_i}$: the direction vector which is a unit vector from the receiver to a satellite
- ρ_i : the pseudorange which is defined by observing the signal from a satellite to the receiver
- c : the velocity of light
- Δt : the error time which depends on the clock bias between the satellite clock and the receiver clock

Our aim is to obtain simultaneously the receiver position vector $\vec{v}$ and the direction vector $\vec{\sigma_i}$. To do this, we introduce the auxiliary position vector $\vec{a_i}$ so as that all $\vec{a_i}$ are on the shpere, then

$$\vec{a_i} = \vec{s_i} - \rho_i \vec{\sigma_i} \quad (i = 1 - N) \tag{1}$$

The above equation is derived directly from the definition and the geometric interpretation in Fig.1. To determine the sphere in three-dimensional space, the four auxiliary position vector $\vec{a_i}$ are needed, so $N = 4$ in eq.(1). To make our discussion easy, we define $\vec{a_0} = \vec{s_0}$ such as to $\vec{s_0} = \min_i \rho_i$. From the definition, using $\vec{a_1}, \vec{a_2}, \vec{a_3}$, the reciever position is derived as following (see Appendix)

$$\vec{v} = \frac{a_1^2(\vec{a_2} \times \vec{a_3}) + a_2^2(\vec{a_3} \times \vec{a_1}) + a_3^2(\vec{a_1} \times \vec{a_2})}{2(\vec{a_1} \times \vec{a_2}) \cdot \vec{a_3}} \tag{2}$$

where " $\cdot$ " is inner product and " $\times$ " is outer product.

In eqs.(1) and (2) $\vec{a_i}$ and $\vec{\sigma_i}$ are unknown, so we introduce the direction vector such as

$$\vec{\sigma_i} = \frac{(\vec{s_i} - \vec{v})}{\rho_i + c\Delta t} \tag{3}$$

The above equation can be derived easily from Fig.1, and is concerned with the pseudorange and the corrected range $c\Delta t$, where $c\Delta t$ has the following relationship between $\vec{a_i}$ and $\vec{v}$

$$c\Delta t = |\vec{a_i} - \vec{v}| \quad for \ all \ \ i \tag{4}$$

Using eqs.(1)-(4), the proposed algorithm is stated below

STEP 1: Let $\vec{\sigma_i} = -\vec{s_i}/|\vec{s_i}|$ and set $k = 1$, $\Delta t(0) = 0$ and ϵ =an arbitrary positive value.

STEP 2: Calculate $\vec{a_i}$ from eq.(1)

STEP 3: Calculate $\vec{v}$ from eq.(2)

STEP 4: Calculate $\vec{\sigma_i}$ from eq.(3)

STEP 5: Update $\Delta t(k)$ from eq.(4)

STEP 6: If $|\Delta t(k) - \Delta t(k-1)| < \epsilon$ then $k = k + 1$ and GOTO STEP 2, else STOP

Note that, in the algorithm, the calculated $\Delta t(k)$ is a variable and unknown because its true value is not observable from the receiver, so the calculated $\Delta t(k)$ may change at each iteration time.

3 Simulation

Comparing the proposed algorithm with the linearization method based on Newton-Raphson calculation, we verify the effectiveness of that through experimental simulations.

In the simulations, the actual receiver position is given previously for our verification

$$\vec{v} = (-3933122.888616, 3377926.862563, 3715284.287880)$$

and we use the satellite position which are observed from the actual measurment in the case of 5 visible satellites. From the actual reveiver position and the observed satellite positions, the true ranges and the true $c\Delta t$ are calculated (Fig.1). We define $\Delta t = 1.5[\mu\text{sec}]$ for the verification, then the pseudorange can be also calculated.

Using the datum, the GDOP values are calculated for each selected 4 satellites (Table 2). A low GDOP value should mirror good satellite configuration, so obtain the receiver position in high accuracy, and vice versa. From the fact, we will consider the two cases in Table 2 as following

(a) The configuration of the selected satellites is poor in the case of No.4.

(b) The configuration of the selected satellites is good in the case of No.2.

For the two cases above, we examine two method ; the proposed algorithm and the linearization method, and the results which shows the number of the iteration in completing the algorithm are shown in Table.3. The iterative situations using the proposed algorithm are shown in Fig.2. From the results, it is found that the iteration increases in use of the proposed method when the geometric configuration is the case (a). On the other hand, the iterative situations by the linearization method shows no information about the configration for the same case.

From the investigation, if the iterative situation of the algorithm shows poor, it is found that the high accuracy for the receiver position is not expected. Therefore, we can stop carrying out the algorithm halfway, and select the another configuration for the other satellites until the expected situation be obtained. This leads to save the computational effort to get the receiver position.

Table 1 Satellite positions used in the simulation and those true values of the range and $c\Delta t$.

	Satellite position [m]	True range [m]	True $c\Delta t$ [m]
$\vec{s_1}$	-15896246.98002887 21237102.46107696 1751992.024069211	21585203.44120545	450
$\vec{s_2}$	-3395203.436112383 23373949.46232741 12139860.97314664	21704925.0050199	450
$\vec{s_3}$	-26429481.17605432 -2688357.807378323 -3034538.920169972	24257907.14867098	450
$\vec{s_4}$	-17341772.44390838 -3895669.423616398 19615606.99326976	22034458.19543649	450
$\vec{s_5}$	-12733967.99739484 8241145.010058916 21696047.23338308	20601300.96574331	450

Table 2 Selected satellites and the GDOP values.

No.	Satellite	GDOP
1	$\vec{s_1}, \vec{s_2}, \vec{s_3}, \vec{s_4}$	6.373321
2	$\vec{s_1}, \vec{s_2}, \vec{s_3}, \vec{s_5}$	5.316043
3	$\vec{s_1}, \vec{s_2}, \vec{s_4}, \vec{s_5}$	6.796607
4	$\vec{s_1}, \vec{s_3}, \vec{s_4}, \vec{s_5}$	66.823178
5	$\vec{s_2}, \vec{s_3}, \vec{s_4}, \vec{s_5}$	9.166053

Table 3 The number of the iteration for the methods, L.M.:the linearization method, P.M.:the Proposed method.

No.	L.M.	P.M.
1	5	5
2	5	5
3	5	5
4	5	7
5	5	5

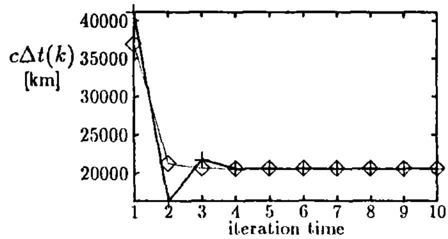

Fig.2 Change of radius of sphere, + : the case (a), ◇ : the case (b).

4 Conclusion

This paper presents the new algorithm which obtains simultaneously the receiver position and the direction vector, and requires no complex initial values. Furthermore, investigating the iterative situation in completing the algorithm, it is found that the algorithm is able to evaluate directly whether the geometric configuration of the selected satellites among the visible satellites is good or poor. On the other hand, in use of the linearlization method, it is difficult to do so in completing its algorithm. As a result, by excluding the case of poor iterative situation, we can efficiently get the receiver position by using the proposed algorithm.

References

[1] B.H.Wellenhof, H.Lichtenegger and J.Collins, "GPS theory and practice", Springer Verlag, 1992.

[2] M.Higashiguchi and M.Nakagawa, "Position Determination Algorithm in satellite navigation", Proc. of the 1994 IEICE spring conference, No.2, March,pp.731-32,1994.

Appendix

The auxiliary position vector $\vec{a_0}, \vec{a_1}, \vec{a_2}, \vec{a_3}$ is defined so as to be on the sphere, so the relationship between $\vec{a_i}$ and the center of the sphere (is equivalent to the receiver position) is given by

$$(\vec{a_i} - \vec{a_j}) \cdot \vec{v} = \frac{1}{2}(a_i^2 - a_j^2) \ \ (i \neq j)$$

Now, to make the derivation easy, we define $\vec{a_0}$ as the basis vector. From the definition, we get the following relationships

$$2\frac{\vec{a_1} - \vec{a_2}}{a_1^2 - a_2^2} \cdot \vec{v} = 1$$

$$2\frac{\vec{a_2} - \vec{a_3}}{a_2^2 - a_3^2} \cdot \vec{v} = 1$$

$$2\frac{\vec{a_3}}{a_3^2} \cdot \vec{v} = 1$$

Soving the equations above concerned with $\vec{v}$, the receiver position is given by eq.(2) (For detail see [2]).