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ELECTRICAL COMPUTED TOMOGRAPHY FOR IMAGING OF INTERNAL RESISTIVITY AND PERMITTIVITY DISTRIBUTION

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Abstract

In this paper, reconstructing the internal resistivity and relative permittivity distribution is discussed. The iterative reconstruction method based on Finite Element method and Newton method were used to reconstruct both of resistivity and permittivity distribution. The Finite Element model of impedance distribution is built in complex field of resistivity and capacitive medium. The reconstruction results based on computer simulated data and experimental data are presented.

1. Introduction

Electrical Computed Tomography (ECT) is a new Computerized Tomographic method for reconstructing internal image based on the spatial distribution of electrical parameters such as resistivity or conductivity and relative permittivity. This method reconstructs the internal cross sectional distribution electrical impedance from measured boundary potential when the currents are applied to the periphery of object via a set of electrodes combination. ECT is an imaging technique that uses nonionizing radiation and it requires relatively simple instrumentation. However, the spatial resolution of ECT is limited by the number of potential measurement made on the boundary of object and the accuracy of measurement system.

Most of researchers have been developed the ECT system for reconstructing the resistivity or conductivity distribution[2]. L.F Fuks *et al* [3] proposed a method for imaging of electric conductivity and relative permittivity at low frequency. The purpose of this paper is to present the ECT method for reconstructing the internal resistivity and relative permittivity distribution. Reconstructing the resistivity and relative permittivity distribution, at once, will give us more information about the object for a certain purpose.

In ECT, reconstruction of image is a nonlinear problem. The iterative reconstruction algorithm based on Finite Element Method (FEM) and Newton - Raphson method which is proposed by Yorkey *et al* [4] were used to reconstruct a resistivity and permittivity distribution. The Newton-Raphson method compares the voltage responses of the real impedance distribution and the estimated voltage from Finite Element model of object to find the solution. Since, both of resistivity and relative permittivity images would be reconstructed the Finite Element model is built in complex field of resistive and capacitive medium. To obtain the information of the object for reconstruction process, the currents are applied to the set of electrodes on the surface of the object, then the resulting rms magnitudes of the voltages and its phase are measured.

2. The Principle of The System

Basically, the ECT system consist of a set of electrode combination, a data collection system including injected current distribution, voltage measurement, and difference phase measurement, and image reconstruction computer which is shown in fig.[1]. The electrodes that consist of current electrodes and voltage electrodes are placed on the periphery of object. The data collection system measures the voltage respons at the voltage electrodes when the currents are applied to the current electrodes. The measured voltage data and its phases, the differences phase of voltage response and the phase of the current source, are introduced into the computer for reconstruction process.

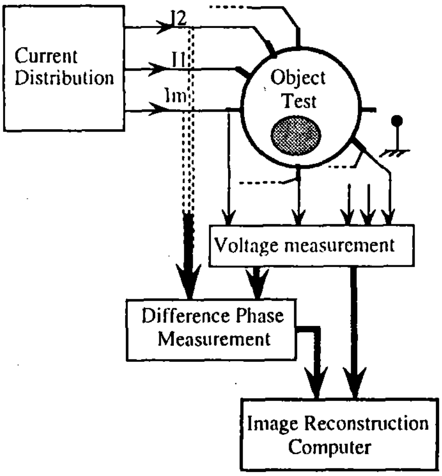


Fig. 1. ECT system

There are numerous ways to obtain the voltage data from measurement by injecting the current distribution. In this work, the currents are simultaneously injected to the current electrodes with one of them is set as reference and we assumed that phase angle is zero. The injected current at kth current electrode can be expressed as follows,

$$i_k = I_k e^{j\omega t} = \frac{I_k}{\sqrt{2}} \angle 0^o \qquad (1)$$

for m current electrodes and for one cycle measurement, the current distribution vector is

$$i = \left[i_1, i_2, \ldots, i_m, \ldots, i_{m\times(m-1)} \right]^T \qquad (2)$$

The voltage response at kth voltage electrode is described as follows

$$v_k = V_k\, e^{j\omega t + j\phi_k} = \frac{V_k}{\sqrt{2}} \angle \phi_k \qquad (3)$$

for m voltage electrodes and for one cycle measurement, we obtain the voltage distribution vector is

$$v = \left| v_1\ v_2 \ldots\ldots v_m, \ldots\ v_{m\times(m-1)} \right|^T \qquad (4)$$

where,

$k = 1,2,\ldots,m$
m = Number of electrodes
I_k = Current amplitude at kth electrode
V_k = Voltage amplitude at kth electrode
ϕ_k = Difference phase between injected curent and voltage response
T = Transpose of vector or matrix
ω = Angular frequency
t = Time

3. Modeling

The calculation of potential distributions on a body of complex impedance is to solve the governing equation for electric fields in complex of resistive and capacitive medium.

$$\nabla \cdot \frac{1}{r} \nabla \Phi = 0 \qquad (5)$$

where,

$\frac{1}{r} = \sigma + i\,\omega\varepsilon = \frac{1}{\rho} + i\,\omega\varepsilon$
$i = \sqrt{-1}$
σ = condutivity
ρ = resistivity
ε = permittivity

with boundary conditions

$$\Phi = \Phi_o \quad on\ \partial A_1 \qquad (6.a)$$

$$\frac{1}{r}\frac{\partial \Phi}{\partial \eta} = J_o \quad on\ \partial A_2 \qquad (6.b)$$

where, Φ_o is voltage and J_o is current density at the boundary on each electrode that can be obtained through physical measurement. From the equation (5), the complex field of resistivity and permittivity can be rewritten as follows,

$$r = \frac{\sigma}{\sigma^2 + (\omega\varepsilon)^2} - i\frac{\omega\varepsilon}{\sigma^2 + (\omega\varepsilon)^2} \qquad (7)$$

since $\sigma^2 >> (\omega\varepsilon)^2$, the equation (7) can be approximated to

$$r \approx \frac{1}{\sigma} - i\,\frac{\omega\varepsilon}{\sigma^2} \qquad (8)$$

Thus, two images can be obtained from the quantity of resistivity distribution (ρ) and from the quantity of reactivity or relative permittivity $(-\omega\varepsilon / \sigma^2)$.

The FEM is a numerical technique

approximating the solution to the equation (5). Using the FEM, the medium is divided into a finite numbers of small triangular element, and assumed that the impedance, i.e, resistivity and permittivity, in each element is homogeneous and isotropic. To make the FEM model in complex medium, the equation (5) is divided into resistive model and capacitive reactive model as follows,

$$\nabla \cdot \frac{1}{\rho}\, \nabla \Phi = 0 \qquad (9.a)$$

$$\nabla \cdot i\, \omega \varepsilon\, \nabla \Phi = 0 \qquad (9.b)$$

The above equations are to be said as the real part of Laplace equation (equation 9.a) or Laplace equation for resistive medium and the imaginary part of Laplace equation (equation 10.b) or Laplace equation for capacitive medium.

Modeling of the medium by FEM method in triangular element yields the (3 x 3) matrix for each element as follows,

$$y_e = \frac{1}{4\Delta}\left(\frac{1}{\rho} - i\,\frac{\sigma^2}{\omega\varepsilon}\right)\lfloor y_{3x3} \rceil \qquad (10)$$

where, Δ is area of element, $\lfloor y_{3x3} \rceil$ is [3 x 3] matrix triangular element for constructing the FEM model. The FEM defines the algebraic equation as follows,

$$\mathbf{Y\,V} = \mathbf{I} \qquad (11)$$

where, $\mathbf{Y}$ is complex admittance matrix that contains both of resistivity and relative permittivity, $\mathbf{V}$ is voltage distribution vector, and $\mathbf{I}$ is current vector.

4. Reconstruction Method

The iterative reconstruction algorithm based on Newton-Raphson method or Gauss-Newton method would be used to reconstruct both of internal resistivity and permittivity distribution, compares the voltage responses of the real object and the estimated one from FEM model by minimizing of the objective function. The objective function is defined as follows

$$F(r) = \frac{1}{2}\left(f(r) - v_o\right)^T\left(f(r) - v_o\right) \qquad (12)$$

where, $f(r) = \mathbf{TV}$

$f(r)$ is the complex estimated boundary voltage from Finite Element model, $\mathbf{T}$ is a transformation matrix, and v_o is the complex measured boundary voltage. The reconstruction equation of r is obtained by minimizing of equation (12), that can be described in the following expression

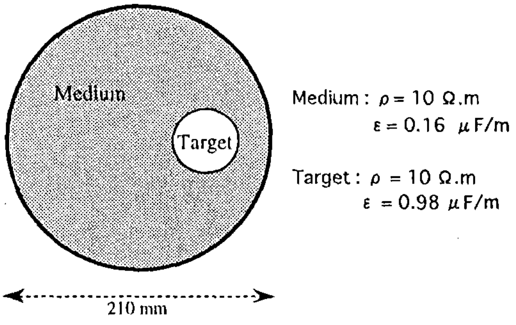
$$r^{k+1} = r^k + \Delta r^k \qquad (13.a)$$

$$\Delta r^k = -\frac{\left[f'(r^k)\right]^T \left\lfloor f(r^k) - v_o \right\rfloor}{\left\lfloor f'(r^k)^T\, f'(r^k) \right\rfloor} \qquad (13.b)$$

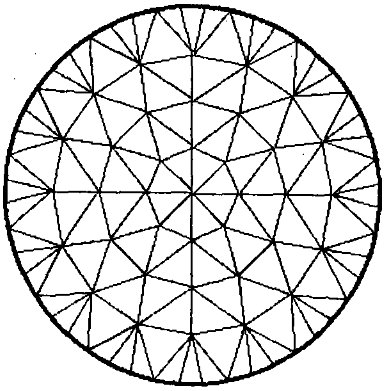
where, k is number of iteration. We guess a distribution of resistivity and relative permittivity, and calculate its theoretical voltage responses for the given injected current data using Finite Element method. Then we calculate the objective function. When the objective function value is less then a criterion error, the guessed distribution or calculated distribution of resistivity and relative permittivity are claimed as the desired solution. For reconstructing of resistivity distribution, only the real part of measured voltage and estimated voltage are used, and variable r is replaced with ρ. For reconstructing of permittivity, r is replaced with $\left(-\omega\varepsilon / \sigma^2\right)$ and only the imaginary part of voltages are used in the reconstruction equations.

5. Reconstruction Result

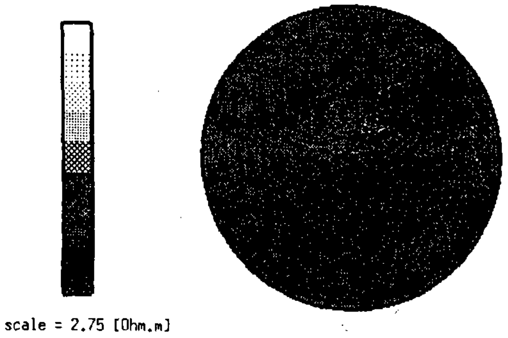
To illustrate the performance of the ECT system, we implemented the ECT system with two dimensional computer simulation and real circular object. Fig.[2] shows the computer model of object and its finite element model with 120 element pixels. As a numerical example, the resistivity of medium is the same with the resistivity of target but the relative permittivity is difference. Fig.[3] shows the result for both of resistivity distribution image and relative permittivity distribution image



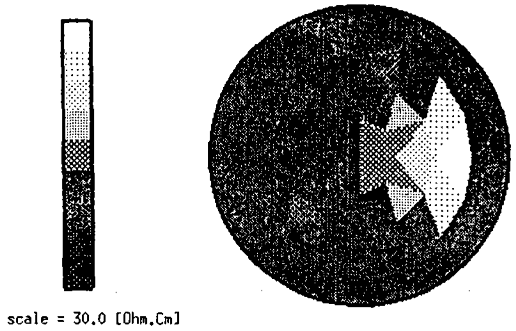
(a). Object Test



(b). Finite Element Model
Fig.2. Object test and its FEM Model



(a). Resistivity Image

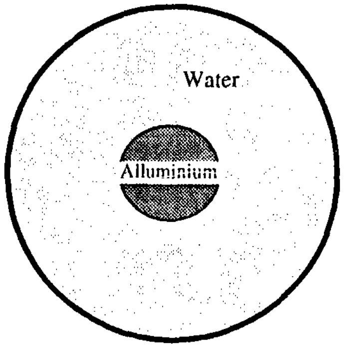


(b). Relative Permittivity Image
Fig.3. Reconstruction Result 1

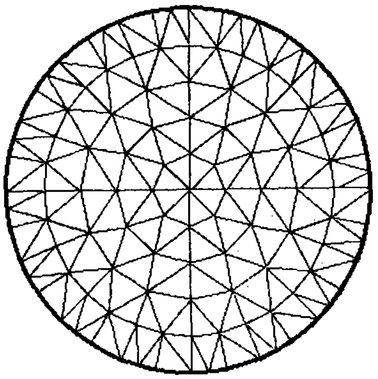
To reconstruct the internal resistivity and permittivity from the real object, the object test and the electrodes are arranged to compatible with the Finite Element model. The object would be tested, is made from cylinder tank with 210 [mm] diameter which is filled with water. 16 of current electrodes with 16 [mm] width and 16 of voltage eletrodes with 0.5 [mm] width are placed at the periphery of object.

To collect the data for reconstruction algorithm, the data collection system is assembled as block diagram in fig.[1]. The currents with amplitude 1.0 [mA] and 10 [kHz] frequency are injected to all of electrodes simultaneously and the voltage responses and its difference phase are measured at voltage electrodes with one of them is set as reference (ground). We used 16 pairs of electrode at the boundary, thus we obtained 256 number of measurement data.

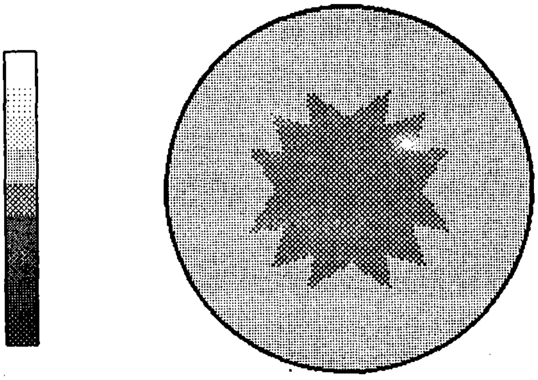
We placed alluminium with 70 [mm] diameter as the target inside the cylinder tank, and we used the Finite Element model with 184 element pixels. Fig[4] shows the object test and its finite element model, and Fig.[5] shows its reconstruction result.



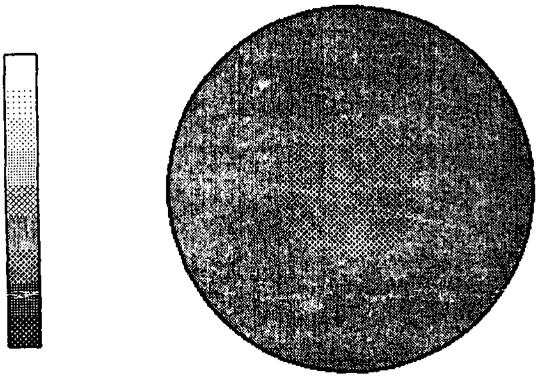
(a) Object Test



(b). Finite Element Model
Fig. 4. Object Test and Its FEM Model



(a). Resistivity Image



(b). Relative Permittivity Image
Fig. 5. Reconstruction Result 2

From the result, the mean reconstructed resistivity for the medium was approximately 23.70 [Ω.cm] and for the alluminium target was 18.66 [Ω.cm]. From relative permittivity image, the 'reactivity' for the medium was -12.96 [Ω.cm] and the target was -11.70 [Ω.cm]. From both of results, although the difference value of medium and target in the reactivity result is smaller than the resistivity result, the alluminium target can be identified in the reconstruction.

6. Conclusion

This paper presented the development of the ECT for resistivity and relative permittivity imaging system. We have developed the Finite Element model of object test in complex field of resistive and capacitive medium. The measured voltage and its difference phase with current source are applied for the reconstruction process. We have shown that the relative permittivity could, in principle, also be reconstructed by means of reconstructing the imaginary component of complex impedance.

7. References

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